ColOpt Winter school
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Many-body physics in Quantum Nonlinear Optics

Francesco Piazza
(Max-Planck Institute for the Physics of Complex Systems)

Nonlinear Optics

Observation:
Photons do not interact

Nonlinear Optics need a medium
which mediates interactions

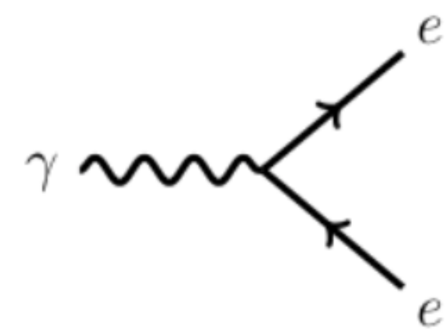
First thing to study:

Interaction between light and matter

Coupling between light and matter

@ fundamental level: QED

Electron-photon scattering in vacuum



$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \simeq \frac{1}{137}$$

Coupling is weak

Set by the fine-structure constant

QUESTION

Is the light-matter coupling always weak?

Strong coupling between light and matter

One solution:

- Use finite-density medium (to catch a photon)
- Use strong light fields (to affect the medium)

Lightning strike is a plasma channel. Source: Wikipedia

Example 1: Plasma

Ionised matter strongly coupled with electromagnetic field.

Very complex many-body light-matter system

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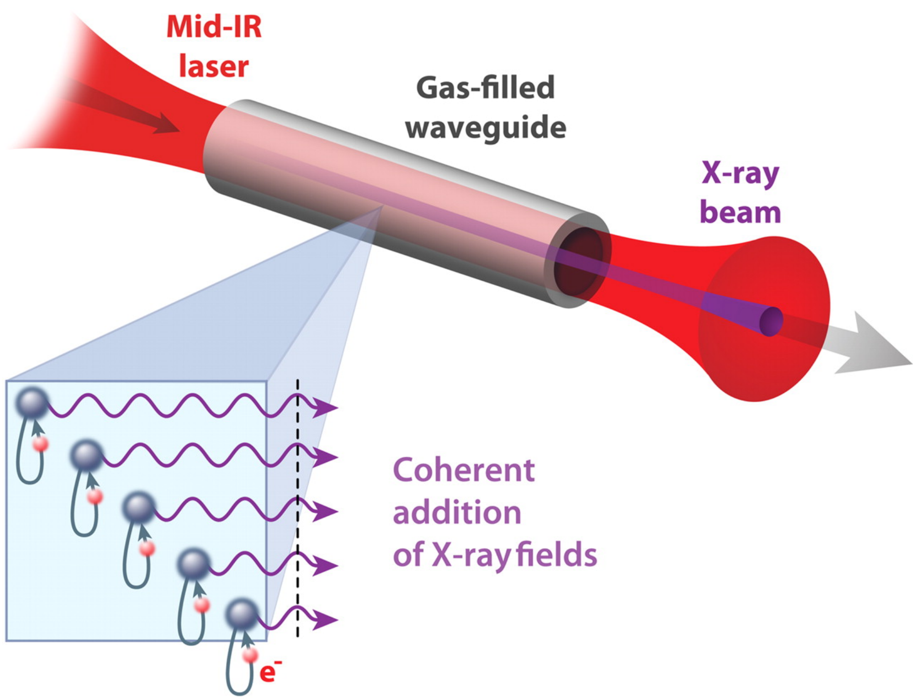


High-harmonic generation with femtosecond pulses on gases [PNAS]

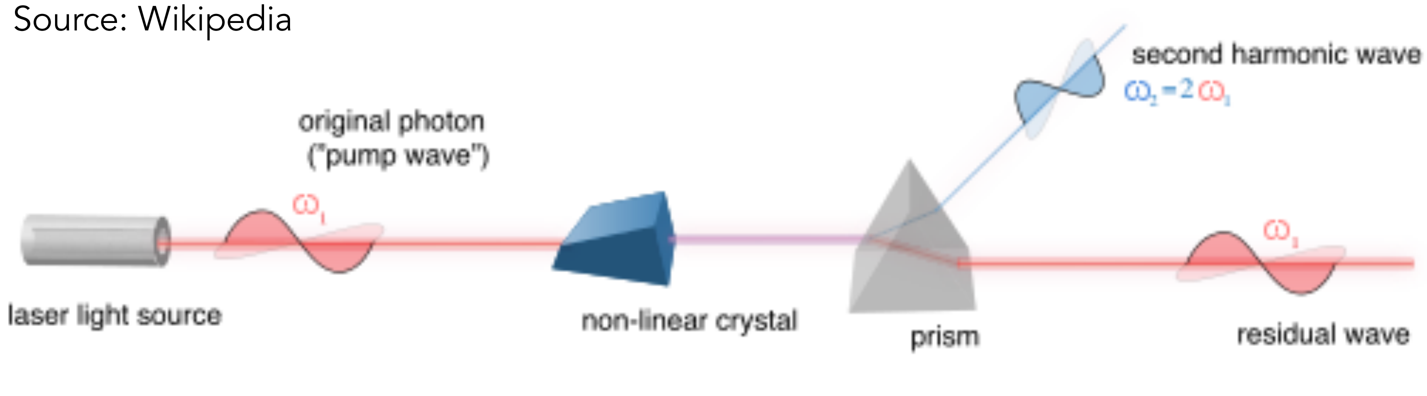
Example 2: Strong laser pulses

Nobel Prize 2018:
Gérard Mourou and Donna Strickland for their method of generating high-intensity, ultra-short optical pulses.

Nonlinear optics

Example: second harmonic generation

Source: Wikipedia



Fundamental principle:

a strong enough beam
modifies medium (polarisability)
which in turn modifies beam propagation

Equivalent picture:

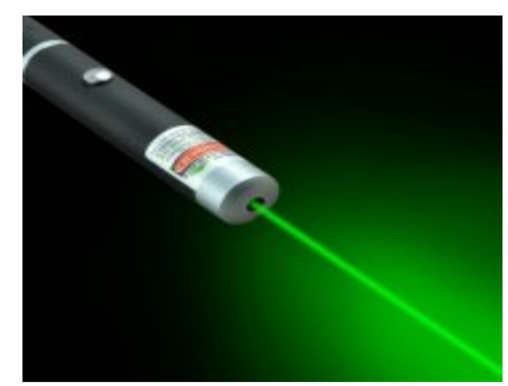
The medium mediates
interactions between photons

Classical version: nonlinear polarisability in Maxwell's equations

$$P_i = \epsilon_0 \sum_j \chi_{ij}^{(1)} E_j + \sum_{jk} \chi_{ijk}^{(2)} E_j E_k + \ldots$$

Technology:

Photon interactions are useful for **signal processing** in optics
(optical modulation/switching, frequency conversion, …)
So far in the **classical regime**!



Strong coupling between light and matter

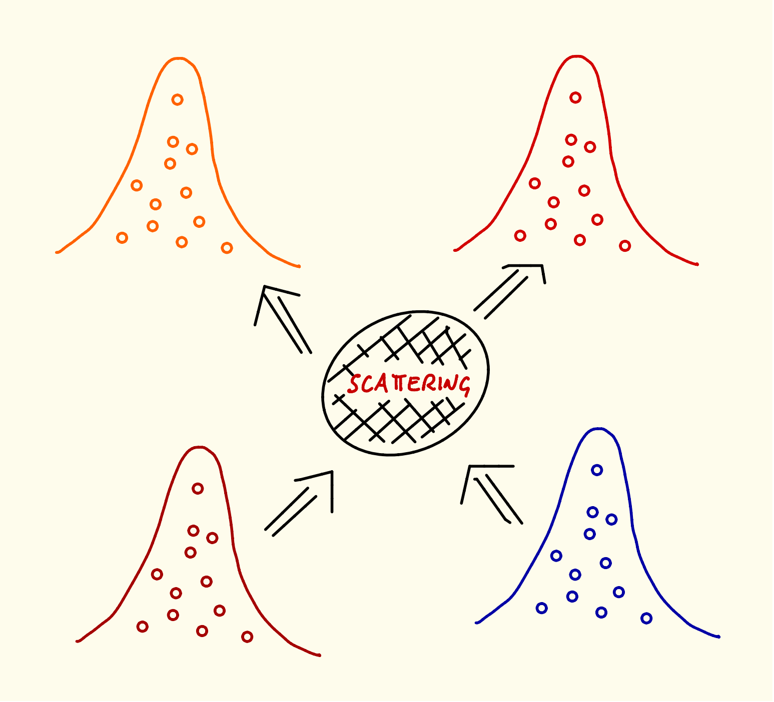
- Use finite-density medium (to catch a photon)
- Use strong light fields (to affect the medium)

Nonlinear Optics tends to be classical

- Light is classical at high intensities

Interactions between coherent "lumps" of photons

Photon-lump scattering



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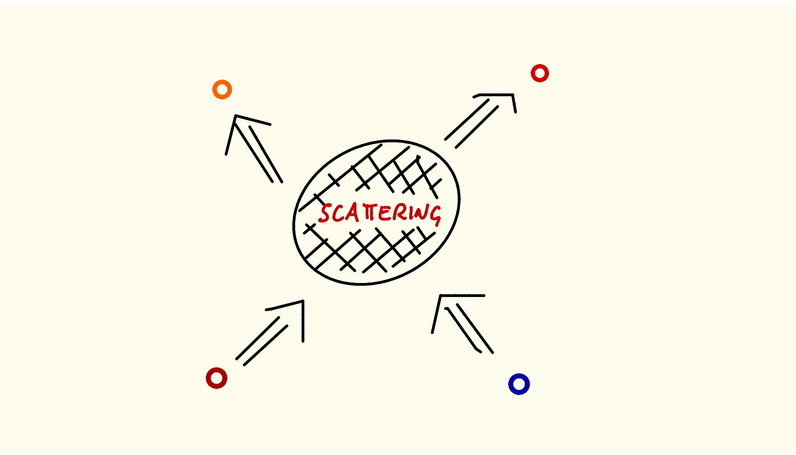
Interactions between coherent "lumps" of photons

Can quantum mechanics be important?

- Interactions need to take place between single photons

Photon-by-photon scattering?



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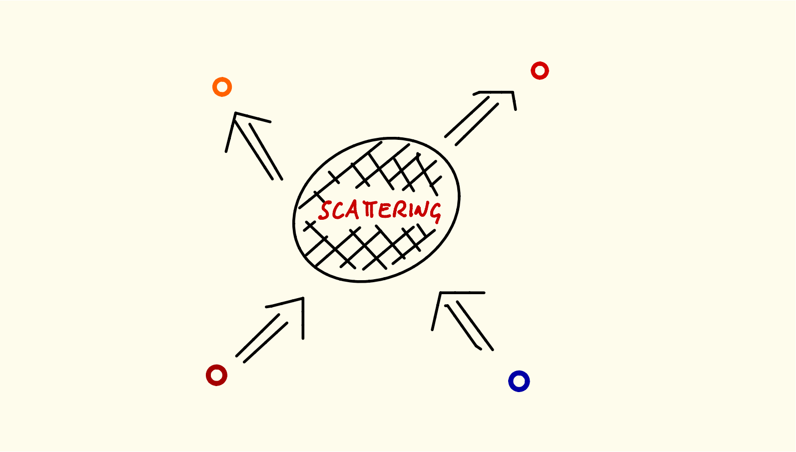


Is Quantum Nonlinear Optics interesting?

Quantum nonlinear optics: technological promise

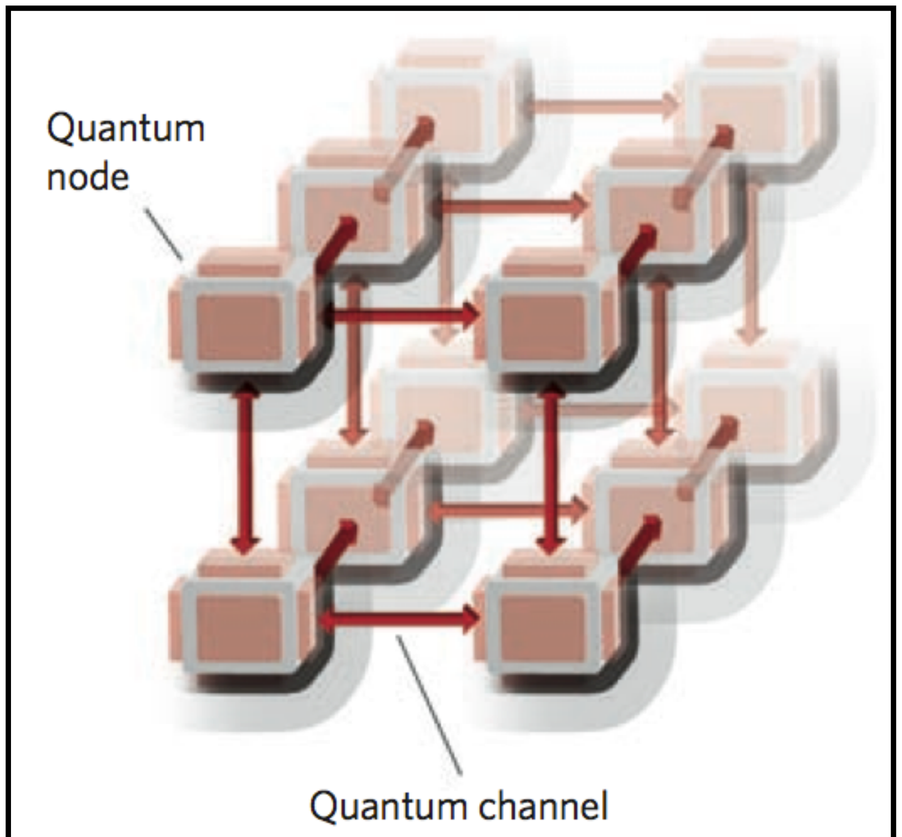
PROSPECT:

Quantum information processing and communication

- Photons are optimal quantum information carriers
- Photon-interactions could allow for quantum information processing

The Quantum Internet

Kimble, et al., Nature 453, 1023 (2008).



- Photons transport information across channels
- Photon-interactions at the nodes process information
- Interactions rely on nonlinear optics in the quantum regime

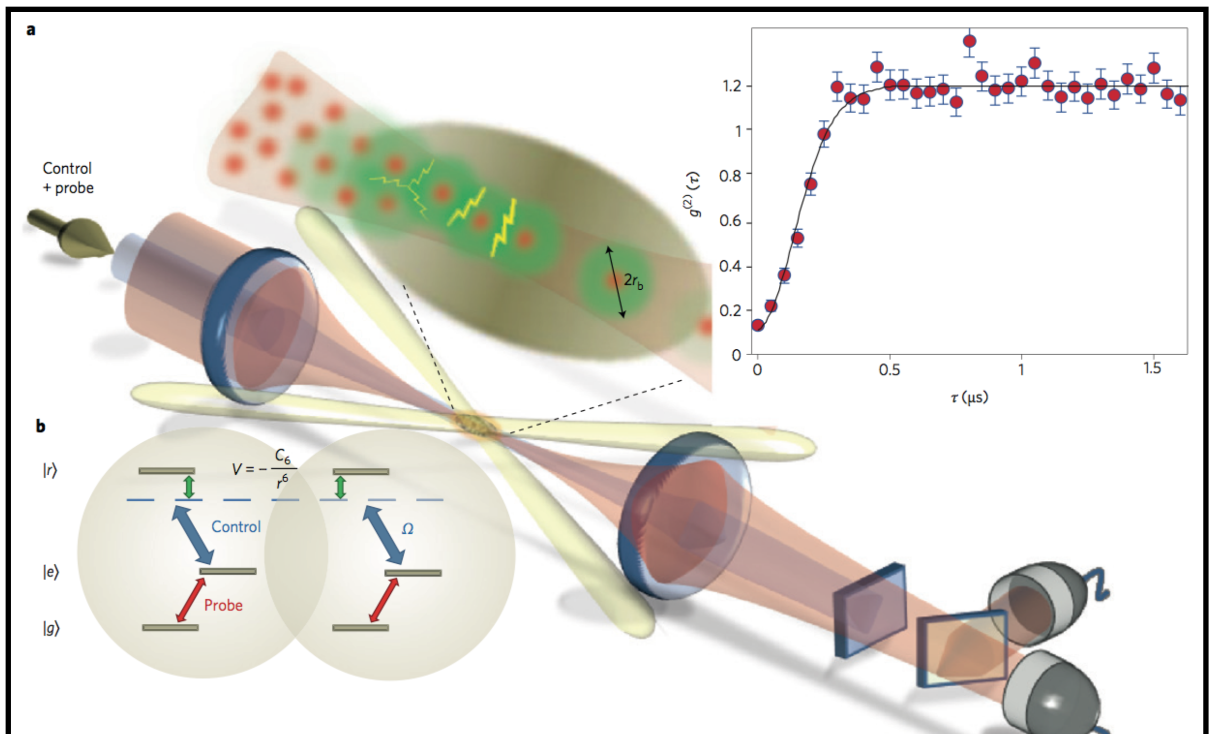
Quantum nonlinear optics: fundamental interest

PROSPECT:

Novel many-body phenomena in quantum plasmas

- Material's degree of freedom strongly interacting with light at the level of single quanta
- Take a finite excitation density of both light and matter where collective phenomena appear

Example: Quantum atom-photon "plasma"



Peyronel, et al., Nature 488, 57 (2012).

Example: single Rydberg atoms interacting with single photons

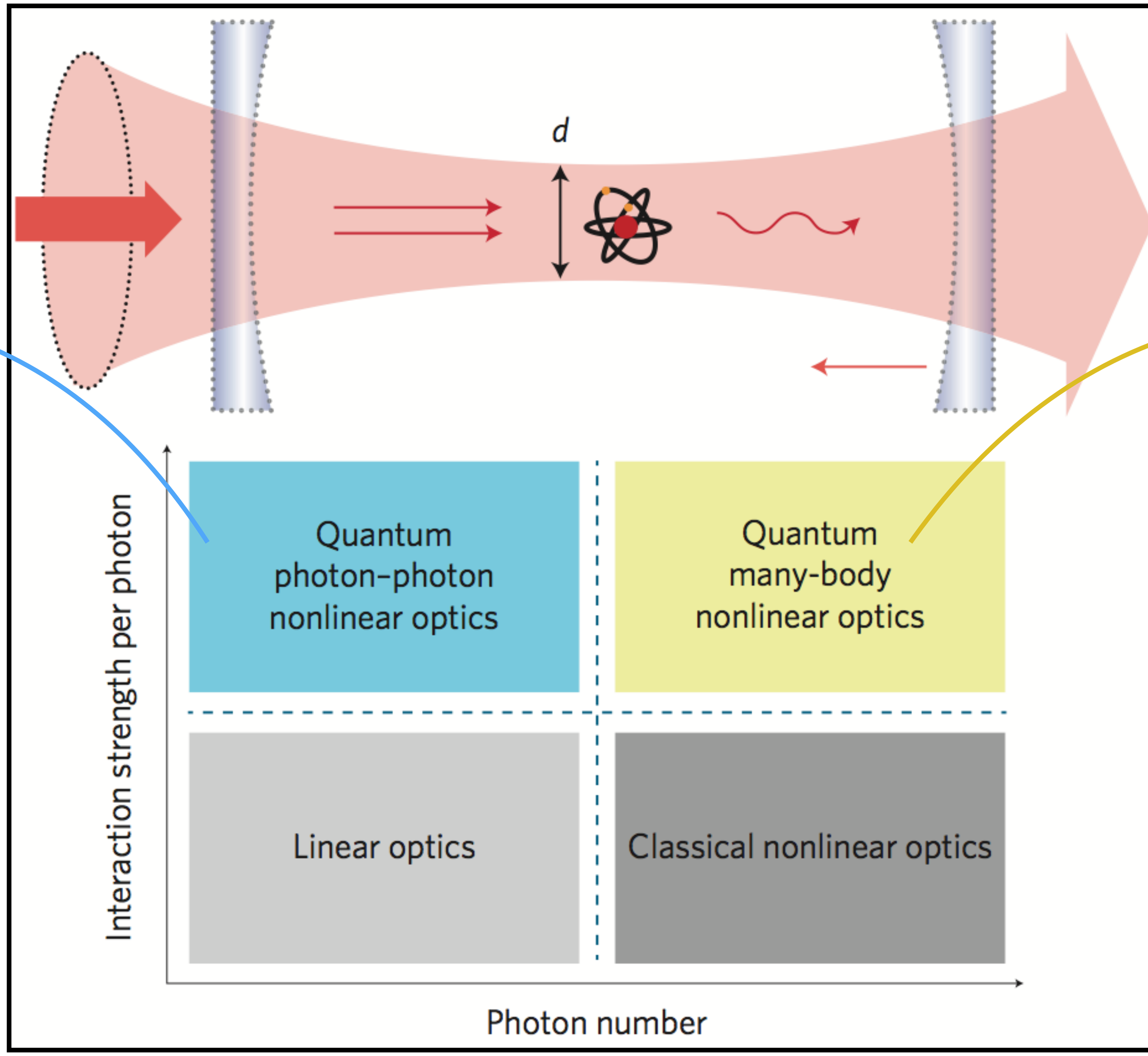
Peculiar features:

- Not in thermal equilibrium (driven-dissipative)
- Exotic interactions (retardation, sign-change, long-range)

Quantum nonlinear optics: overview

Review: Chang, Vuletic, Lukin, Nat. Phot. 8, 685 (2014).

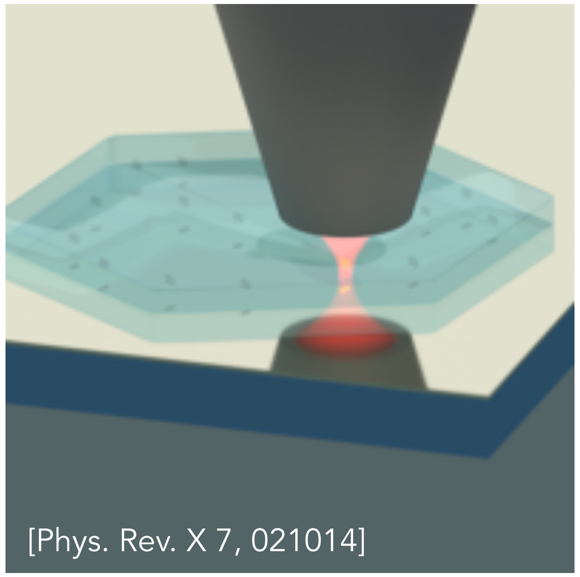
Technological motivation:
Quantum Internet



Fundamental motivation:
Many-body phenomena In quantum plasmas

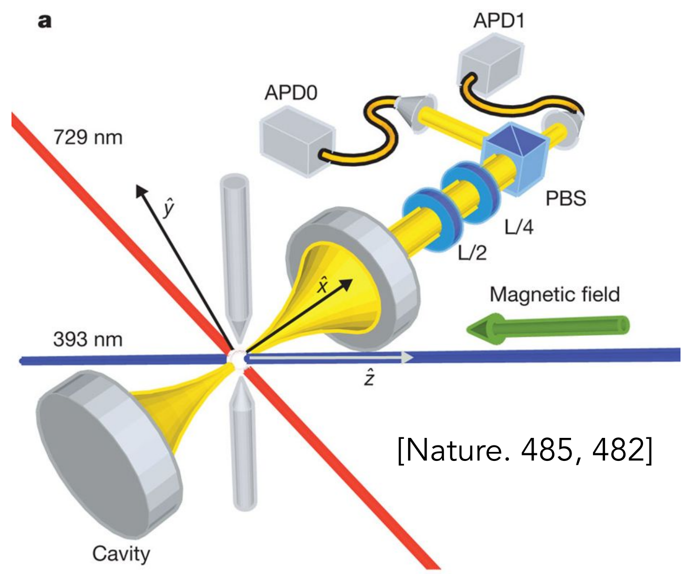
Quantum nonlinear optics: overview of materials

Molecules



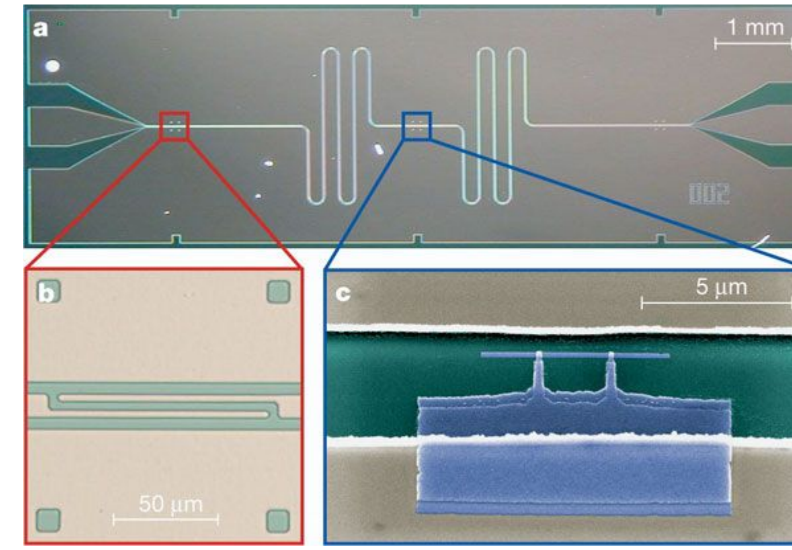
[Phys. Rev. X 7, 021014]

Ions



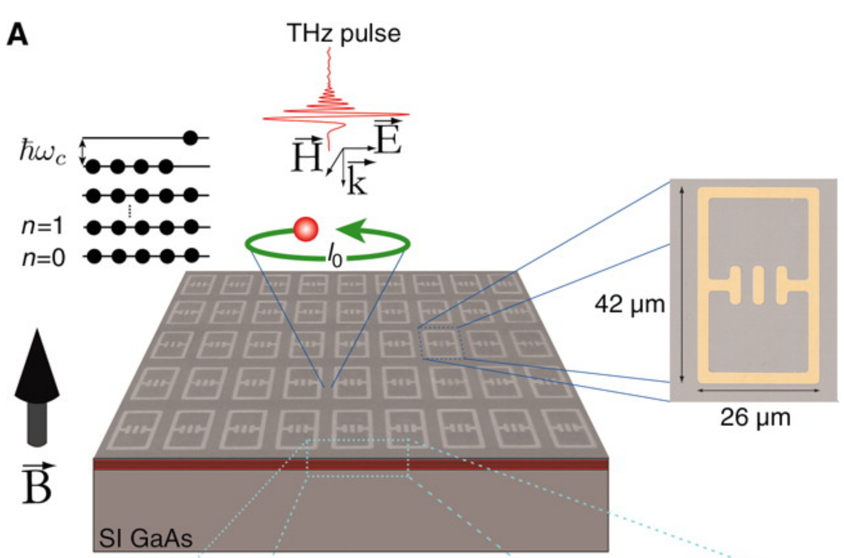
[Nature. 485, 482]

Superconducting circuits



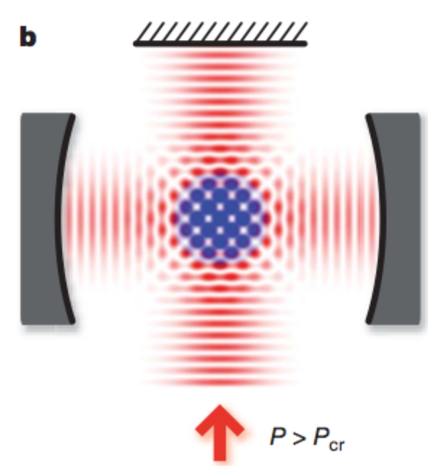
[Nature 431, 162]

2D electron gases

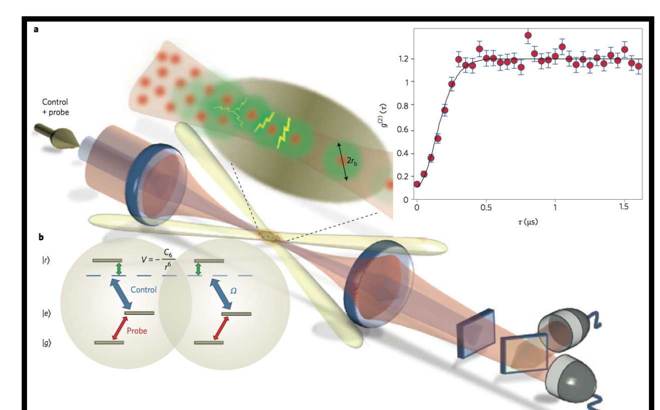


[Science 335, 1323]

Neutral atoms



[Nature 464, 1301]



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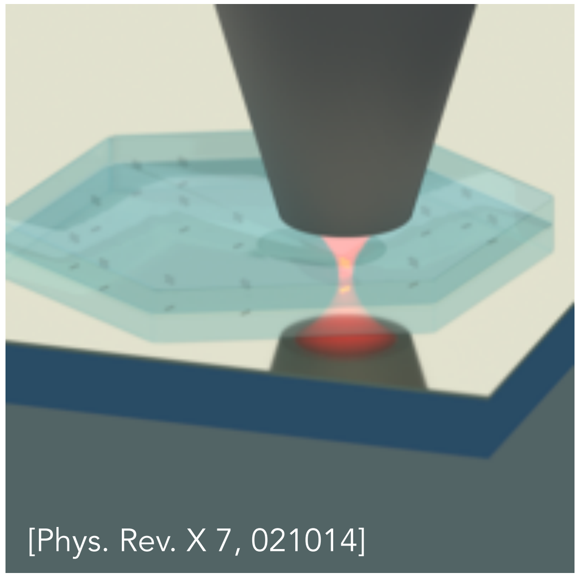

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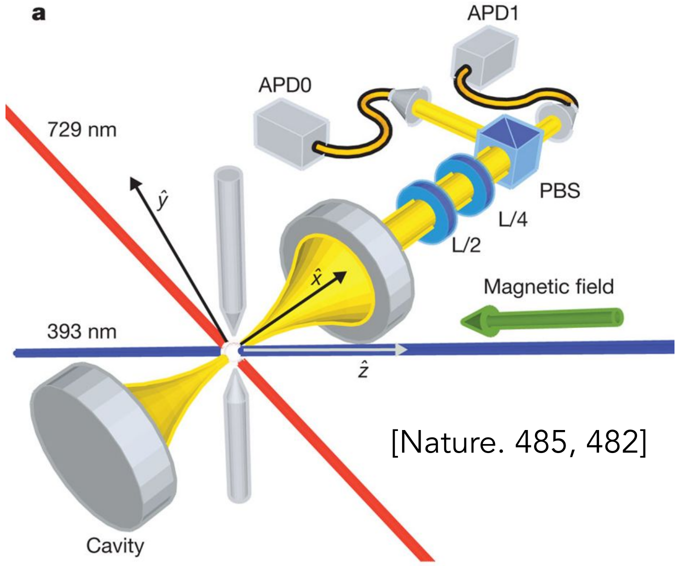

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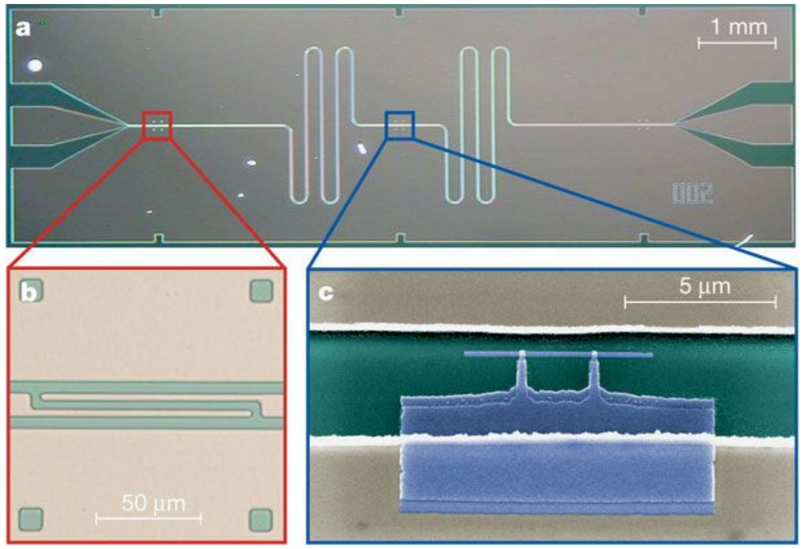
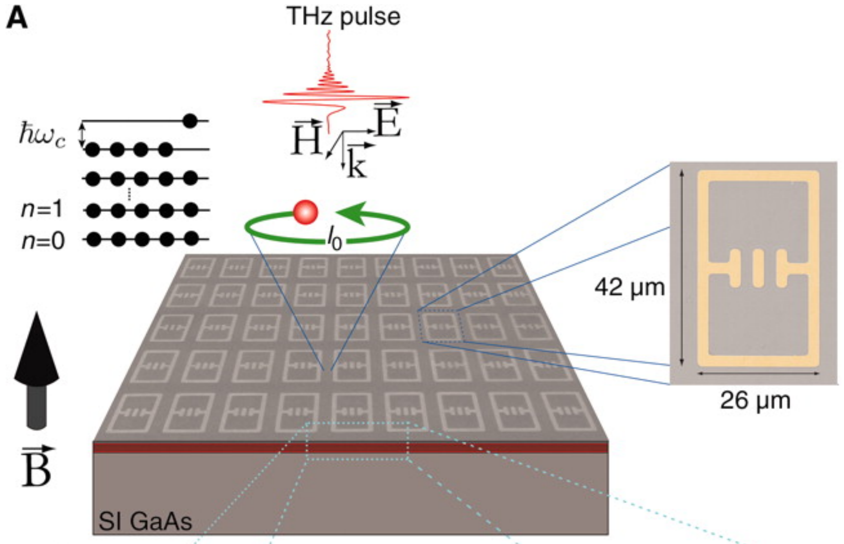


[Nature 431, 162]

QUANTUM DEGENERATE MATTER

2D electron gases

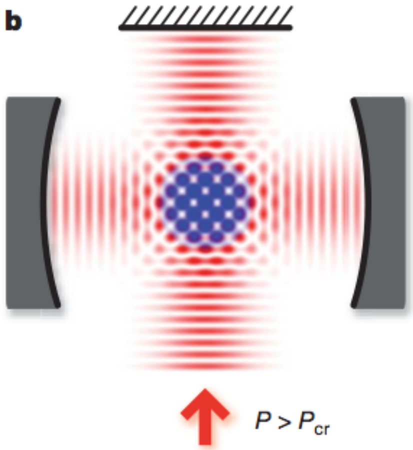
(strongly correlated fermions)



[Science 335, 1323]

Neutral atoms

(ultracold bosons/fermions)



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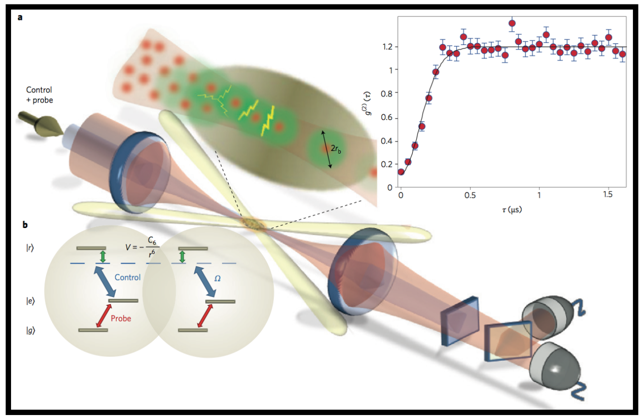

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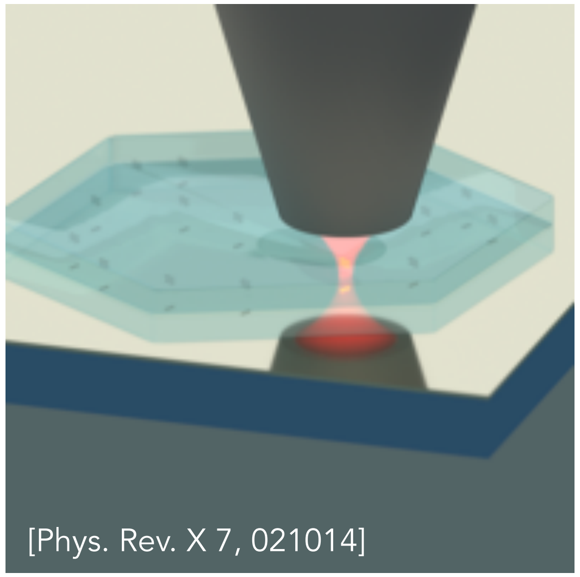

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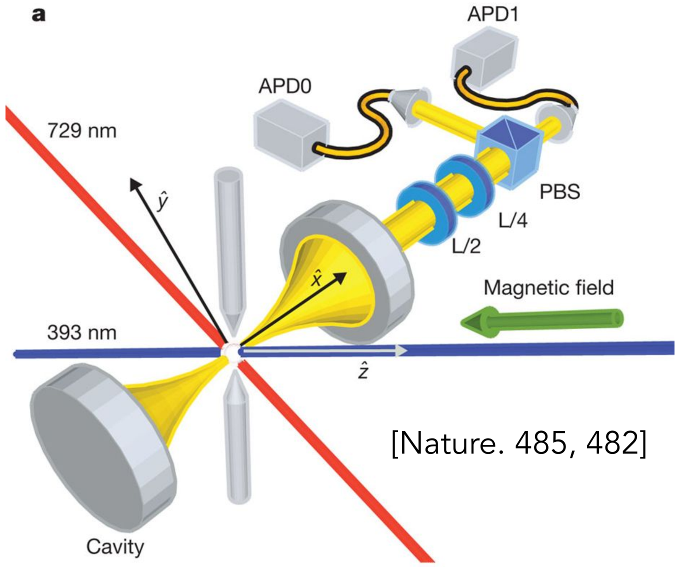

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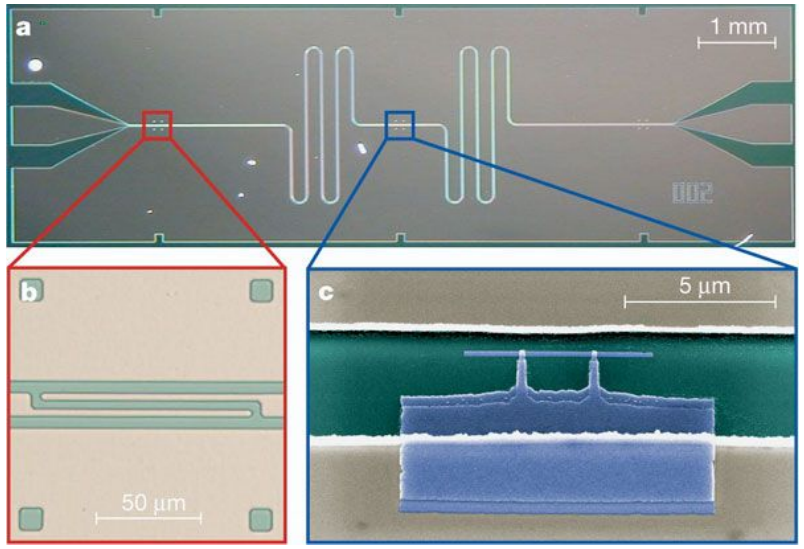

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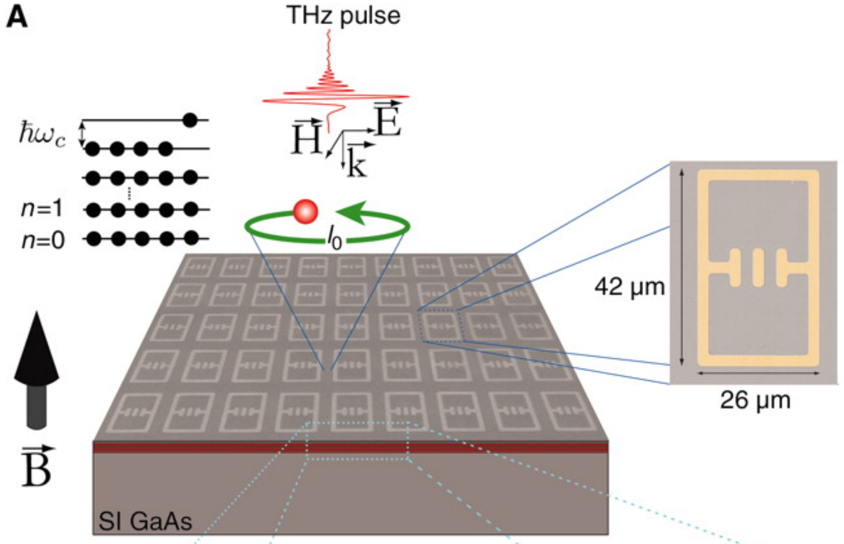

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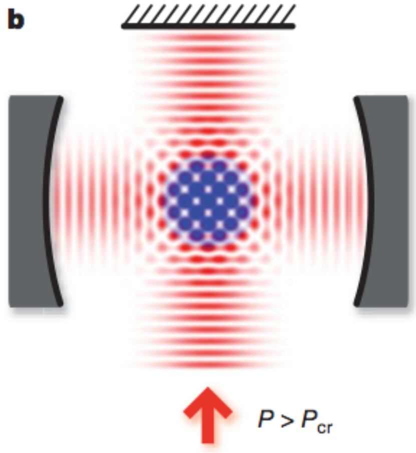
THIS TALK

Neutral atoms

(ultracold bosons/fermions)



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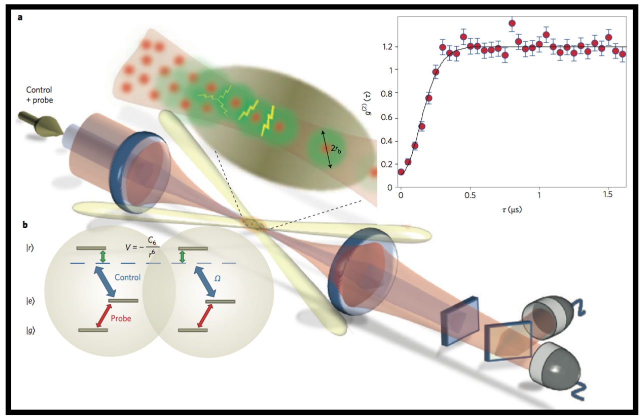


[Nature 488, 57]

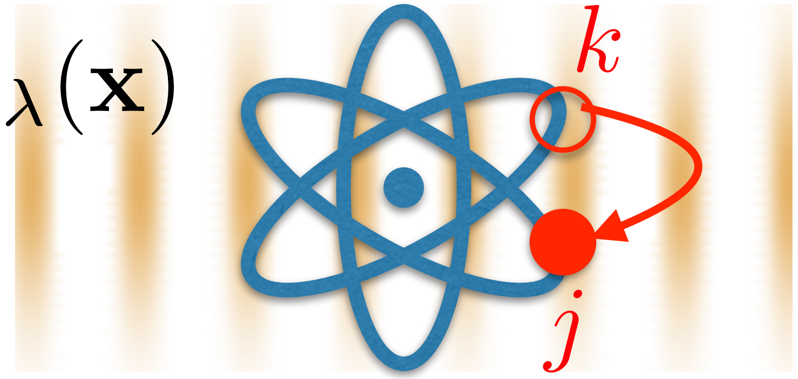
Outline

1. Interaction between atoms and photons
2. Implementing Quantum nonlinear optics
3. Quantum nonlinear optics with quantum degenerate matter
4. Many-body physics with quantum atom-photon plasmas

1. Interaction between atoms and photons

Interaction of electromagnetic fields with (artificial) atoms

$\mathbf{u}_\lambda(\mathbf{x})$



EM field interacts with the **electrons in the atom**

Minimal coupling: $\frac{1}{2m}(\mathbf{p}-e\mathbf{A})^2$

Hamiltonian: $\hat{H}=\hat{H}_{\mathrm{el}}+\hat{H}_{\mathrm{int}}+\hat{H}_{\mathrm{field}}$

$$\hat{H}_{\mathrm{el}}=\int\hat{\psi}^\dagger(\mathbf{x})\Big[-\frac{\nabla^2}{2m}+eV(\mathbf{x})\Big]\hat{\psi}(\mathbf{x})d\mathbf{x}=\sum_j\epsilon_j\hat{c}_j^\dagger\hat{c}_j$$

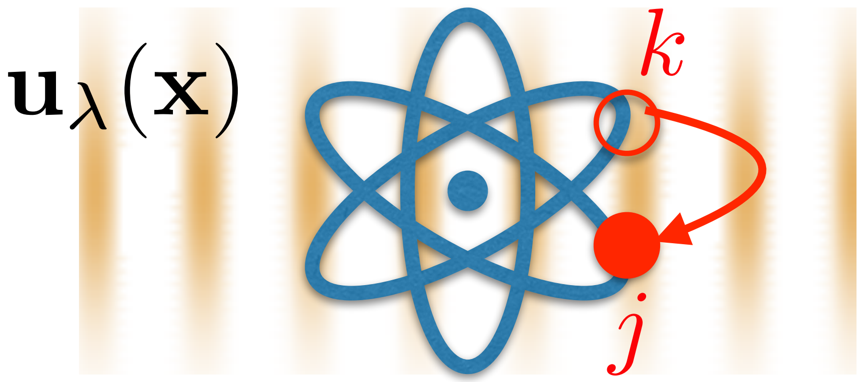
$$\hat{H}_{\mathrm{int}}\simeq\int\hat{\psi}^\dagger(\mathbf{x})\Big[-\frac{e}{m}\mathbf{A}\cdot\mathbf{p}\Big]\hat{\psi}(\mathbf{x})d\mathbf{x}=\sum_{j,k,\lambda}\hat{c}_j^\dagger\hat{c}_k\big(g_{\lambda jk}\hat{a}_\lambda+\mathrm{h.c.}\big)$$

Neglected A^2 term

$$\hat{\psi}(\mathbf{x})=\sum_j\phi_j(\mathbf{x})\hat{c}_j$$ Electronic states in the atom (without EM field)

$$g_{\lambda jk}=-\frac{e}{m}\sqrt{\frac{1}{2\omega_\lambda\varepsilon_0}}\int\phi_j^*(\mathbf{x})[\mathbf{u}_\lambda(\mathbf{x})\cdot\mathbf{p}]\phi_k(\mathbf{x})d\mathbf{x}$$ Coupling strength

Interaction of electromagnetic fields with (artificial) atoms



$$g_{\lambda jk} = -\frac{e}{m}\sqrt{\frac{1}{2\omega_\lambda \varepsilon_0}} \int \phi_j^*(\mathbf{x})[\mathbf{u}_\lambda(\mathbf{x}) \cdot \mathbf{p}]\phi_k(\mathbf{x})d\mathbf{x}$$

Dipole approximation: $\mathbf{u}_\lambda(\mathbf{x}) \simeq \mathbf{u}_\lambda(\mathbf{x_0})$

(Photon wavelength cannot resolve electron wave function)

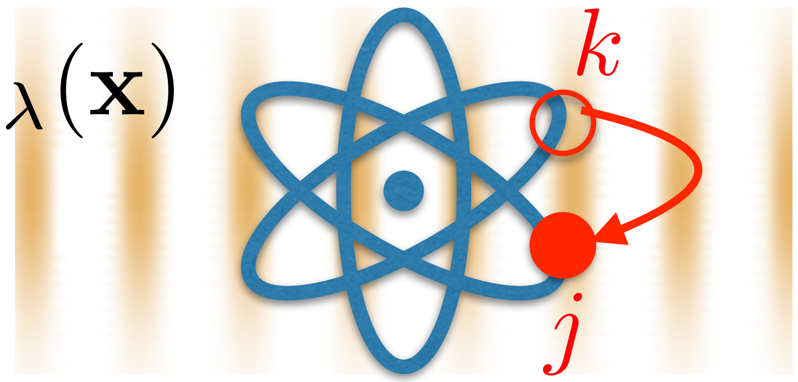
$$g_{\lambda jk} \simeq -i\sqrt{\frac{1}{2\omega_\lambda \varepsilon_0}}(\epsilon_j - \epsilon_k)\mathbf{u}_\lambda(\mathbf{x}_0) \cdot \mathbf{d}_{jk}$$

[Exercise: show it]

Transition dipole moment: $\mathbf{d}_{jk} = \int \phi_j^*(\mathbf{x}) e\mathbf{x} \phi_k(\mathbf{x}) d\mathbf{x}$

Interaction of electromagnetic fields with (artificial) atoms

$\mathbf{u}_\lambda(\mathbf{x})$



$$\hat{H}_{\rm int} = \sum_{j,k,\lambda} \hat{c}_j^\dagger \hat{c}_k \left(g_{\lambda jk} \hat{a}_\lambda + \text{h.c.} \right)$$

Rotating frame:

$$\hat{U} = e^{-i(\hat{H}_{\rm el} + \hat{H}_{\rm field})t}$$

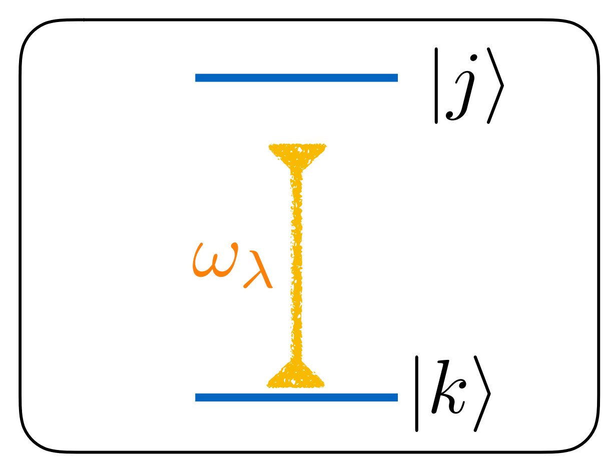
$$\hat{H}_{\rm int} \to \sum_{j,k,\lambda} \hat{c}_j^\dagger \hat{c}_k e^{i(\epsilon_j - \epsilon_k)t} \left(g_{\lambda jk} \hat{a}_\lambda e^{-i\omega_\lambda t} + \text{h.c.} \right)$$

Rotating wave approximation:

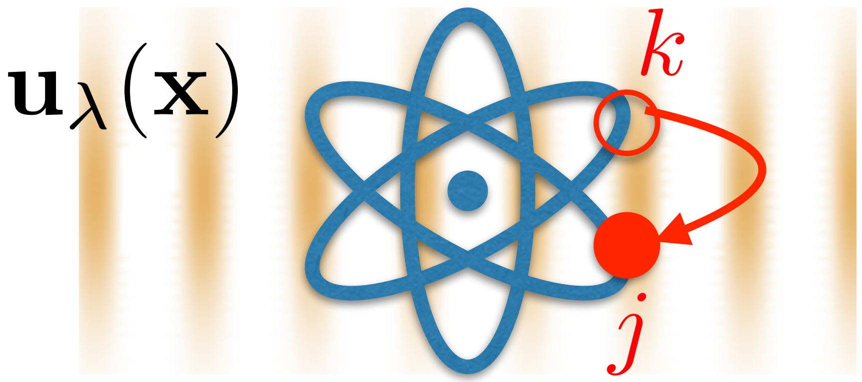
$$\hat{H}_{\rm int} \simeq \sum_{\epsilon_j > \epsilon_k, \lambda} \hat{c}_j^\dagger \hat{c}_k g_{\lambda jk} \hat{a}_\lambda e^{-i(\omega_\lambda - \epsilon_j + \epsilon_k)t} + \text{h.c.}$$

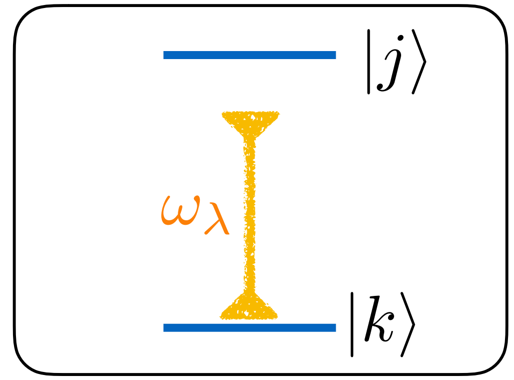
Neglect non-energy-conserving processes: oscillating fast (valid close enough to resonance)

$$e^{-i(\omega_\lambda + \epsilon_j - \epsilon_k)t}$$



Quantifying the strength of light-matter coupling





Materials degrees of freedom modelled a set of transitions (e.g. electron orbitals in atom)

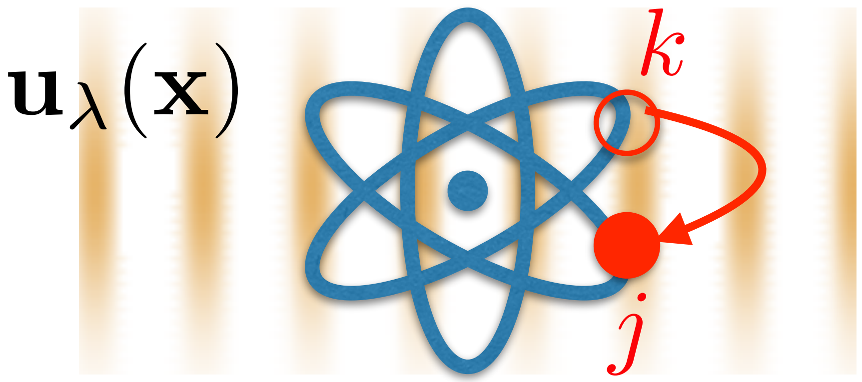
Susceptibility

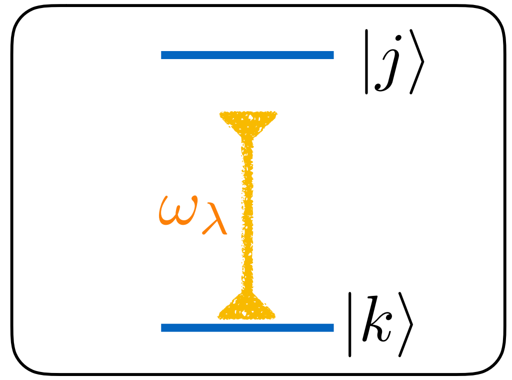
Measures the linear response of the material to a photon

$$\chi(\omega_\lambda) = \sum_{jk} \frac{(n_k - n_j)\left|\langle k|V_\lambda^{\mathrm{pert}}|j\rangle\right|^2}{\omega_\lambda - \epsilon_j + \epsilon_k + i\gamma_{jk}}$$

- Must vanish for equally populated states $n_j = n_k$
- Must have a maximum on resonance $\omega_\lambda = \epsilon_j - \epsilon_k$
- Must contain the interaction matrix-element $\langle k|V_\lambda^{\mathrm{pert}}|j\rangle = g_{\lambda kj} = -\frac{e}{m}\sqrt{\frac{1}{2\omega_\lambda\varepsilon_0}}\int \phi_k^*(\mathbf{x})[\mathbf{u}_\lambda(\mathbf{x})\cdot\mathbf{p}]\phi_j(\mathbf{x})d\mathbf{x}$
- Width of the transition set by γ_{jk}

Quantifying the strength of light-matter coupling





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Measures the linear response of the material to a photon

Dimension of a frequency

$$\chi(\omega_\lambda) = \sum_{jk} \frac{(n_k - n_j)\left|\langle k|V_\lambda^{\text{pert}}|j\rangle\right|^2}{\omega_\lambda - \epsilon_j + \epsilon_k + i\gamma_{jk}}$$

Cooperativity

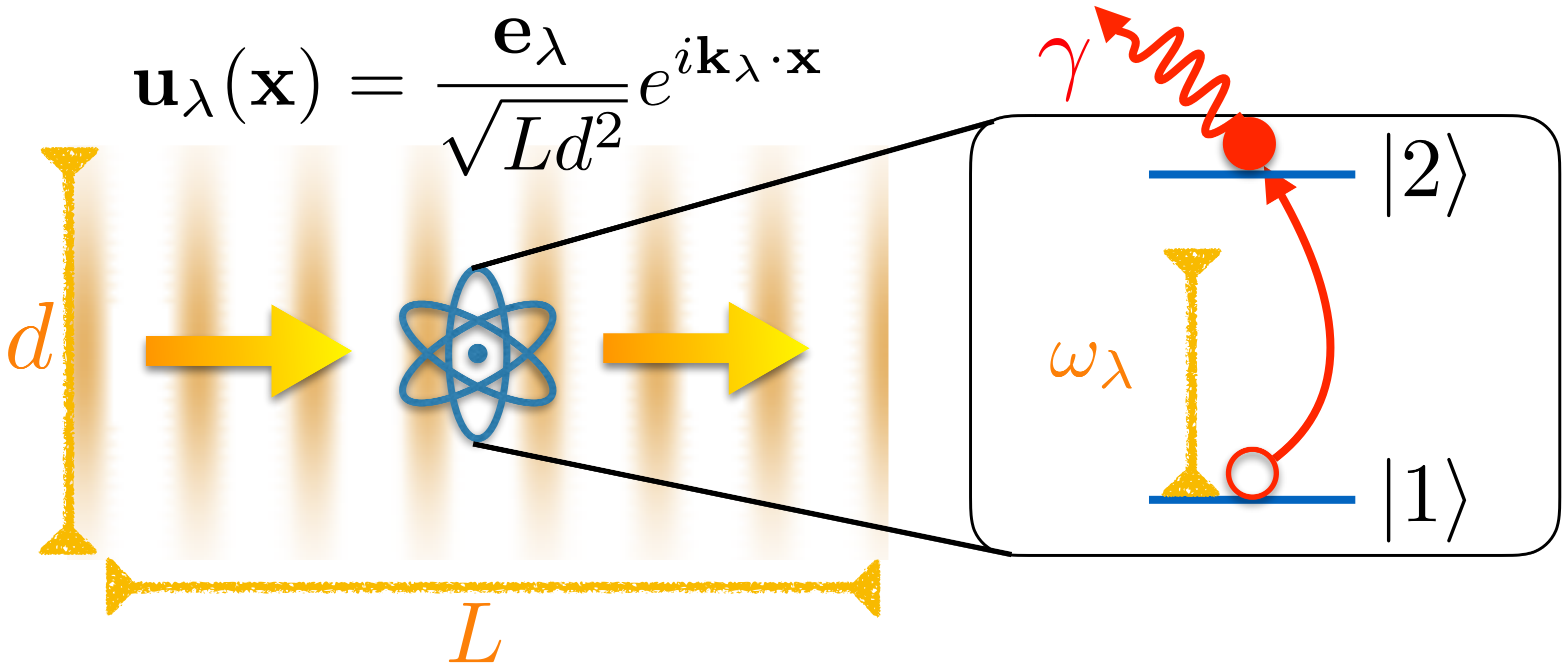
Dimensionless measure of Light-matter coupling

Multiply susceptibility by characteristic interaction time

$$C(\omega_\lambda) = \chi(\omega_\lambda)\tau_{\text{int}}$$

Quantifying the strength of light-matter coupling: atom in free space

$$\mathbf{u}_\lambda(\mathbf{x}) = \frac{\mathbf{e}_\lambda}{\sqrt{Ld^2}} e^{i\mathbf{k}_\lambda \cdot \mathbf{x}}$$



$$g_\lambda \simeq \sqrt{\frac{1}{2\omega_\lambda \varepsilon_0 L d^2}} \omega_{12} \tilde{d}_{12}$$

$$\tau_{\text{int}} = \frac{L}{c}$$

Susceptibility

Measures the linear response of the material to a photon

Dimension of a frequency

$$\chi(\omega_\lambda) = \sum_{jk} \frac{(n_k - n_j)\left|\langle k|V_\lambda^{\text{pert}}|j\rangle\right|^2}{\omega_\lambda - \epsilon_j + \epsilon_k + i\gamma_{jk}}$$

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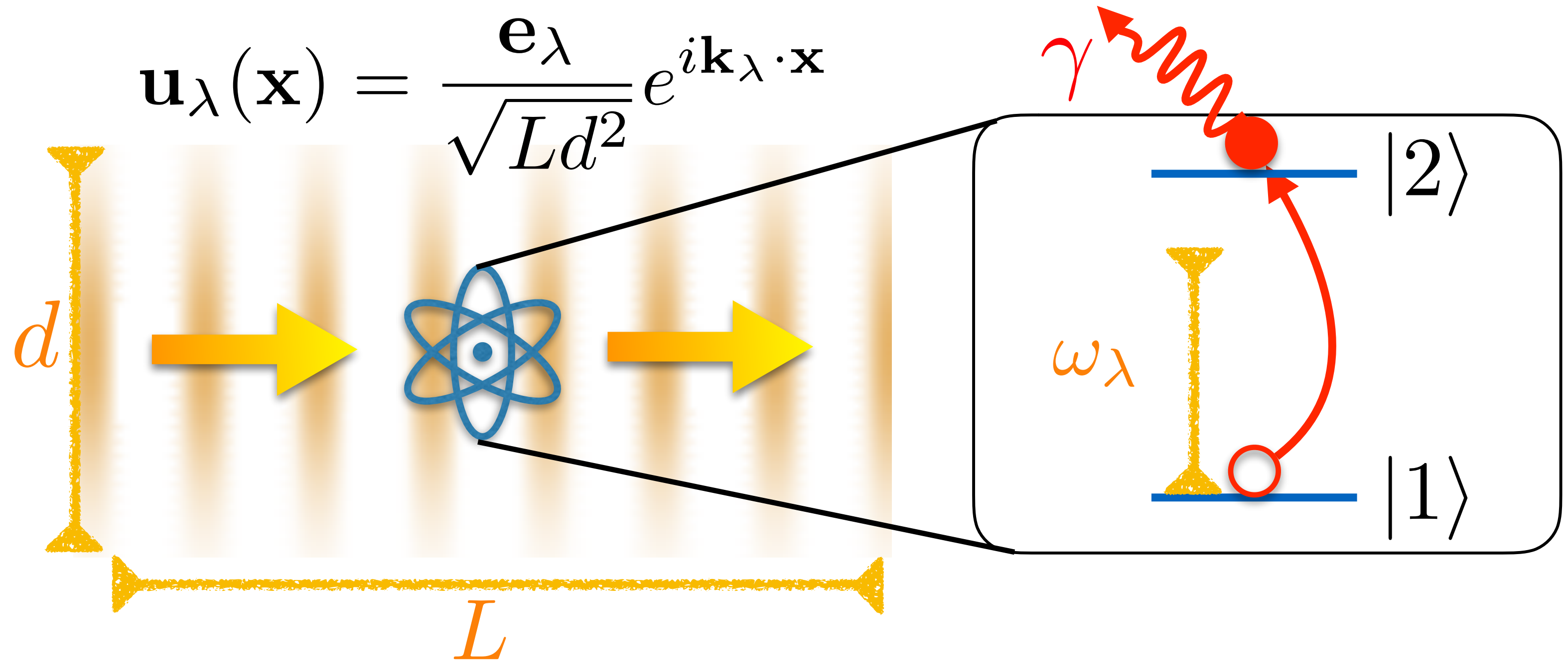

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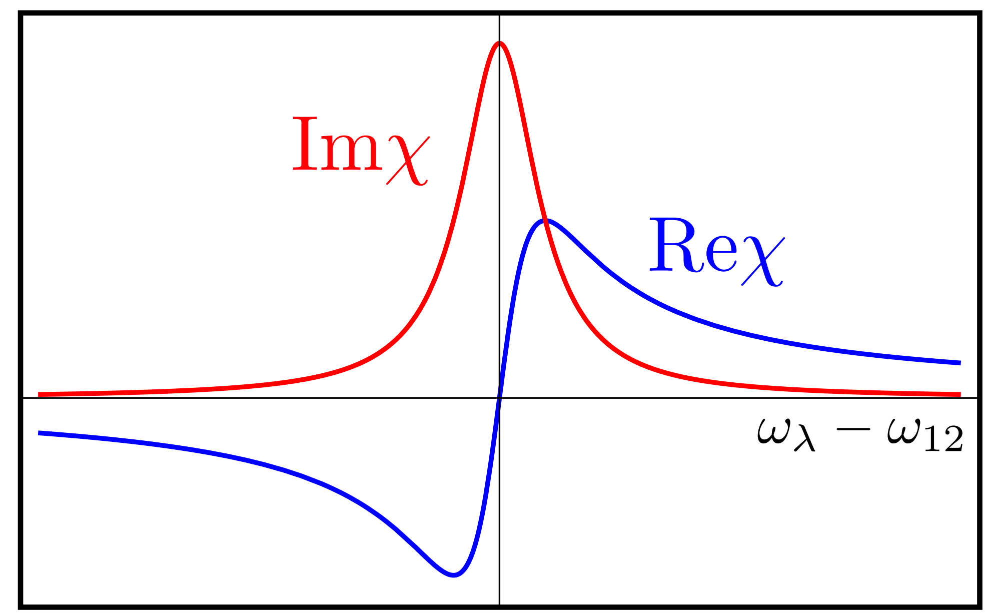

Susceptibility

Real part: coherent interaction (photon dispersion)

Imag. part: incoherent interaction (photon absorption)

$$\chi(\omega_\lambda) = \frac{g_\lambda^2}{\omega_\lambda - \omega_{12} + i\gamma}$$

Cooperativity

$$C(\omega_\lambda) = \frac{g_\lambda^2 L/c}{\omega_\lambda - \omega_{12} + i\gamma}$$

Quantifying the strength of light-matter coupling: atom in free space

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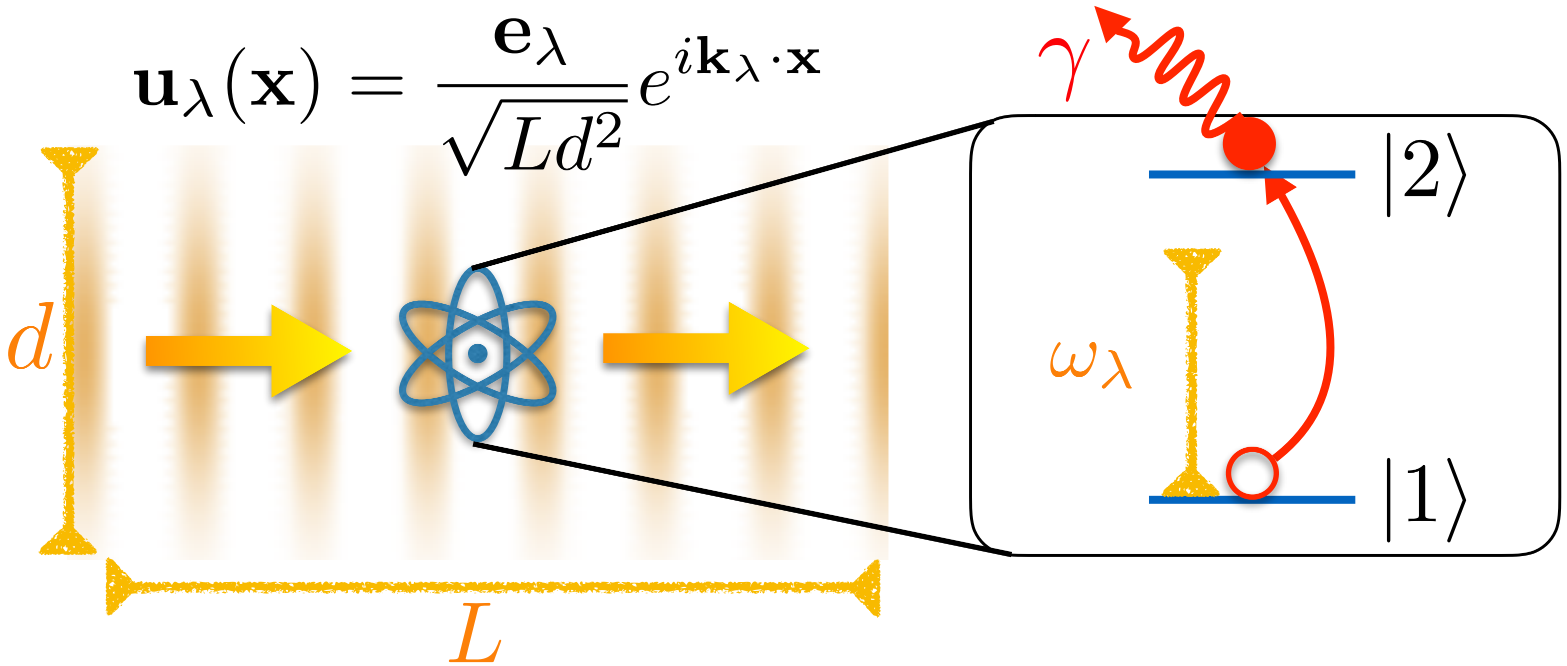


$$g_\lambda \simeq \sqrt{\frac{1}{2\omega_\lambda \varepsilon_0 L d^2}} \omega_{12} \tilde{d}_{12}$$

$$\tau_{\text{int}} = \frac{L}{c}$$

Estimate the cooperativity

(consider resonant absorption for simplicity)

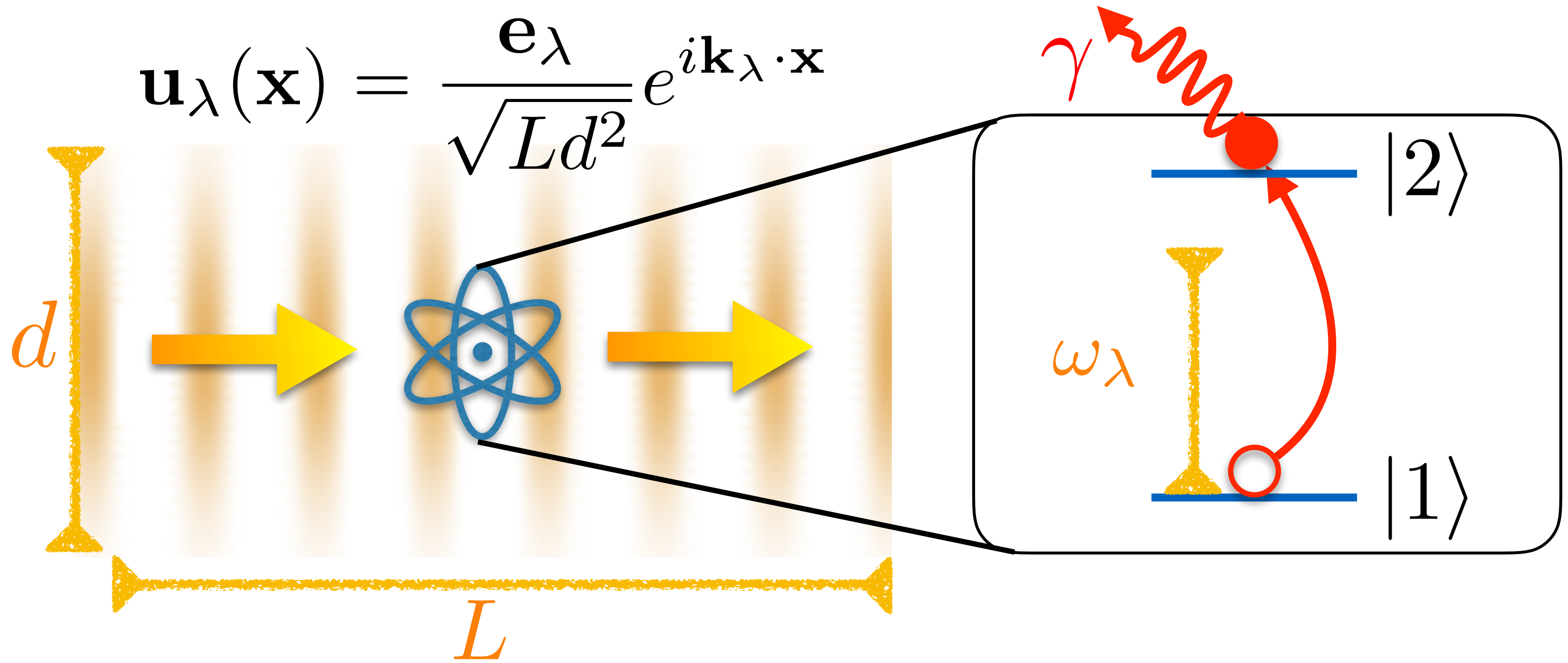
$$\text{Im}C_{\text{res}} = \frac{g_{\text{res}}^2 \frac{L}{c}}{\gamma} = \frac{g_{\text{res}}^2 \frac{L}{c}}{g_{\text{res}}^2 \frac{\omega_{12}^2}{c^3} \frac{d^2 L}{(2\pi)^2}} = \frac{\lambda_{12}^2}{d^2}$$

Exercise: show that

$$\gamma = g_{\text{res}}^2 \frac{\omega_{12}^2}{c^3} \frac{d^2 L}{(2\pi)^2}$$

Hint: compute the incoherent susceptibility
for an atom in a volume=d^2*L
filled with a continuum of EM plane-wave modes
(sum over all modes)

$$\mathbf{u}_\lambda(\mathbf{x}) = \frac{\mathbf{e}_\lambda}{\sqrt{Ld^2}} e^{i\mathbf{k}_\lambda \cdot \mathbf{x}}$$



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Estimate the cooperativity

(consider resonant absorption for simplicity)

$$\text{Im}C_{\text{res}} = \frac{g_{\text{res}}^2 \frac{L}{c}}{\gamma} = \frac{g_{\text{res}}^2 \frac{L}{c}}{g_{\text{res}}^2 \frac{\omega_{12}^2}{c^3} \frac{d^2 L}{(2\pi)^2}} = \frac{\lambda_{12}^2}{d^2}$$

Diffraction limit

light cannot be focused below its wavelength

$d \gg \lambda$ (typically)

→

PROBLEM IN FREE SPACE:

$$|C| \ll 1$$

Atom-photon coupling small!

2. Implementing Quantum Nonlinear Optics

Increasing the cooperativity

$$\mathrm{Im}C_{\mathrm{res}} = \frac{\tau_{\mathrm{int}}}{L/c}\frac{\lambda_{12}^2}{d^2}$$

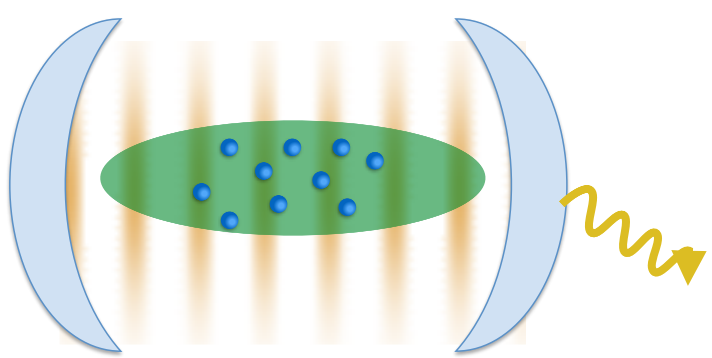
Free space problem:

$$\tau_{\mathrm{int}} = \frac{L}{c}$$

Increase interaction time

Cavities

Multipass enhancement



Atoms in an optical resonator

Increasing the cooperativity

$$\mathrm{Im}C_{\mathrm{res}} = \frac{\tau_{\mathrm{int}}}{L/c}\frac{\lambda_{12}^2}{d^2}$$

Free space problem:

$$\tau_{\mathrm{int}} = \frac{L}{c}$$

Increase interction time

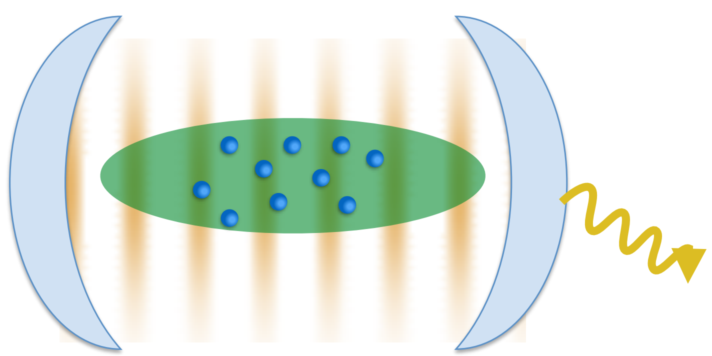
Cavities

Multipass enhancement

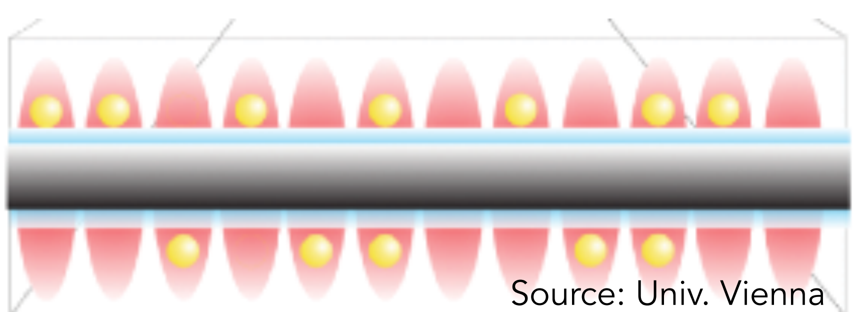
Beat the diffraction limit

Evanescent fields

Confinement @wavelength level



Atoms in an optical resonator



Atoms close to a waveguide

Increasing the cooperativity

$$\mathrm{Im}C_{\mathrm{res}} = N \frac{\tau_{\mathrm{int}}}{L/c} \frac{\lambda_{12}^2}{d^2}$$

Free space problem:

$$\tau_{\mathrm{int}} = \frac{L}{c}$$

Increase interaction time

Cavities

Multipass enhancement

Beat the diffraction limit

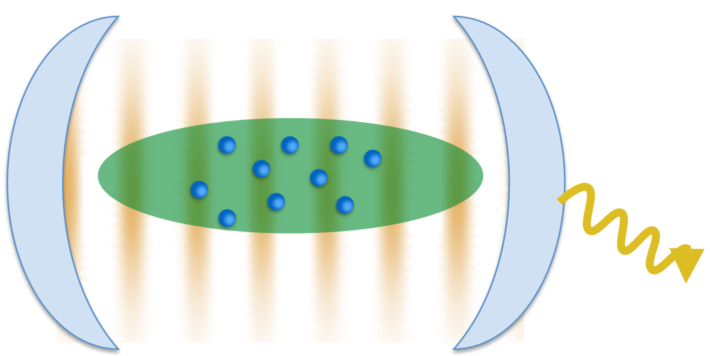
Evanescent fields

Confinement @wavelength level

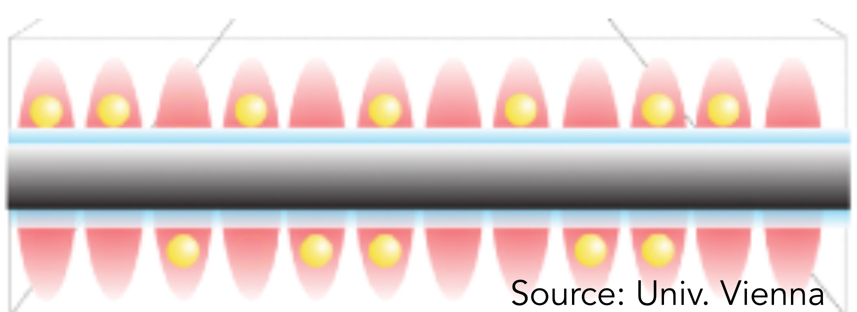
Use collective effects

Strong interatomic interactions

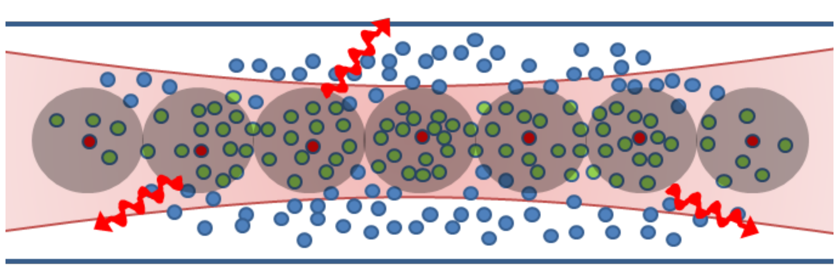
"Superatom"



Atoms in an optical resonator



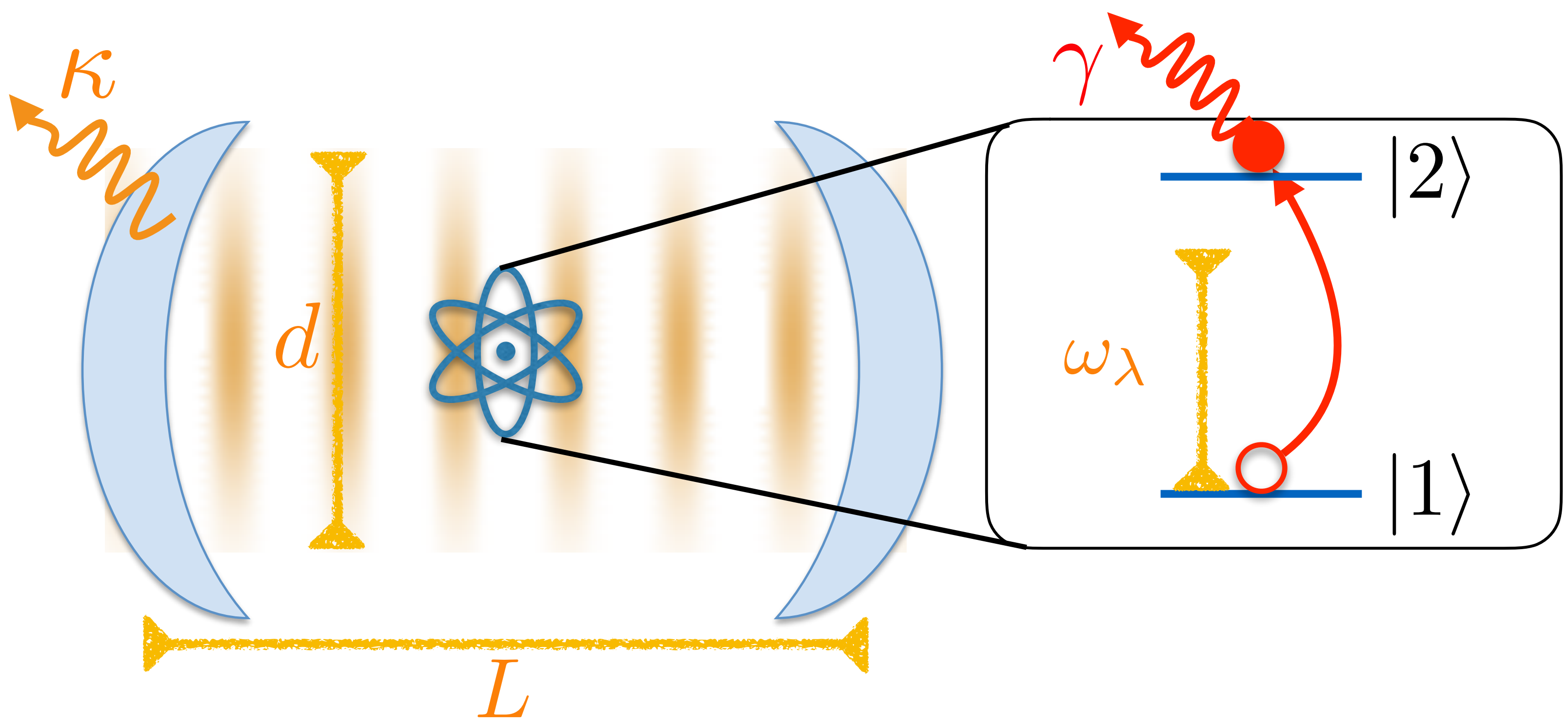
Atoms close to a waveguide



Source: Univ. Stuttgart

Rydberg atoms

Option 1 - Cavities



Interaction time

Loss rate out of the mirrors

$$\tau_{\rm int} = \frac{1}{\kappa}$$

$$\mathrm{Im} C_{\rm res} = \frac{\tau_{\rm int}}{L/c}\frac{\lambda_{12}^2}{d^2} = F\frac{\lambda_{12}^2}{d^2}$$

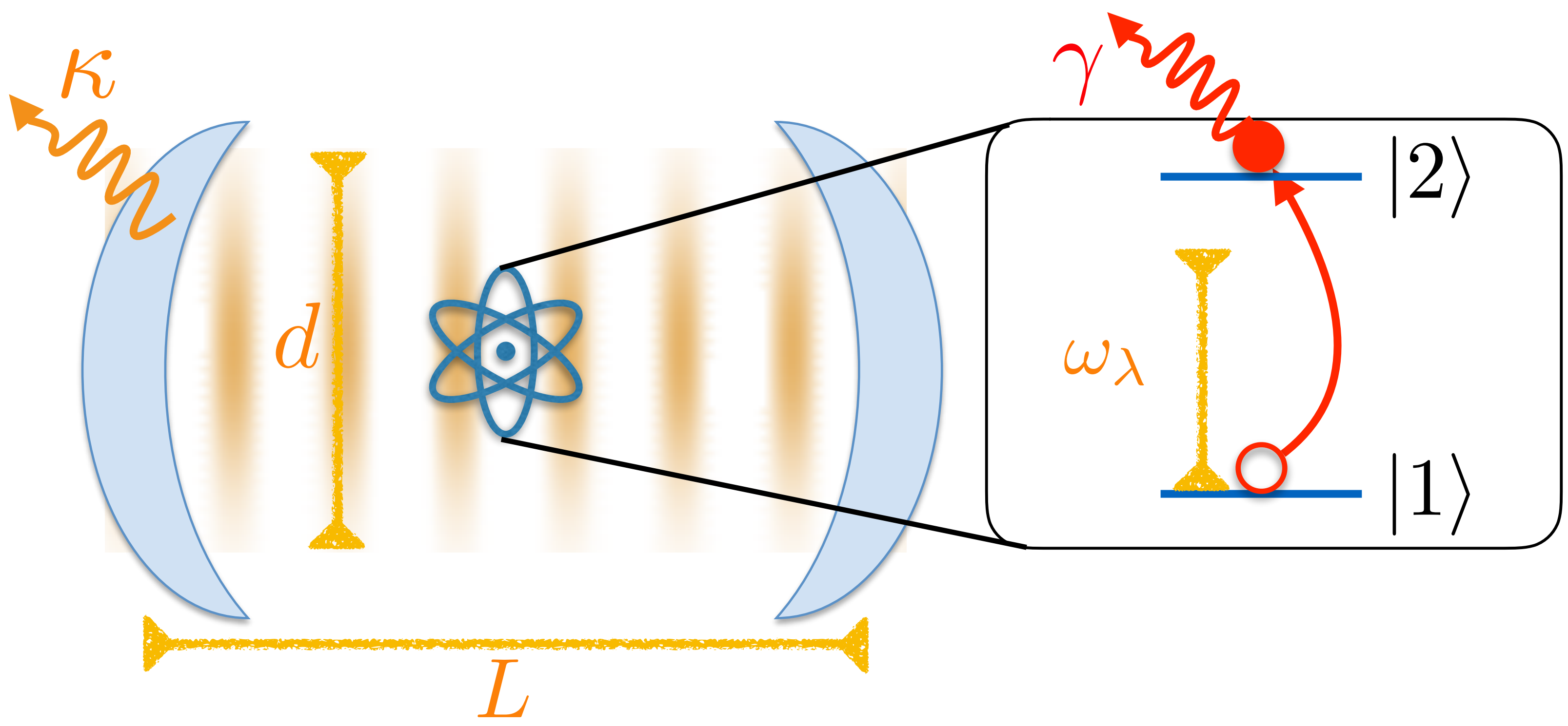
Multipass enhancement

Determined by Finesse

(can be as large as 10^6)

$$F = \frac{c}{\kappa L}$$

Option 1 - Cavities



<u>Interaction time</u>

Loss rate out of the mirrors

$$\tau_{\text{int}} = \frac{1}{\kappa}$$

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Multipass enhancement

Determined by <u>Finesse</u>

(can be as large as 10^6)

$$F = \frac{c}{\kappa L}$$

Cooperativity reweritten as:

$$C = \frac{g^2}{\gamma\kappa}$$

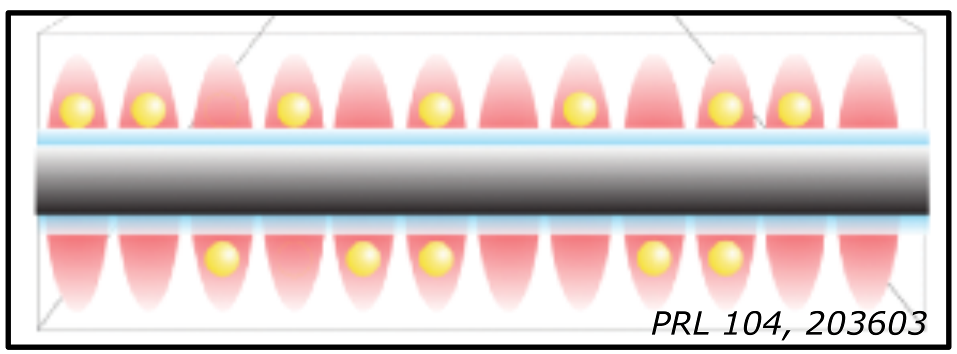
Standard suggestive expression -> longer-lived state seems to help.

<u>Question</u>: why is not true?

Option 2 - Evanescent fields

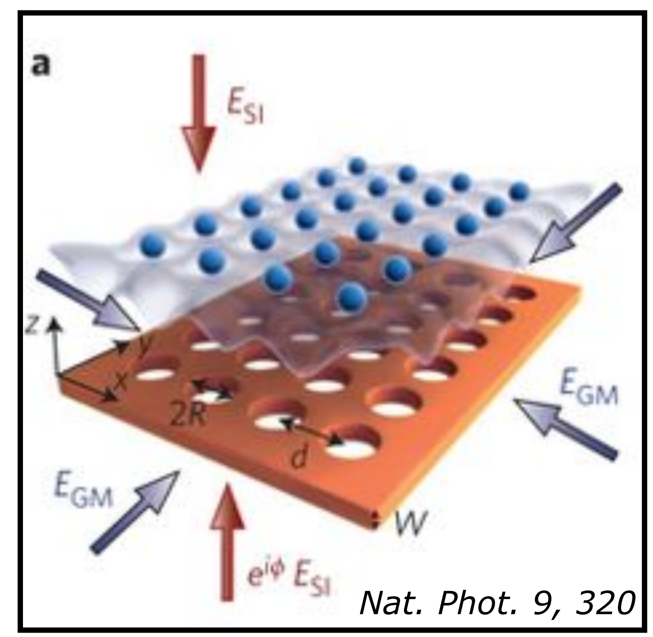
$$\mathrm{Im}C_{\mathrm{res}} = \frac{\tau_{\mathrm{int}}}{L/c}\frac{\lambda_{12}^2}{d^2}$$

Optical Nanofibers



Trapping atoms close to fiber

Photonic Crystals



Trapping atoms close to 2D structure

Exponential confinement

Transverse size can go below wavelength

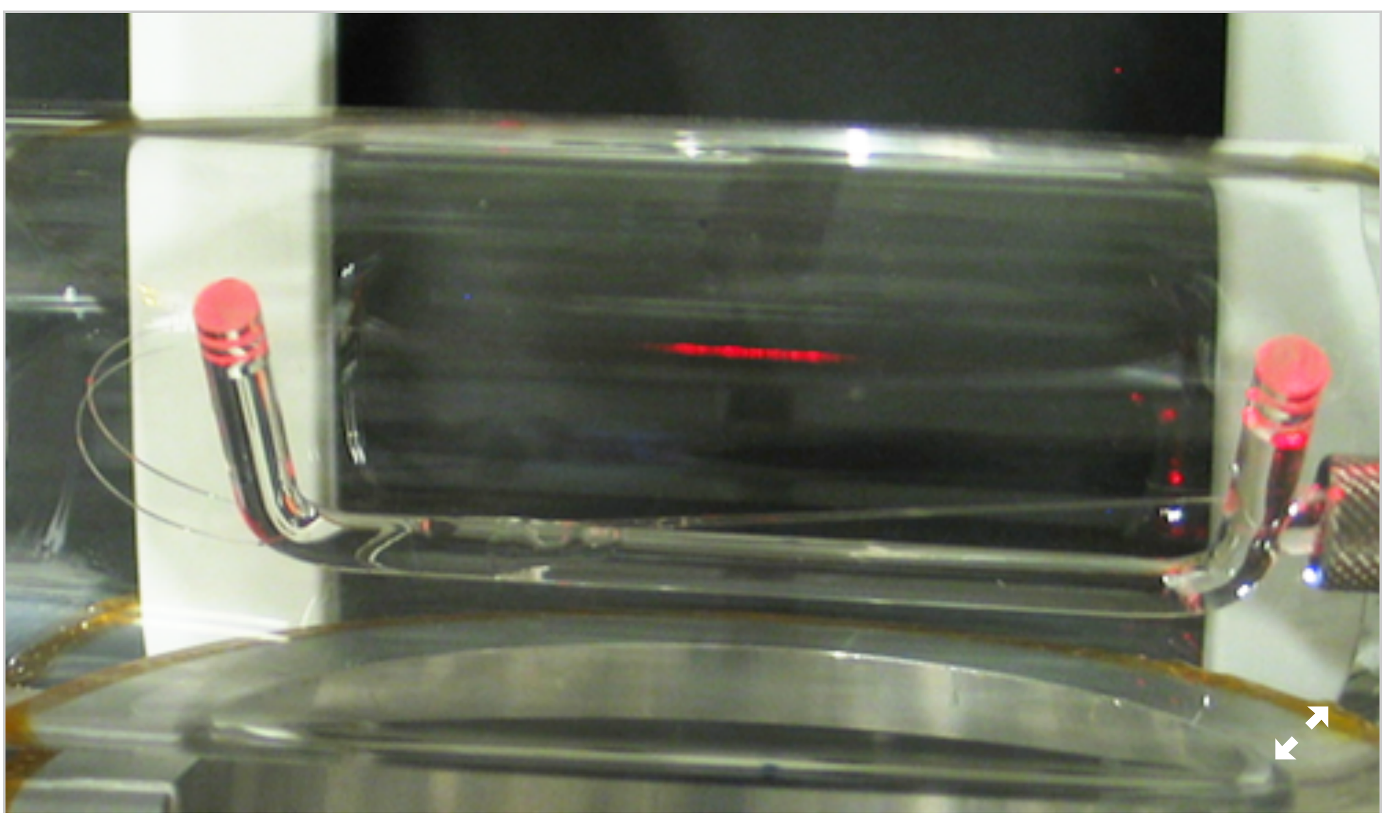
Option 2 - Evanescent fields

APS physics | Physics

Focus: Strong Light Reflection from Few Atoms

September 23, 2016 • *Physics* 9, 109

Up to 75% of light reflects from just 2000 atoms aligned along an optical fiber, an arrangement that could be useful in photonic circuits.



J. Appel/Univ. of Copenhagen

Coherent Backscattering of Light Off One-Dimensional Atomic Strings

H.L. Sørensen, J.-B. Béguin, K.W. Kluge, I. Iakoupov, A.S. Sørensen, J.H. Müller, E.S. Polzik, and J. Appel

Phys. Rev. Lett. 117, 133604 (2016)

Published September 23, 2016

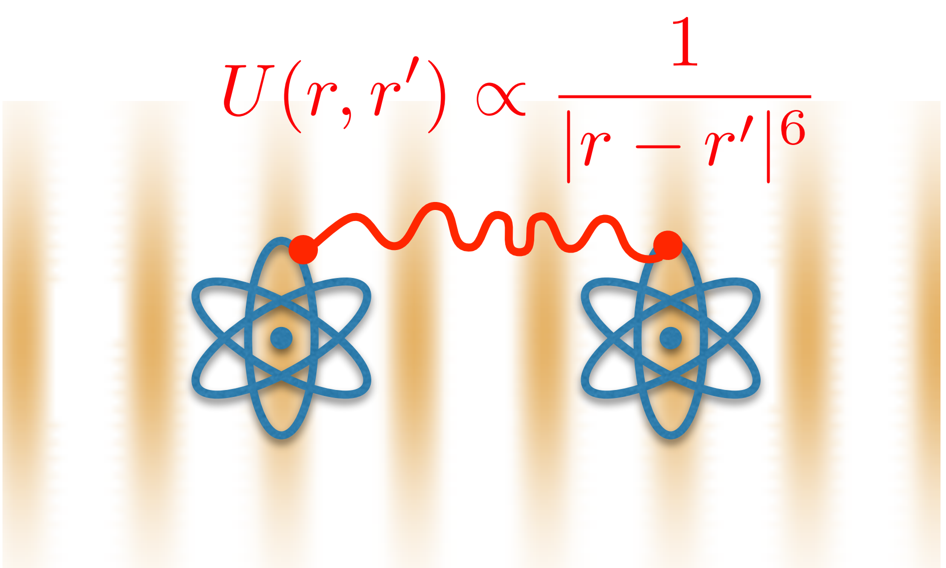
Large Bragg Reflection from One-Dimensional Chains of Trapped Atoms Near a Nanoscale Waveguide

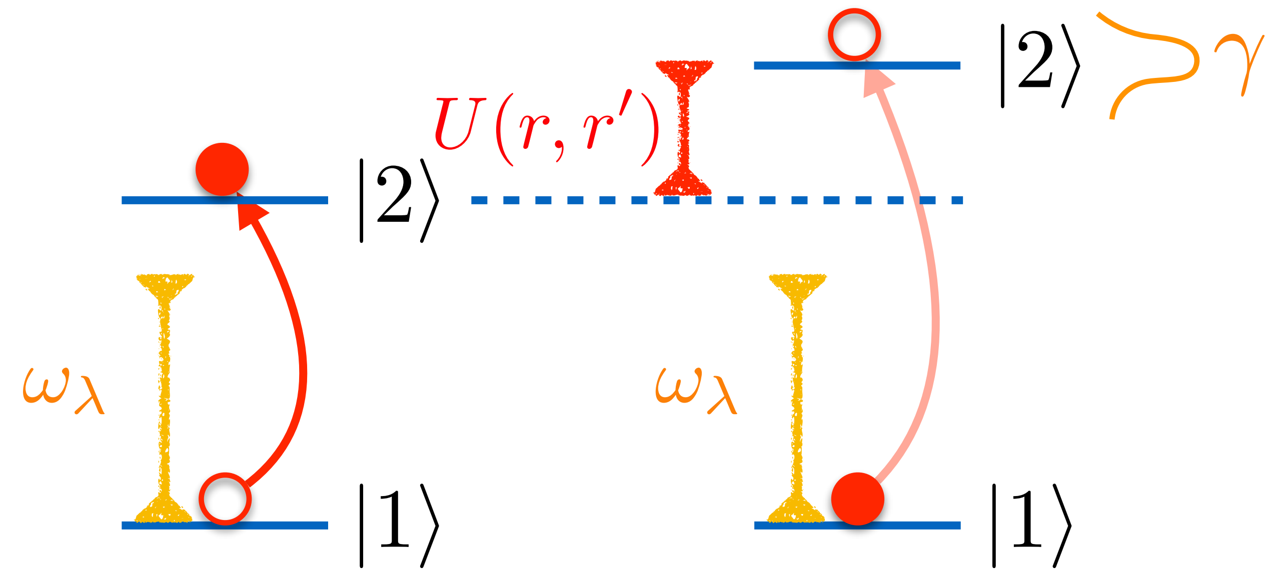
Neil V. Corzo, Baptiste Gouraud, Aveek Chandra, Akihisa Goban, Alexandra S. Sheremet, Dmitriy V. Kupriyanov, and Julien Laurat

Phys. Rev. Lett. 117, 133603 (2016)

Published September 23, 2016

Option 3 - Collective effects in Rydberg atoms



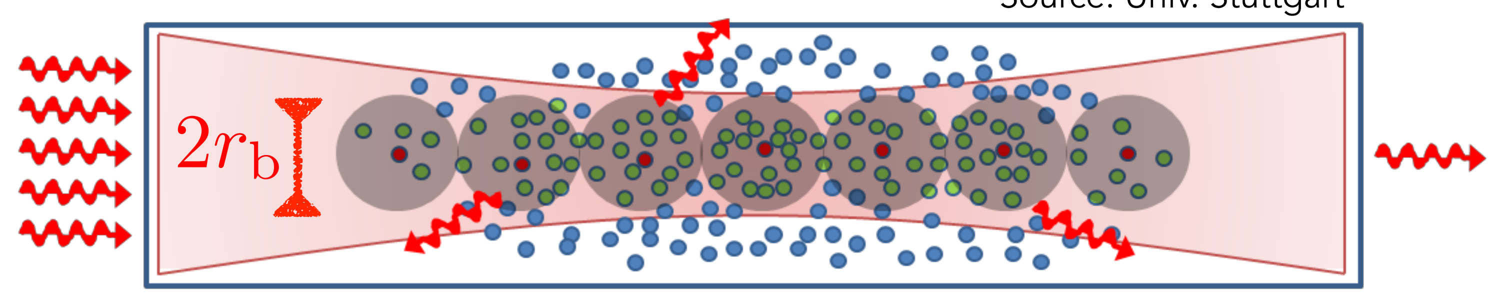


Rydberg Blockade: $U \gg \gamma$

Only one of the two atoms can be excited

Blockade Radius within which only a single atom can be excited

Source: Univ. Stuttgart



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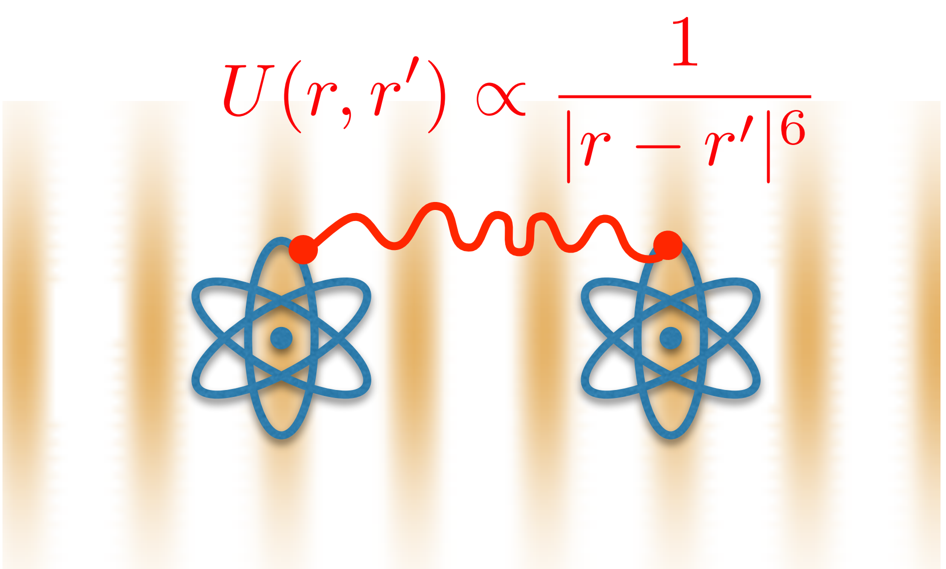

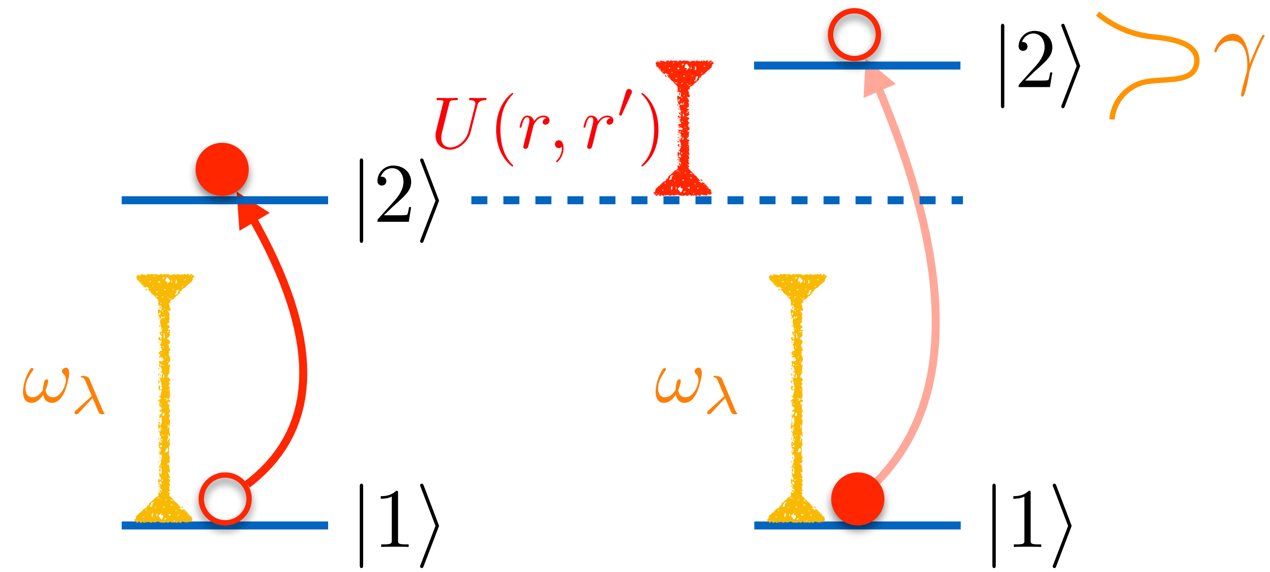

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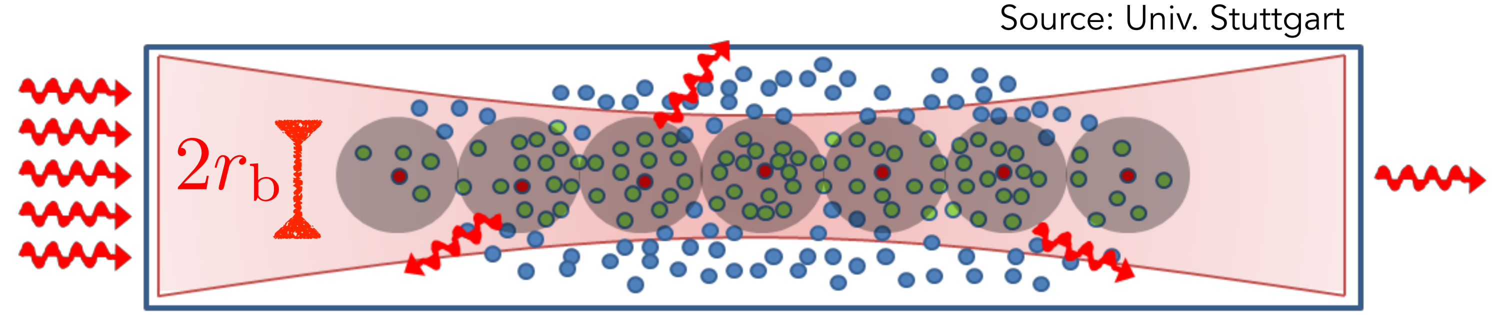


Source: Univ. Stuttgart

A <u>single</u> photon can saturate the whole blockade radius

"Superatom" made of N_b atoms

$$g \rightarrow \sqrt{N_\mathrm{b}}\, g$$

$$\mathrm{Im}C_\mathrm{res} = \frac{N_\mathrm{b} g_\mathrm{res}^2 \frac{L}{c}}{\gamma} = N_\mathrm{b} \frac{\lambda_{12}^2}{d^2}$$

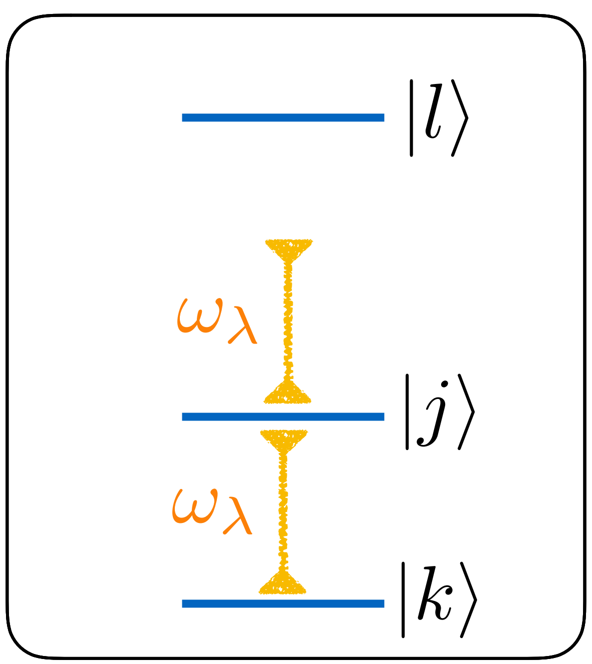
Implementing the nonlinearity

Needs an active element in the medium:

Atomic degree of freedom which is nonlinearly coupled to the photon

Atomic saturation

Multiple excitations avoided
Due to nonlinear level spacing



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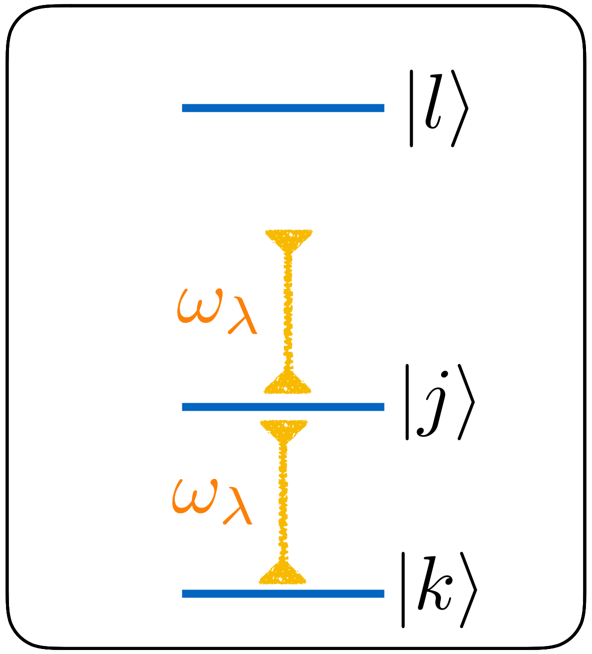
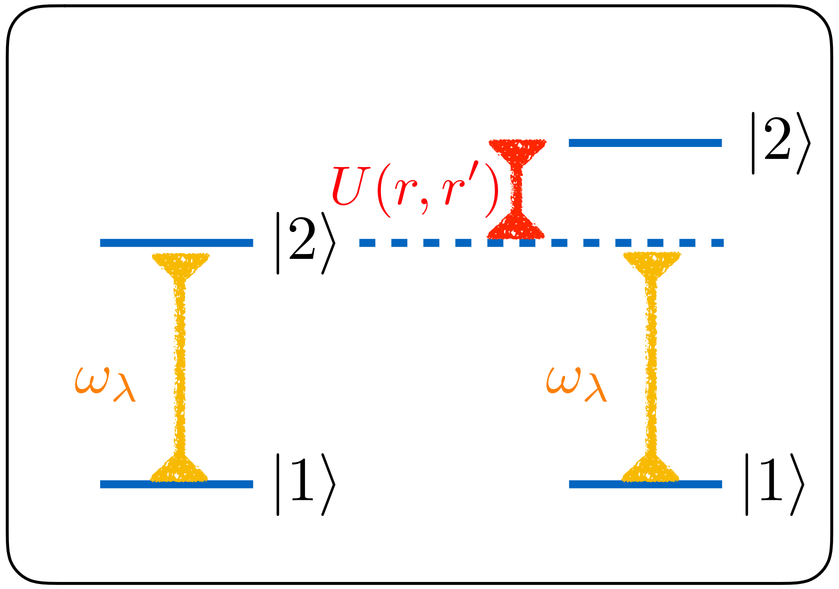


Interatomic interactions

Example: Rydberg interaction



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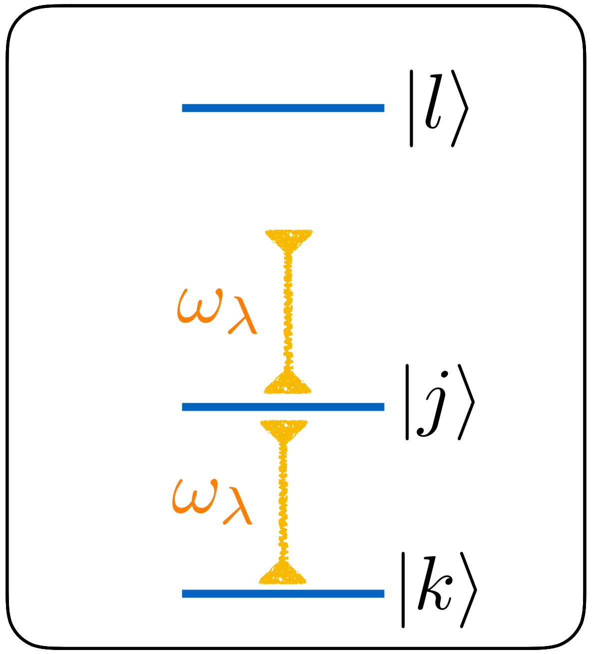

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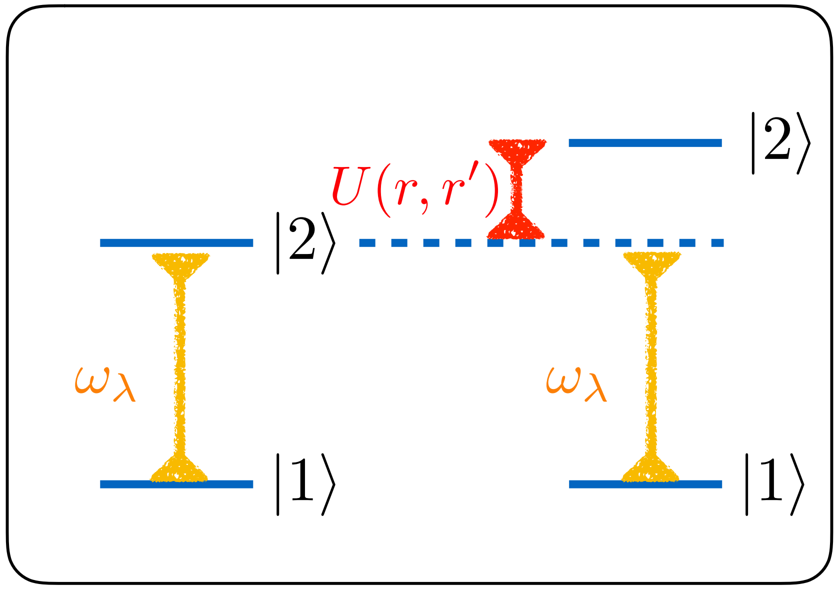
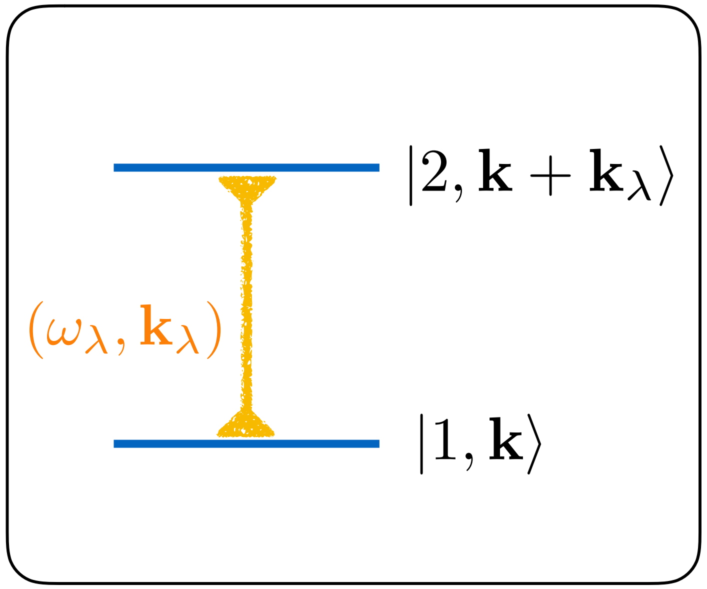


Atomic motion

Feedback between
Internal and motional dynamics



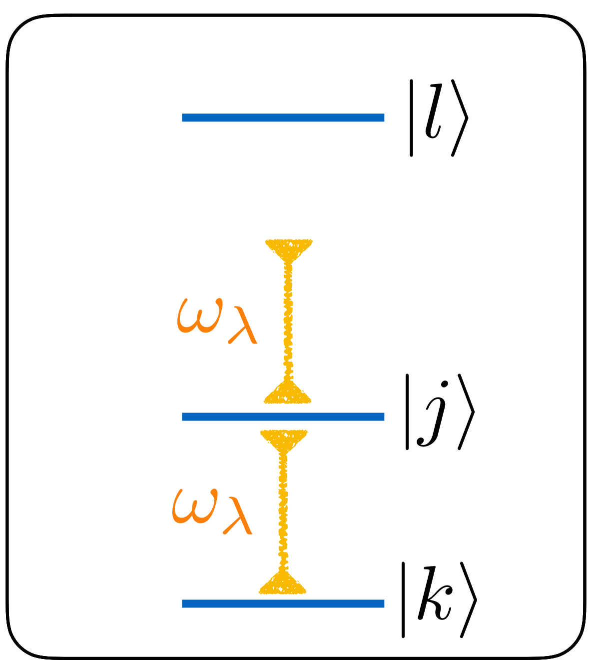
Implementing the nonlinearity

Needs an active element in the medium:
Atomic degree of freedom which is nonlinearly coupled to the photon

Quantum degenerate matter (see part 3)

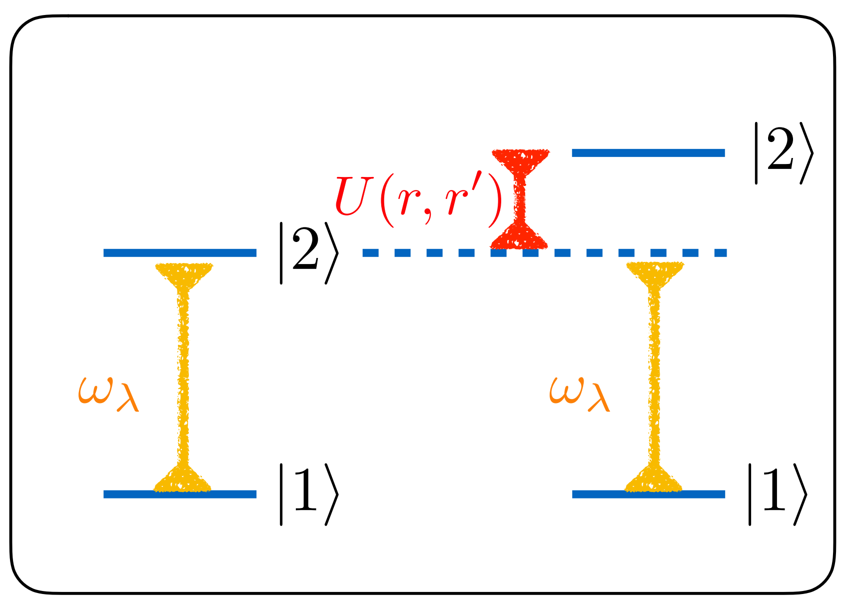
Atomic saturation

Multiple excitations avoided
Due to nonlinear level spacing



Interatomic interactions

Example: Rydberg interaction



Atomic motion

Feedback between
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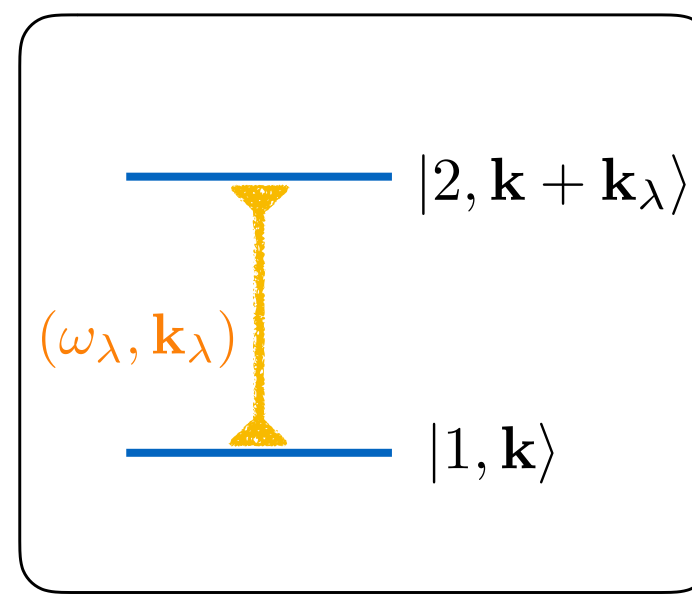
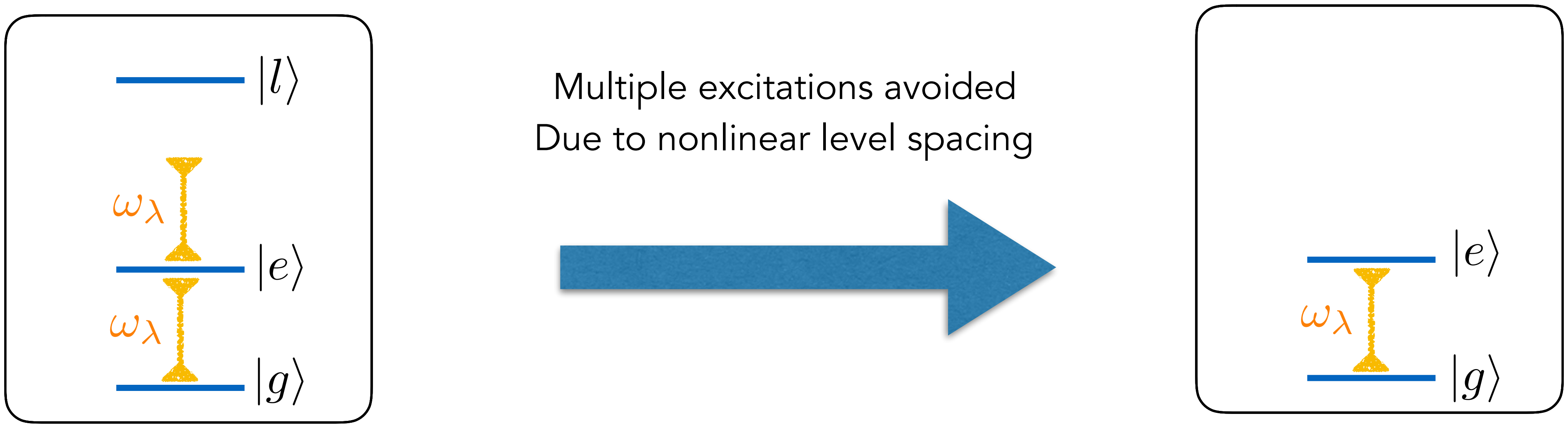


Quantum nonlinear optics from atomic saturation

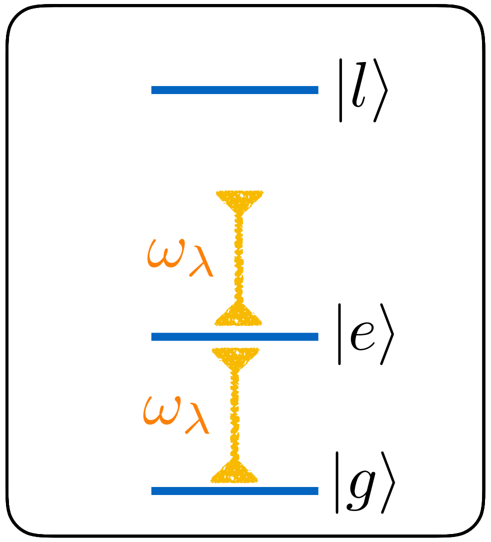


Jaynes-Cummings Hamiltonian

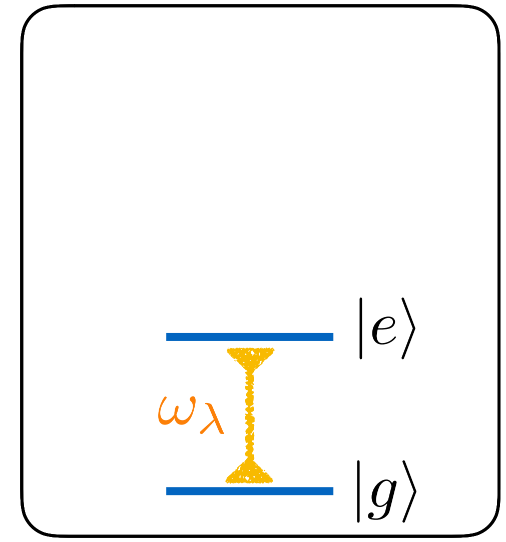
$$\hat{H}_{JC} = \sum_\lambda \omega_\lambda \hat{a}^\dagger_\lambda \hat{a}_\lambda + \omega_{ge}\sigma_z + \sum_\lambda \left(g_\lambda a_\lambda \sigma^+ + \text{h.c.}\right)$$

Restricted Hilbert space	Fermion annihilation/creation operators	Spin operators
	σ^+	$\hat{c}_g \hat{c}^\dagger_e$
$\hat{c}^\dagger_g \hat{c}_g + \hat{c}^\dagger_e \hat{c}_e = 1$	σ_z	$(\hat{c}^\dagger_e \hat{c}_e - \hat{c}^\dagger_g \hat{c}_g)/2$

Quantum nonlinear optics from atomic saturation



Multiple excitations avoided
Due to nonlinear level spacing



Jaynes-Cummings Hamiltonian

$$\hat{H}_{JC} = \sum_\lambda \omega_\lambda \hat{a}^\dagger_\lambda \hat{a}_\lambda + \omega_{ge}\sigma_z + \sum_\lambda (g_\lambda a_\lambda \sigma^+ + \text{h.c.})$$

<u>Restricted</u>
<u>Hilbert space</u>

$$\hat{c}^\dagger_g \hat{c}_g + \hat{c}^\dagger_e \hat{c}_e = 1$$

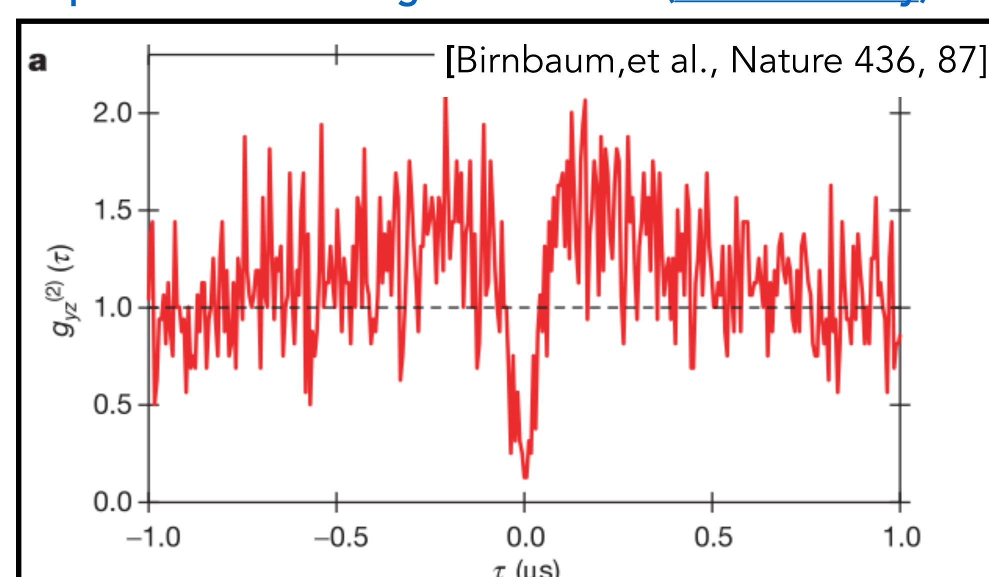
Creates nonlinearity

Like an interaction

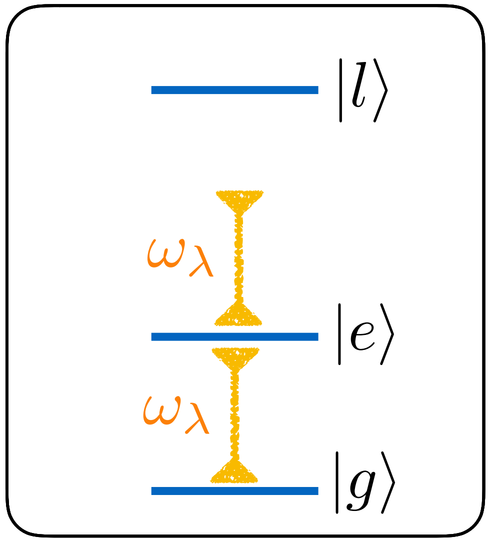
Example: Photon Blockade

Two photons cannot be absorbed/emitted at the same time.

Experiment measuring coincidences (<u>atom in cavity</u>)



Quantum nonlinear optics from atomic saturation



Multiple excitations avoided
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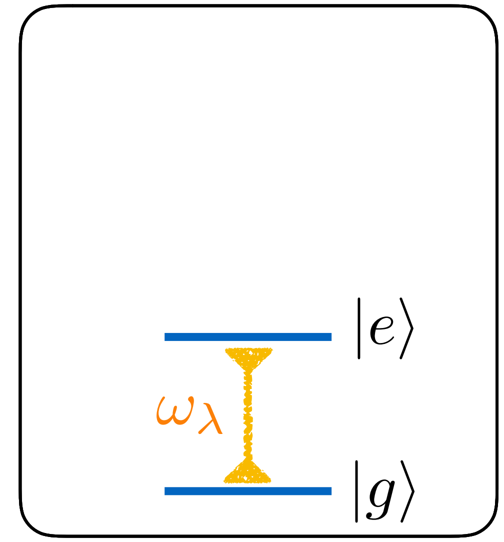


Jaynes-Cummings Hamiltonian

$$\hat{H}_{JC} = \sum_\lambda \omega_\lambda \hat{a}^\dagger_\lambda \hat{a}_\lambda + \omega_{ge}\sigma_z + \sum_\lambda \left(g_\lambda a_\lambda \sigma^+ + \text{h.c.}\right)$$

<u>Restricted Hilbert space</u>

$$\hat{c}^\dagger_g \hat{c}_g + \hat{c}^\dagger_e \hat{c}_e = 1$$

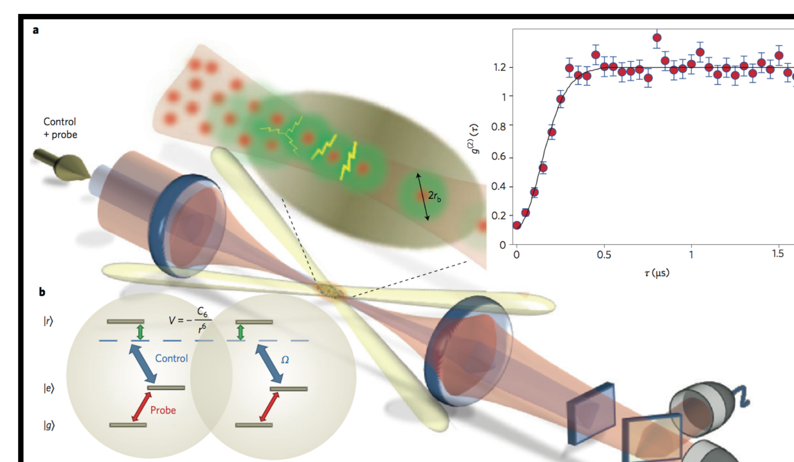
Creates nonlinearity

Like an interaction

Example: Photon Blockade

Two photons cannot be absorbed/emitted at the same time.

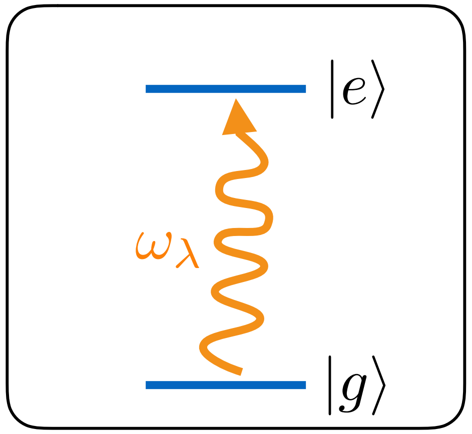
Experiment measuring coincidences (Rydberg ensemble)



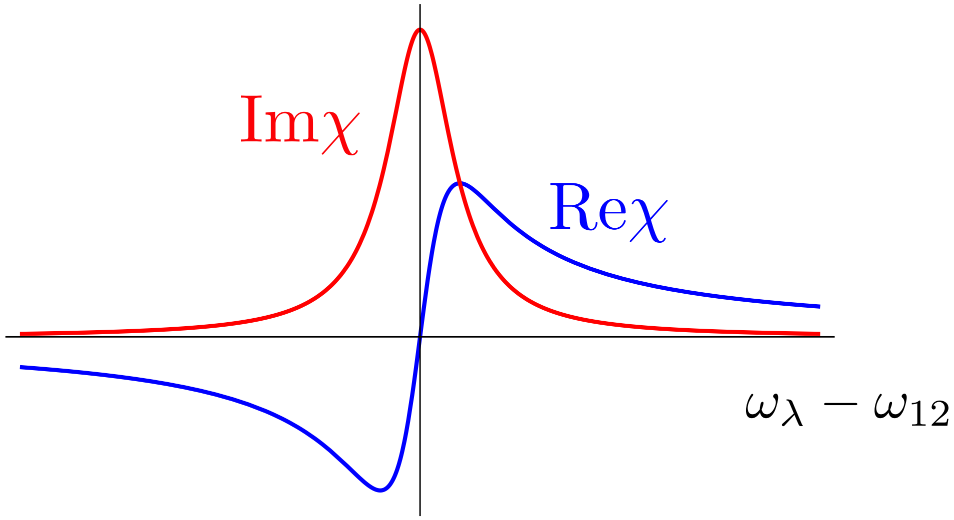
[Peyronel, et al., Nature 488, 57]

Additional tool: Electromagnetically-Induced Transparency (EIT)

Two-level atom

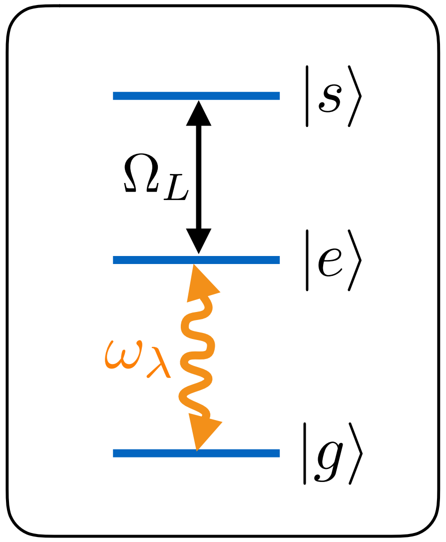


Susceptibility



Strong interactions imply also large absorption

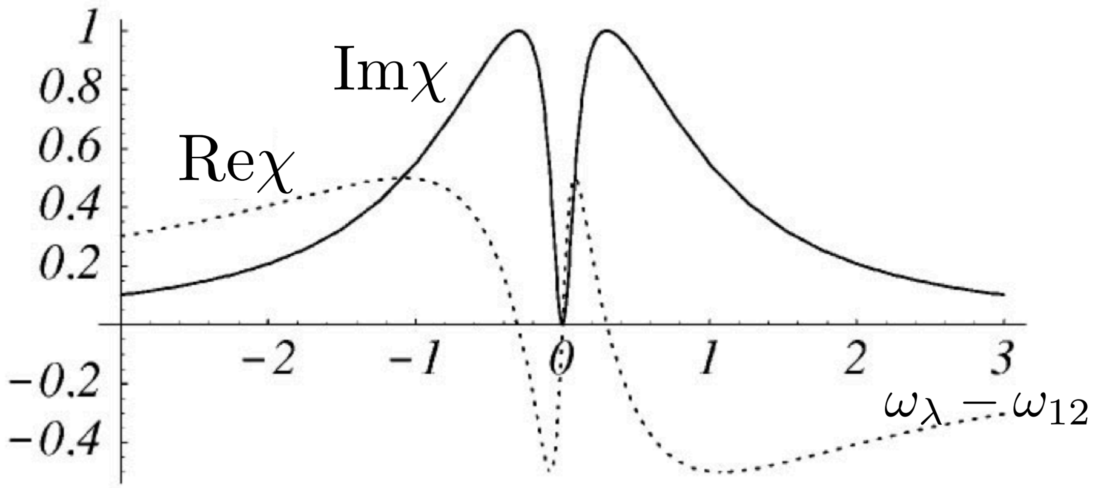
Three-level atom



Single particle effect (linear optics):

destructive interference between excitation pathways

Susceptibility



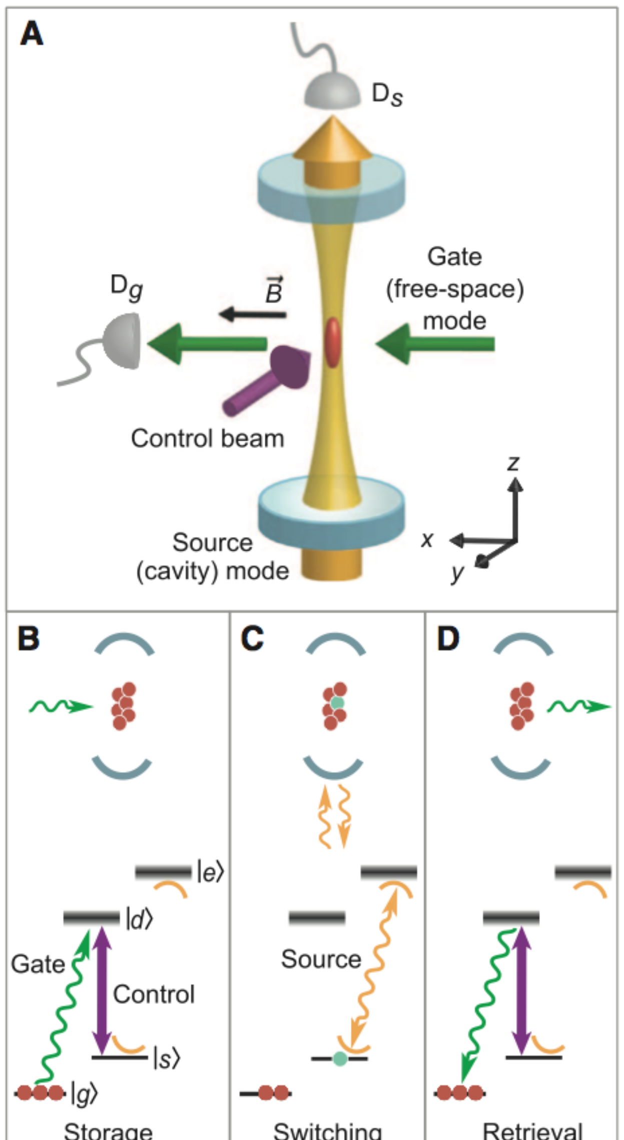
EIT window

Large coherent interactions without absorption

Technological application of quantum nonlinear optics

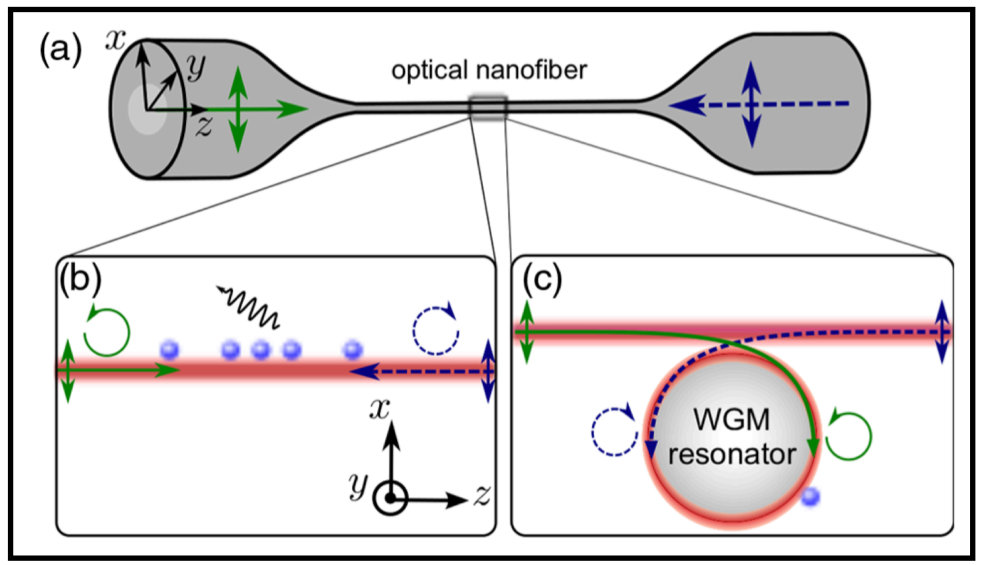
Example: All optical single-photon transistors

Cavities



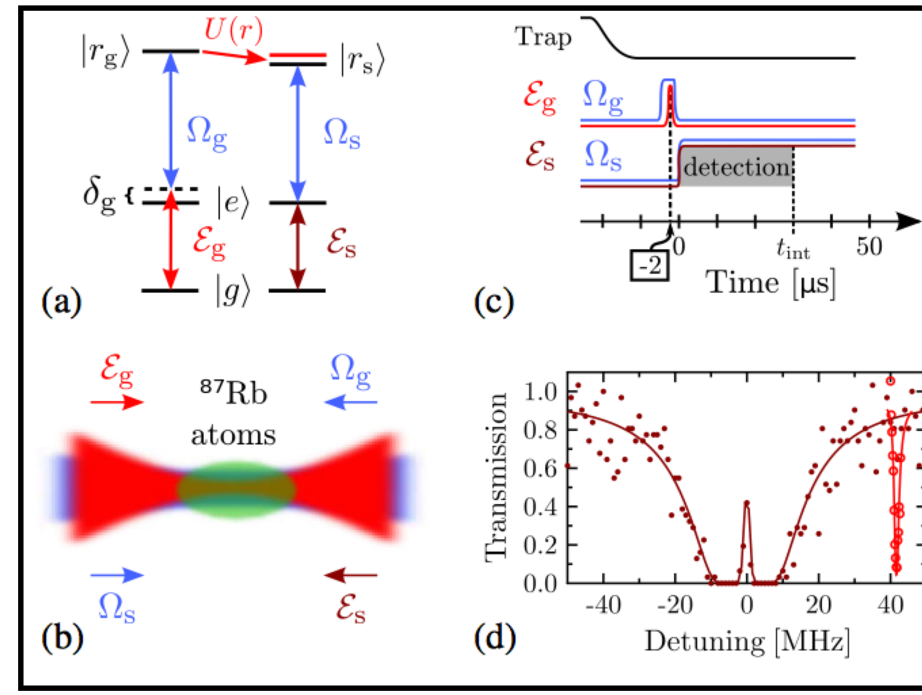
Exp.: Chen ,et al., Science 341, 768 (2013)

Waveguides



Exp.: Sayrin ,et al., PRX 5, 041036 (2015)

Rydberg Ensembles



Exp.: Gorniaczyc, et al., Phys. Rev. Lett. 113, 053601 (2014)

3. Quantum nonlinear optics with quantum degenerate matter

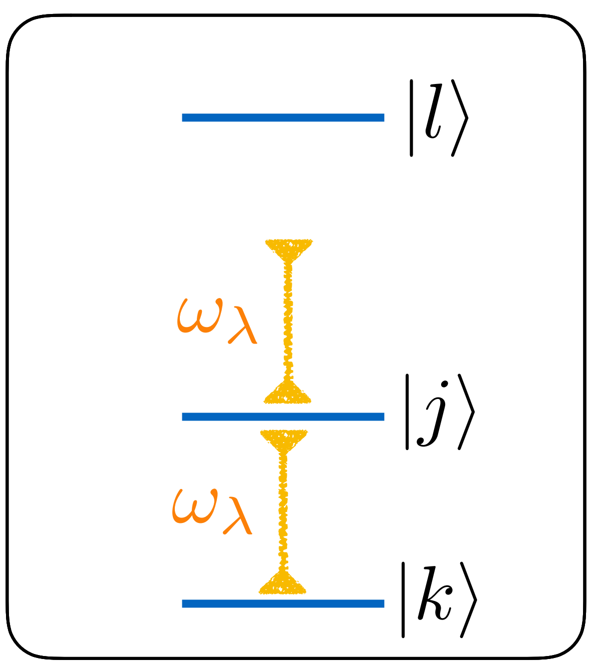
Implementing the nonlinearity

Needs an active element in the medium:
Atomic degree of freedom which is nonlinearly coupled to the photon

Quantum degenerate matter

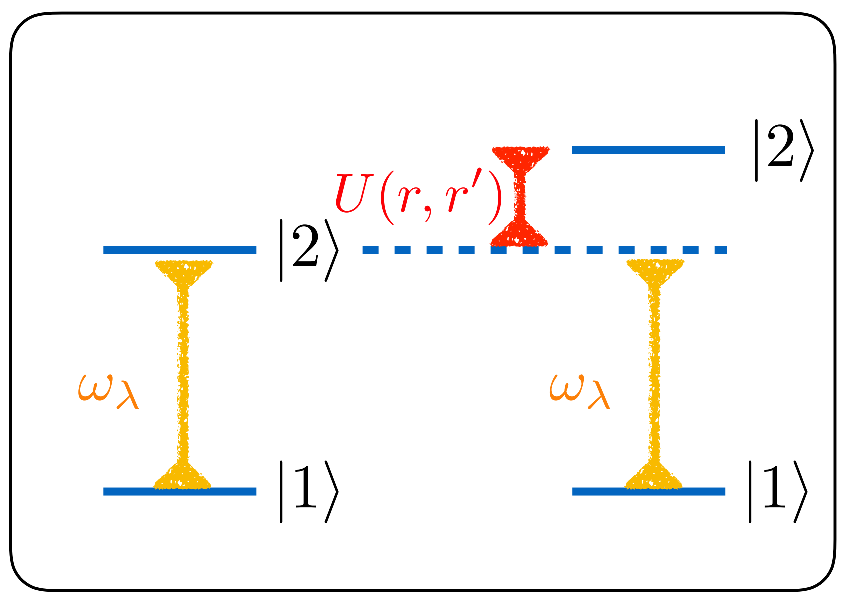
Atomic saturation

Multiple excitations avoided
Due to nonlinear level spacing



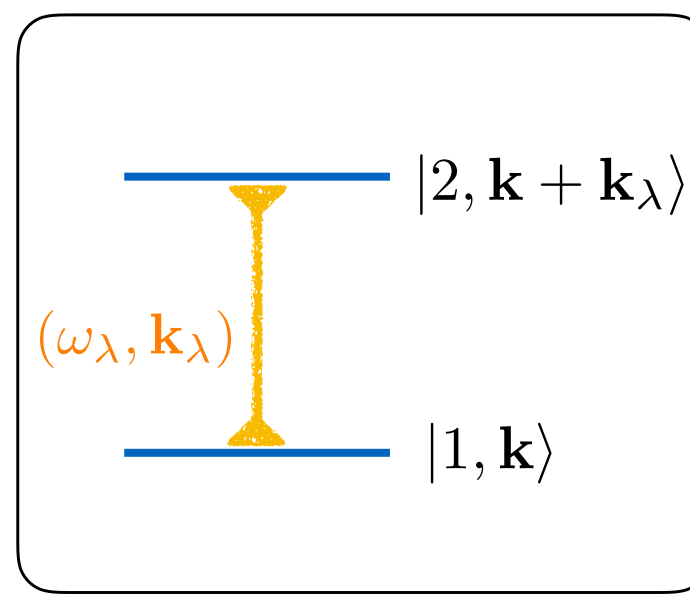
Interatomic interactions

Example: Rydberg interaction

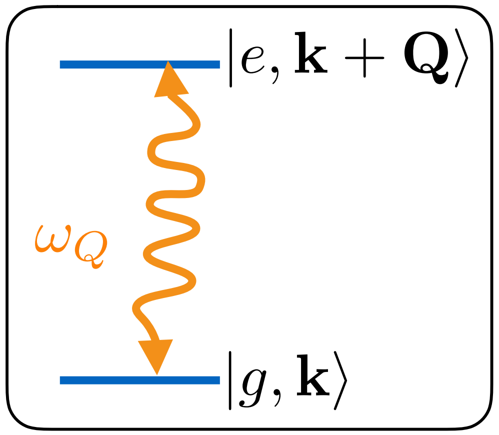


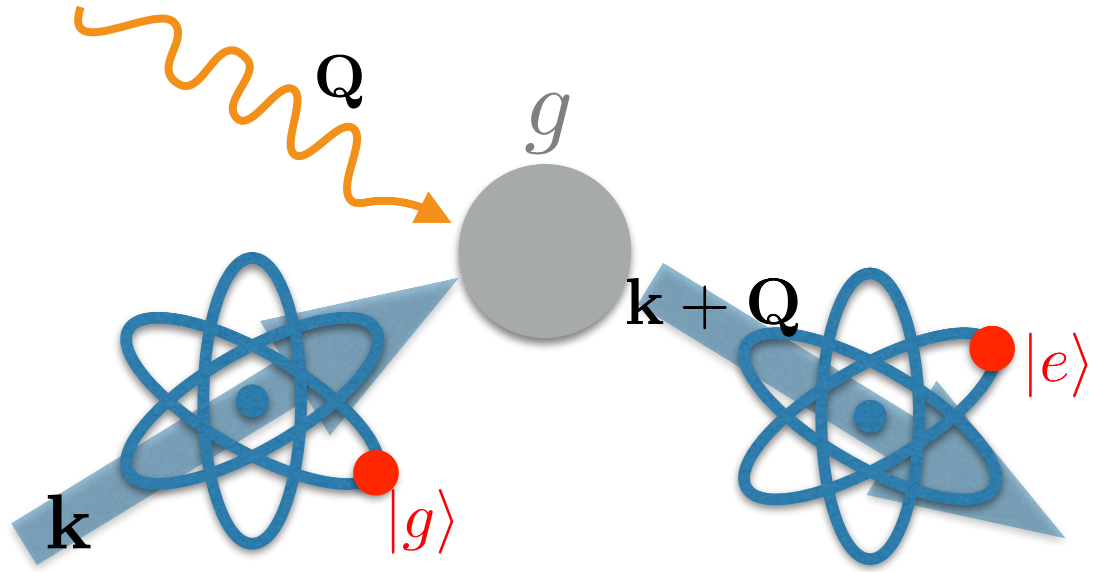
Atomic motion

Feedback between
internal and motional dynamics



Dispersive coupling to the atomic motion



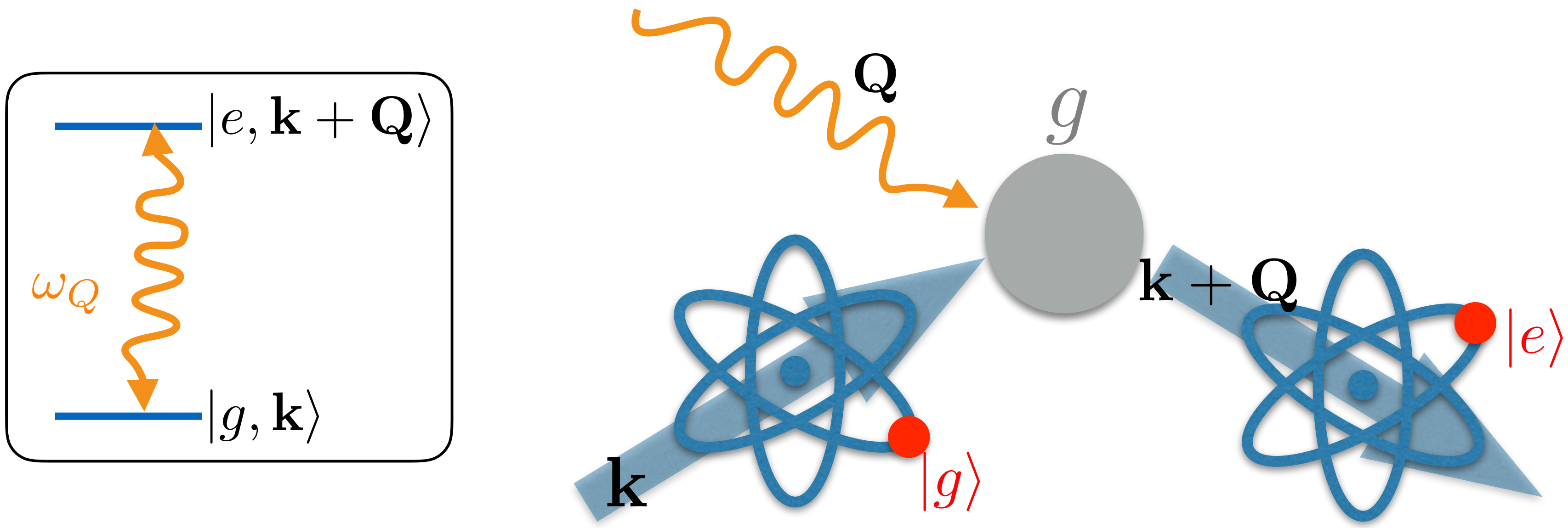


Recoil kick

Atomic center of mass

Still within dipole approx.

Dispersive coupling to the atomic motion



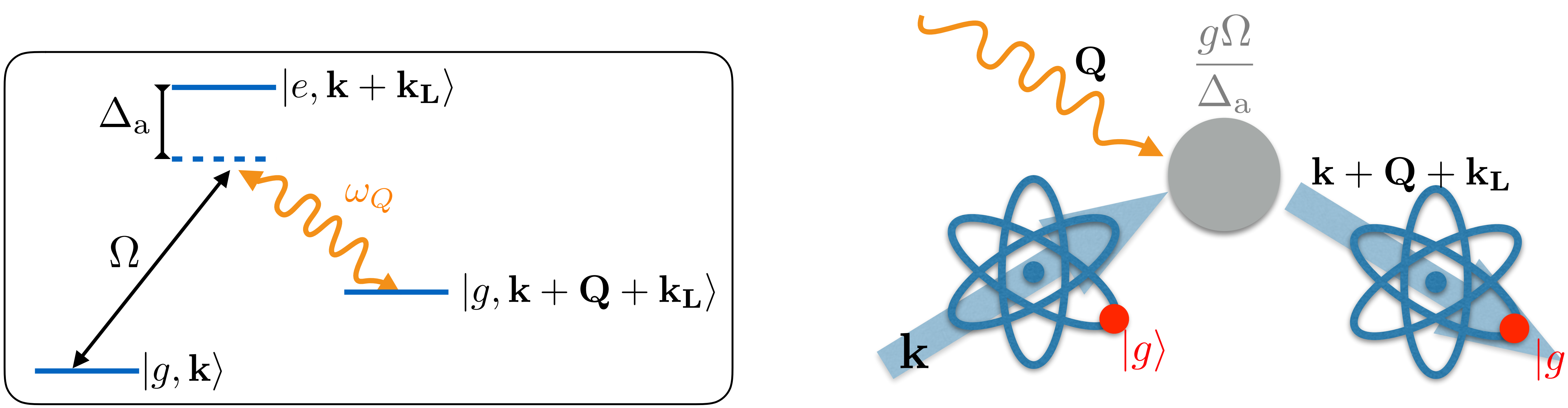
Recoil kick

Atomic center of mass

Still within dipole approx.

Dispersive coupling

Far-off resonant laser (Δ_a is the larges scale): Excited-state-dynamics frozen the two-photon transition



Dispersive coupling to the atomic motion

Far-off resonant laser (Δ_{a} is the larges scale): Excited-state-dynamics frozen the two-photon transition

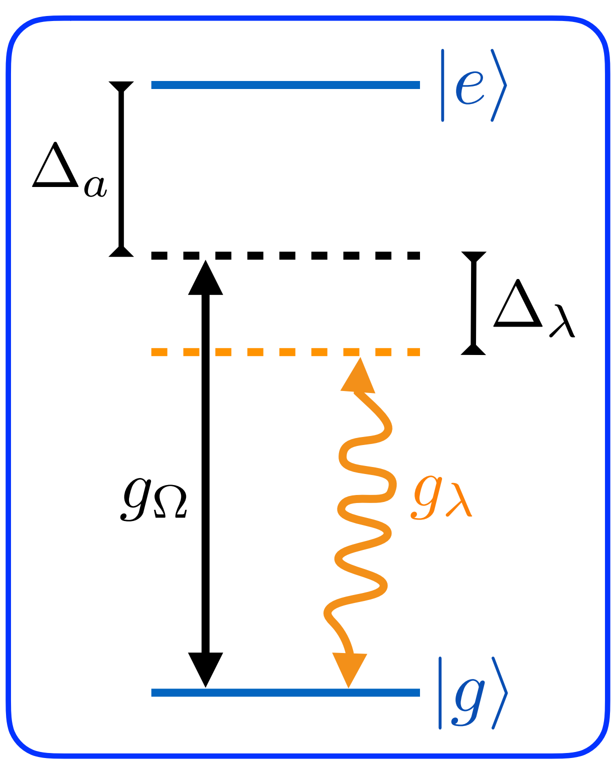
Hamiltonian in real space:

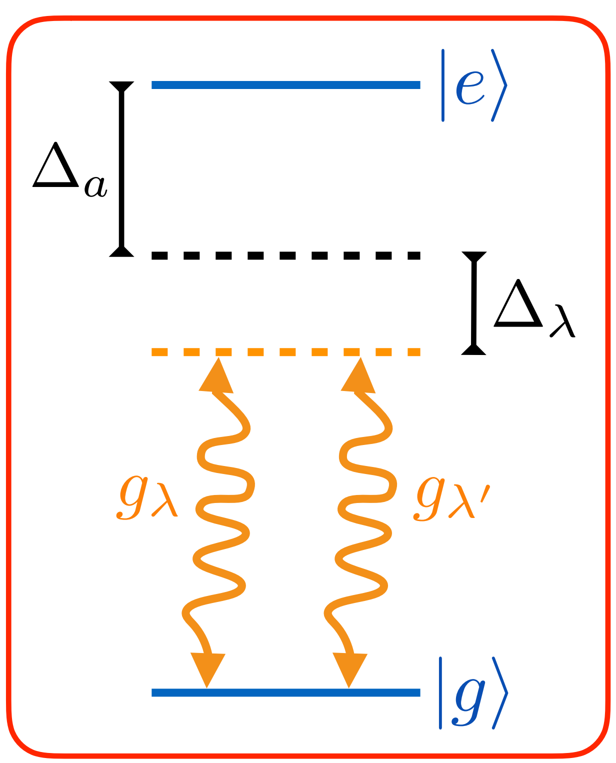
$$\hat{H} = -\sum_\lambda \Delta_\lambda \hat{a}_\lambda^\dagger \hat{a}_\lambda + \int_{\mathbf{r}} \hat{\psi}_g^\dagger(\mathbf{r}) \left(-\frac{\nabla^2}{2m} + \hat{V}(\mathbf{r}) \right) \hat{\psi}_g(\mathbf{r})$$

Coupling is nonlinear

Dynamical Optical potential

$$\hat{V}(\mathbf{r}) = V_{\mathrm{ext}}(\mathbf{r}) + \sum_\lambda \left(\frac{g_\Omega(\mathbf{r}) g_\lambda(\mathbf{r})}{\Delta_{\mathrm{a}}} \sqrt{n_\Omega} \hat{a}_\lambda + \mathrm{h.c.} \right) + \sum_{\lambda,\lambda'} \frac{g_\lambda^*(\mathbf{r}) g_{\lambda'}(\mathbf{r})}{\Delta_{\mathrm{a}}} \hat{a}_\lambda^\dagger \hat{a}_{\lambda'}$$





Recall: the dipole coupling strength

$$g_{\lambda/\Omega}(\mathbf{r}) \propto u_{\lambda/\Omega}(\mathbf{r})$$

Depends on the electromagnetic mode function computed at the position of the atom

Dispersive coupling to the atomic motion

Far-off resonant laser (Δ_a is the larges scale): Excited-state-dynamics frozen the two-photon transition

Hamiltonian in real space:

$$\hat{H} = -\sum_\lambda \Delta_\lambda \hat{a}^\dagger_\lambda \hat{a}_\lambda + \int_\mathbf{r} \hat{\psi}^\dagger_g(\mathbf{r})\left(-\frac{\nabla^2}{2m} + \hat{V}(\mathbf{r})\right)\hat{\psi}_g(\mathbf{r})$$

Dynamical Optical potential

$$\hat{V}(\mathbf{r}) = V_\mathrm{ext}(\mathbf{r}) + \sum_\lambda \left(\frac{g_\Omega(\mathbf{r})g_\lambda(\mathbf{r})}{\Delta_\mathrm{a}}\sqrt{n_\Omega}\hat{a}_\lambda + \mathrm{h.c.}\right) + \sum_{\lambda,\lambda'} \frac{g^*_\lambda(\mathbf{r})g_{\lambda'}(\mathbf{r})}{\Delta_\mathrm{a}}\hat{a}^\dagger_\lambda \hat{a}_{\lambda'}$$

Exercise: derive the above Hamiltonian starting from the Jaynes-Cummings Hamiltonian.

Hint: assuming far-off detuned laser use the approximate representation: $\sigma^+(\mathbf{r}) \simeq \psi_g(\mathbf{r})\psi^\dagger_e(\mathbf{r})$

In Heisenberg picture assume the excited state operator is in the steady state (adiabatic elimination)

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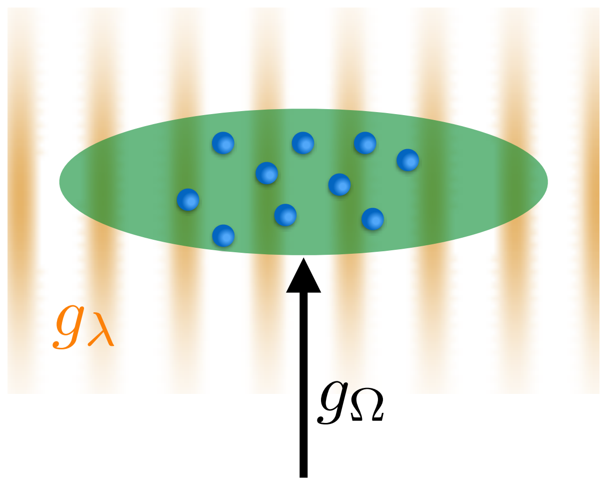
In Heisenberg picture assume the excited state operator is in the steady state (adiabatic elimination)

Note: the \psi-operators describe the motion of the atomic COM and not of the electrons!
The electron dynamics is reduced to a single-particle quantum number e/g.

Directly applicable to N atoms.

Quantum nonlinear optics with atomic motion

Cloud of laser-driven atoms



$$\hat{H} = -\sum_\lambda \Delta_\lambda \hat{a}^\dagger_\lambda \hat{a}_\lambda + \int_{\mathbf{r}} \hat{\psi}^\dagger(\mathbf{r}) \Big(-\frac{\nabla^2}{2m} + \hat{V}(\mathbf{r}) \Big) \hat{\psi}(\mathbf{r})$$

$$\hat{V}(\mathbf{r}) = V_{\mathrm{ext}}(\mathbf{r}) + \sum_\lambda \left(\frac{g_\Omega(\mathbf{r}) g_\lambda(\mathbf{r})}{\Delta_{\mathrm{a}}} \sqrt{n_\Omega} \hat{a}_\lambda + \mathrm{h.c.} \right)$$

Susceptibility

Measures the linear response of the material to a photon

$$\chi(\omega_\lambda) = \sum_{jk} \frac{(n_k - n_j) \left| \langle k | V^{\mathrm{pert}}_\lambda | j \rangle \right|^2}{\omega_\lambda - \epsilon_j + \epsilon_k + i\gamma_{jk}}$$

- n_k : average occupation of the k-th atomic eigenstate in the trap V_ext
- Interaction matrix-element $\langle k | V^{\mathrm{pert}}_\lambda | j \rangle = \frac{\sqrt{n_\Omega}}{\Delta_{\mathrm{a}}} \int_{\mathbf{r}} \phi^*_k(\mathbf{r}) g_\Omega(\mathbf{r}) g_\lambda(\mathbf{r}) \phi_j(\mathbf{r})$
- Width of the transition is negligible in the dispersive regime: $\gamma_{jk} \simeq 0$

Quantum nonlinear optics with atomic motion

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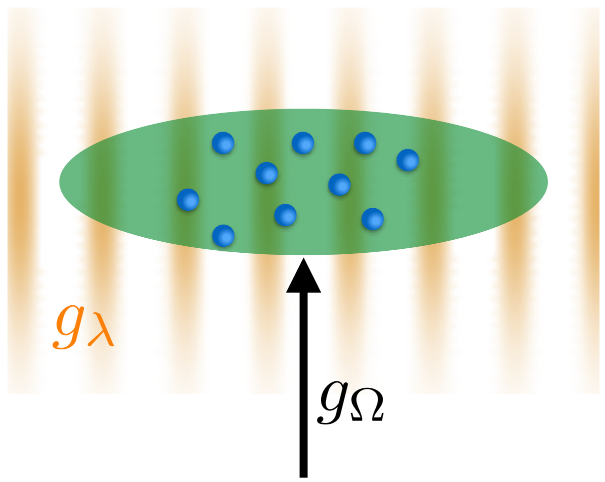

$$\hat{H} = -\sum_\lambda \Delta_\lambda \hat{a}_\lambda^\dagger \hat{a}_\lambda + \int_{\mathbf{r}} \hat{\psi}^\dagger(\mathbf{r}) \Big(-\frac{\nabla^2}{2m} + \hat{V}(\mathbf{r}) \Big) \hat{\psi}(\mathbf{r})$$

$$\hat{V}(\mathbf{r}) = V_{\text{ext}}(\mathbf{r}) + \sum_\lambda \left(\frac{g_\Omega(\mathbf{r}) g_\lambda(\mathbf{r})}{\Delta_{\text{a}}} \Omega \hat{a}_\lambda + \text{h.c.} \right)$$

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Example:

Homogeneous cloud V_ext=0

Plane wave EM mode with momentum **Q**

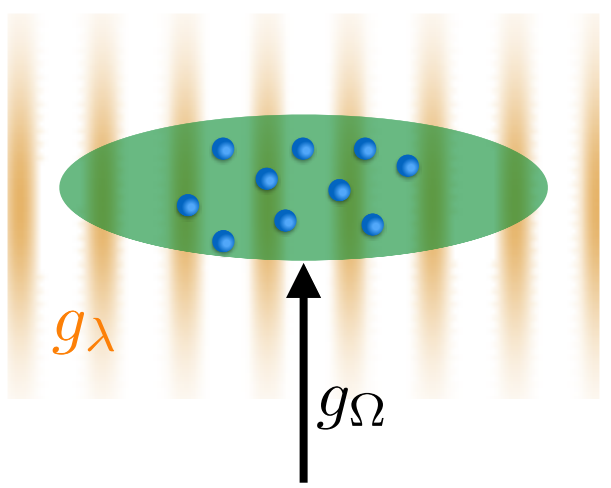
$$g_\Omega(\mathbf{r}) g_\lambda(\mathbf{r}) = g_\Omega g_\lambda e^{i\mathbf{Q}\cdot\mathbf{r}}$$

$$\chi(\omega_\lambda, \mathbf{Q}) = \frac{g_\Omega g_\lambda}{\Delta_{\text{a}}} \sum_{\mathbf{k}} \frac{n_{\mathbf{k}} - n_{\mathbf{k}+\mathbf{Q}}}{\omega_\lambda - \epsilon_{\mathbf{k}+\mathbf{Q}} + \epsilon_{\mathbf{k}} + i0^+}$$

$$\epsilon_{\mathbf{k}} = \frac{k^2}{2m}$$

Susceptibility of atoms in thermal equilibrium

Cloud of laser-driven atoms



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Plane wave EM mode with momentum **Q**

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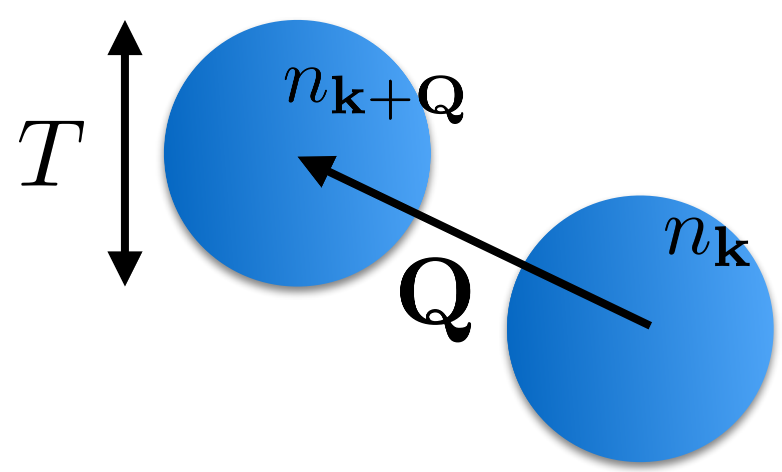
$$n_{\mathbf{k}} = \frac{1}{e^{(\epsilon_{\mathbf{k}}-\mu)/k_B T} \pm 1}$$

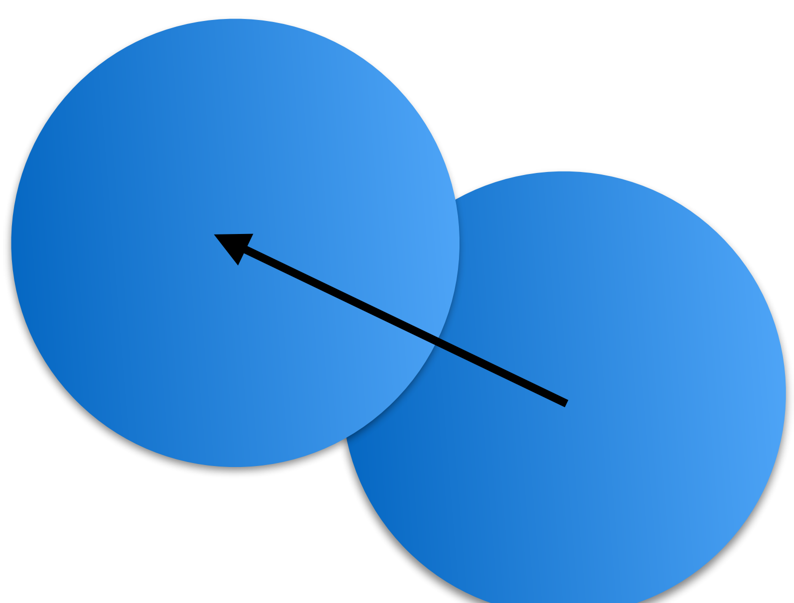
Bose-Einstein/Fermi-Dirac distribution

Temperature decreases the susceptibility

Colder



Hotter

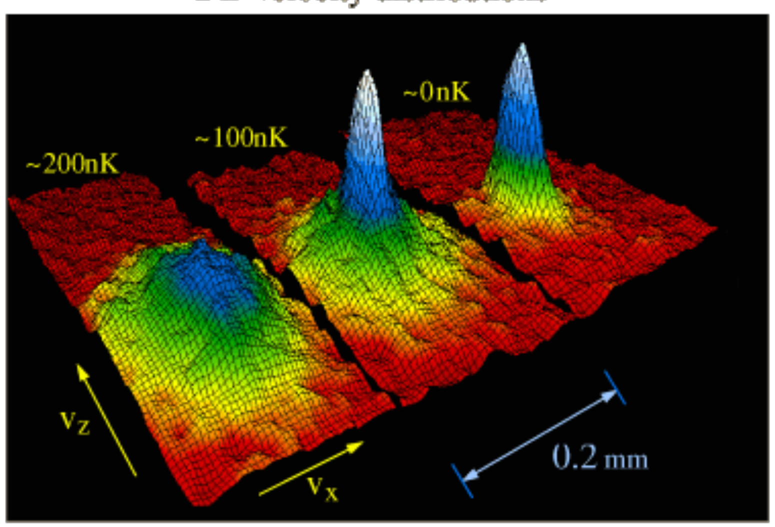


Ultracold matter

Motional degree of freedom of atoms is extremely well controlled → **Trapping** and **cooling** of atoms down to

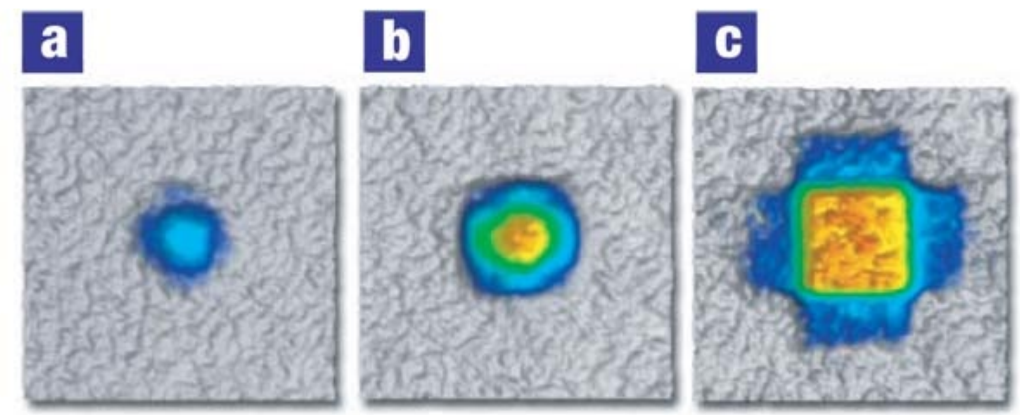
$$k_B T \sim \mathrm{nK}$$

Quantum degenerate bosonic and fermionic **ultracold gases**



Bose-Einstein condensation (BEC)

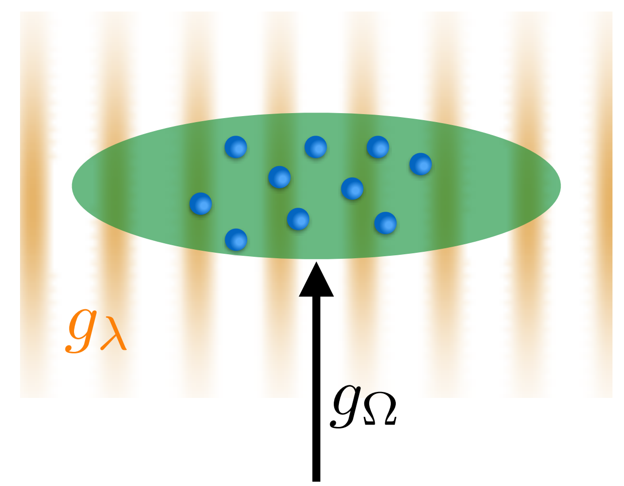
Quantum **many-body** systems
of
controlled complexity
Quantum Simulation



Perfect Fermi-Dirac distribution

Susceptibility of Quantum Matter - BEC

Cloud of laser-driven atoms



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$$n_\mathbf{k} = \frac{1}{e^{(\epsilon_\mathbf{k}-\mu)/k_B T} \pm 1}$$

Bose-Einstein/Fermi-Dirac distribution

Ideal BEC

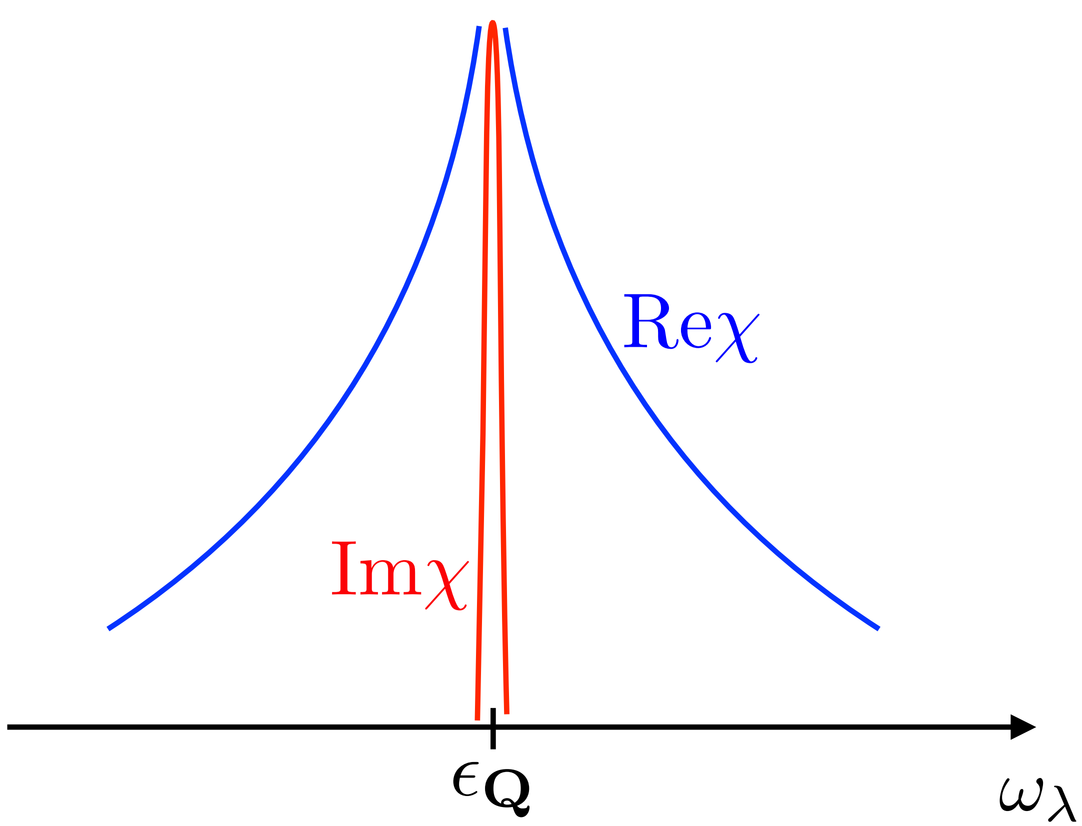
$$n_\mathbf{k} = \delta_{\mathbf{k},0}$$

$$\chi(\omega_\lambda, \mathbf{Q}) = \frac{g_\Omega g_\lambda}{\Delta_\mathrm{a}} \frac{2N\epsilon_\mathbf{Q}}{\omega_\lambda^2 - \epsilon_\mathbf{Q}^2 + 2i\omega_\lambda 0^+}$$

Divergence of both coherent and incoherent susceptibility

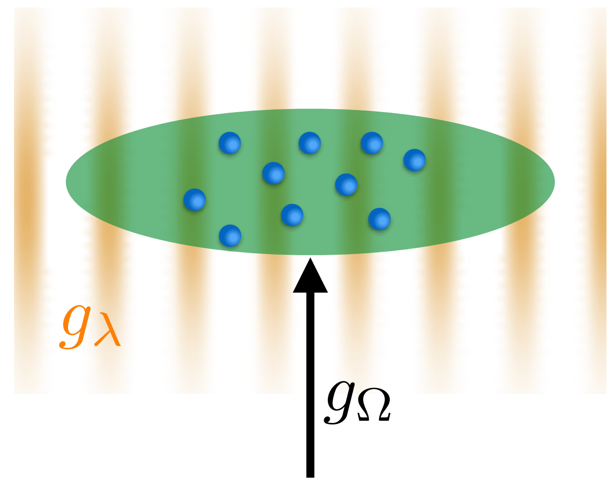
At the recoil energy

Rounded off by interactions



Susceptibility of Quantum Matter - Fermions

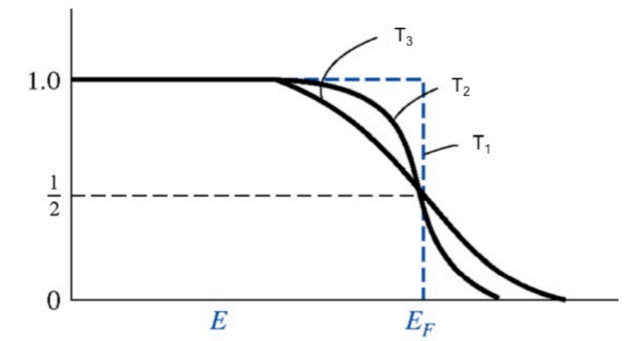
Cloud of laser-driven atoms



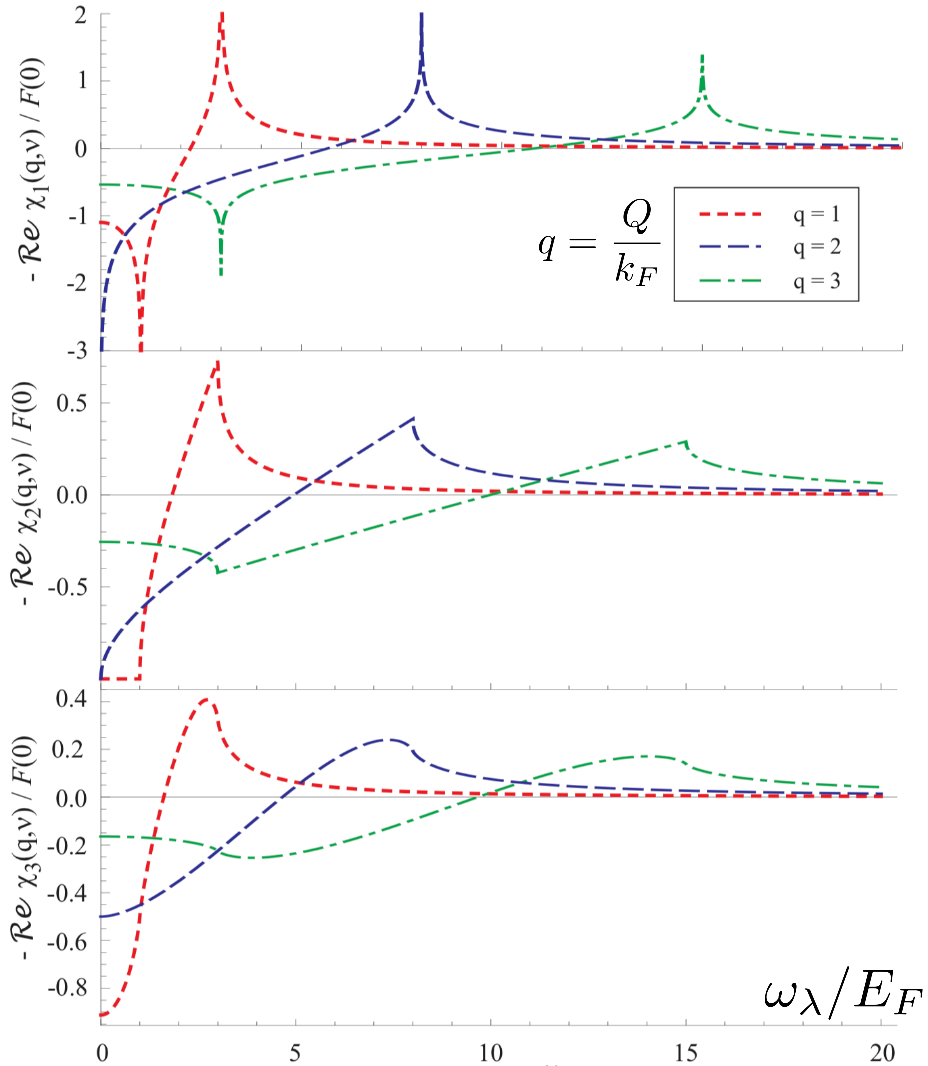
$$\chi(\omega_\lambda, \mathbf{Q}) = \frac{g_\Omega g_\lambda}{\Delta_a} \sum_{\mathbf{k}} \frac{n_{\mathbf{k}} - n_{\mathbf{k}+\mathbf{Q}}}{\omega_\lambda - \epsilon_{\mathbf{k}+\mathbf{Q}} + \epsilon_{\mathbf{k}} + i0^+}$$

Fermi-Dirac distribution

$$n_{\mathbf{k}} = \frac{1}{e^{(\epsilon_{\mathbf{k}} - \mu)/k_B T} + 1}$$



Real Part

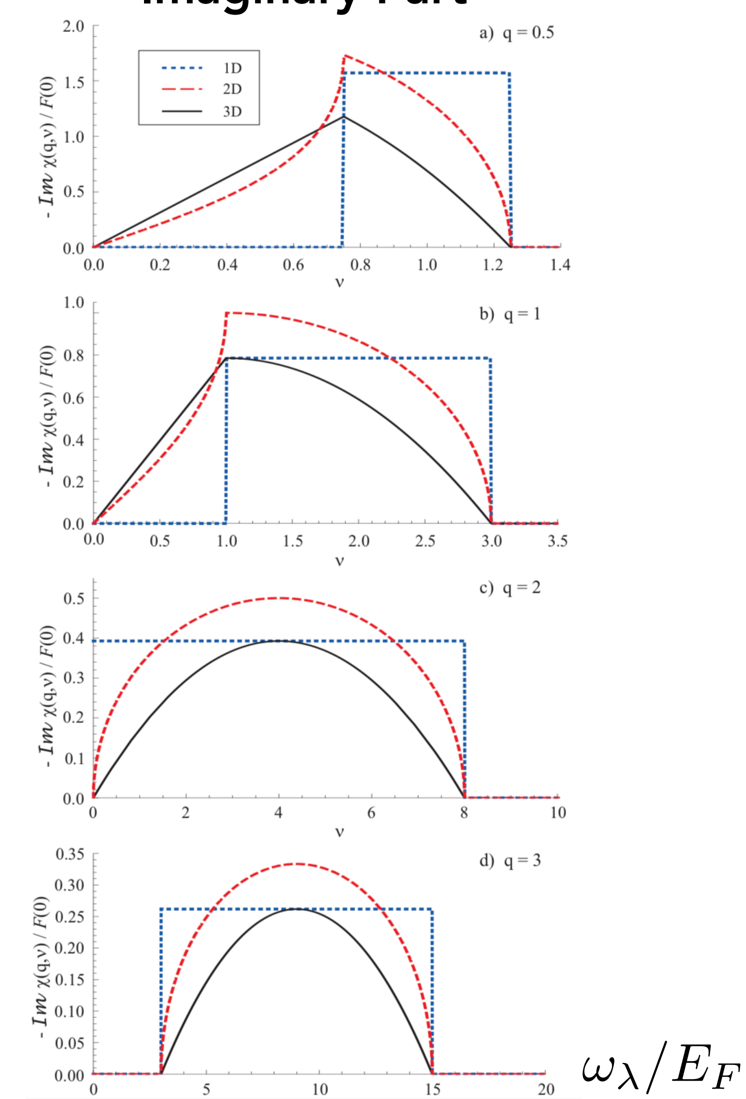


Crucial role of dimensionality

Crucial role of density (E_F)

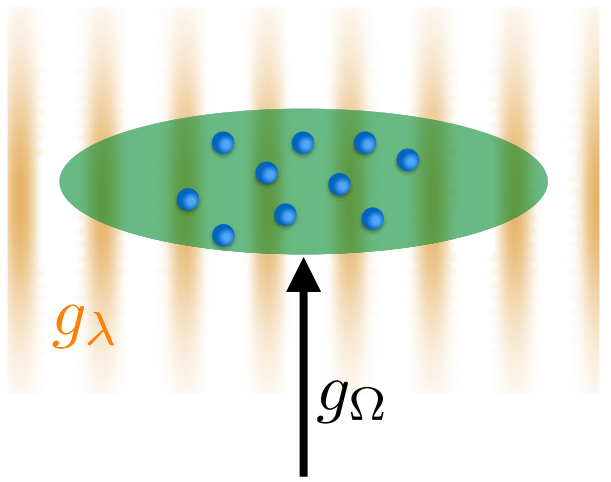
Absorption-less windows

Imaginary Part



Susceptibility of Quantum Matter - Fermions

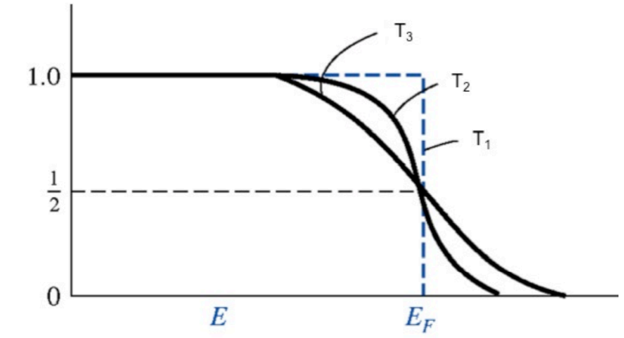
Cloud of laser-driven atoms



$$\chi(\omega_\lambda, \mathbf{Q}) = \frac{g_\Omega g_\lambda}{\Delta_\mathrm{a}} \sum_{\mathbf{k}} \frac{n_{\mathbf{k}} - n_{\mathbf{k}+\mathbf{Q}}}{\omega_\lambda - \epsilon_{\mathbf{k}+\mathbf{Q}} + \epsilon_{\mathbf{k}} + i0^+}$$

Fermi-Dirac distribution

$$n_{\mathbf{k}} = \frac{1}{e^{(\epsilon_{\mathbf{k}} - \mu)/k_B T} + 1}$$



Example in d=1

Divergent dispersion

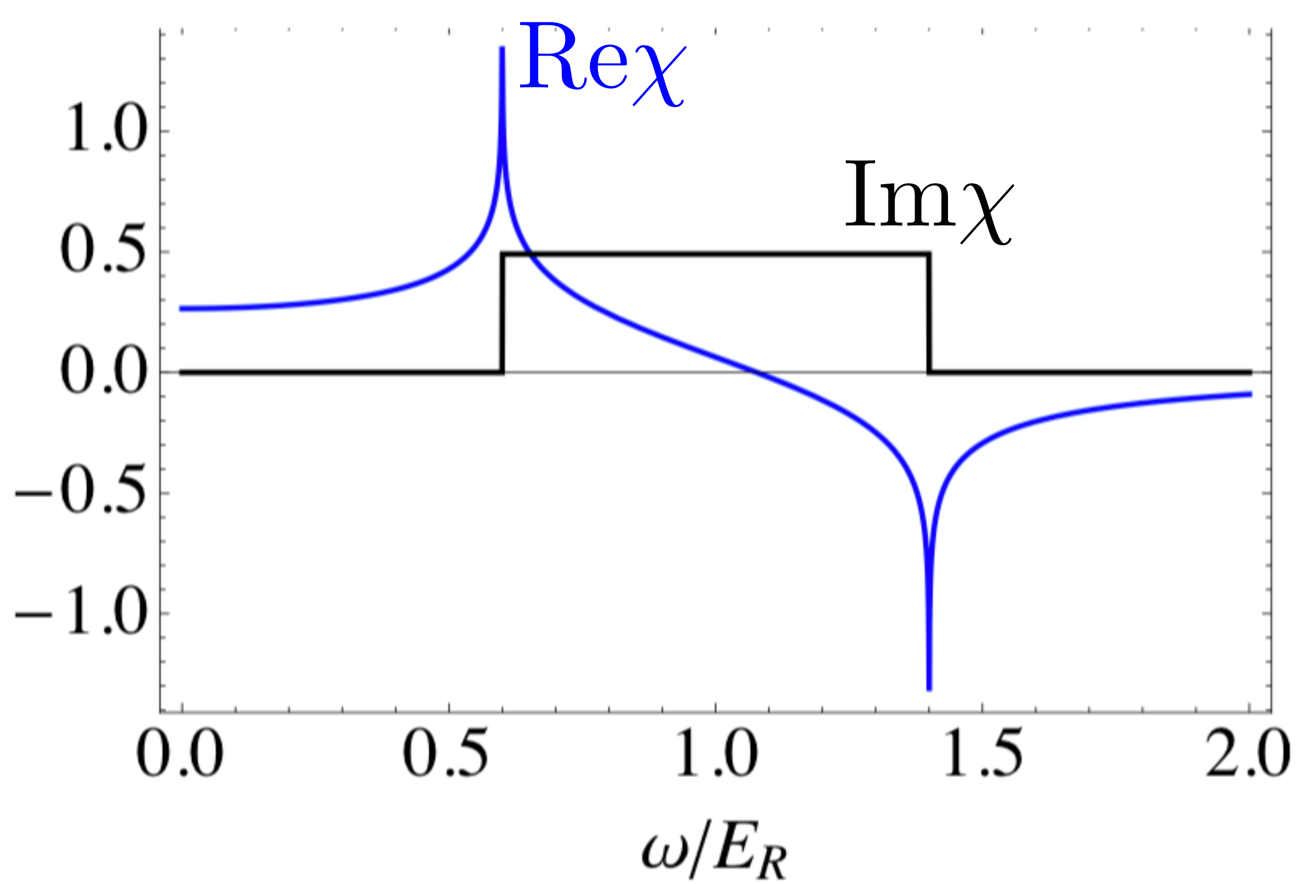
Without absorption

Cfr. BEC-case

+

fermions can be effectively non-interacting

Potential for extreme single-photon nonlinearity



4. Many-body physics with quantum atom-photon plasmas

Quantum plasma of photons and neutral atoms

	Ultracold Atoms	Photons	Plasma
Boundary Conditions	Isolated	Driven Dissipative	Driven+Dissipative with atom number cons.
Interactions	Short range	X	Long range, Retarded, Non-conservative
Tuneability	Trapping, Inter. strength	Boundary conditions	Inter. shape (time&space), Boundary conditions

Unusual combination of complex features:

Wealth of unexplored phenomena

Optimal **methods need to be developed** combining :

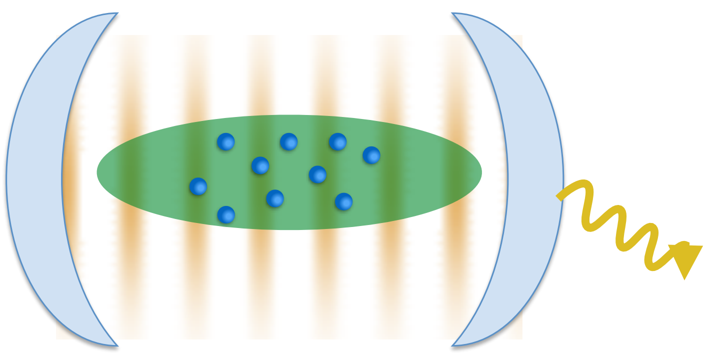
quantum field theory, quantum optics, non-equilibrium open system theory

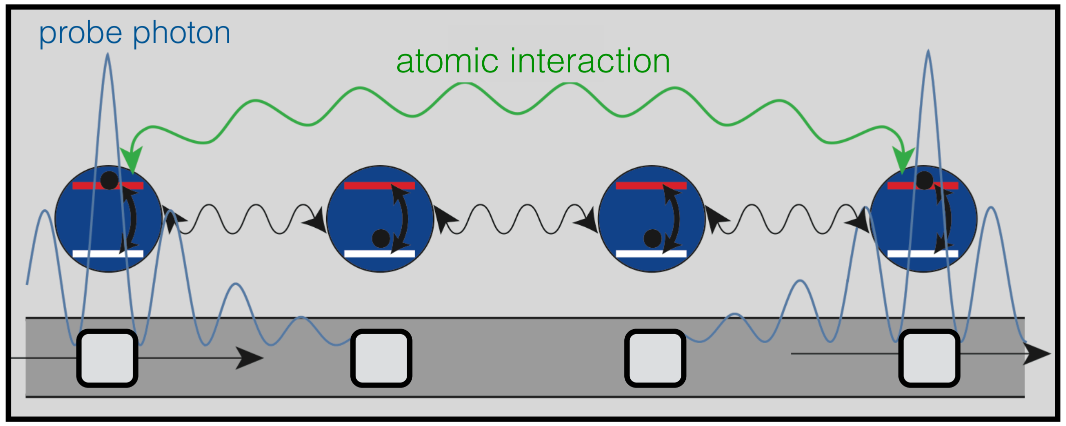
Many-body nonlinear quantum optics

Two perspectives

Quantum many-body phases of matter
with light-mediated interactions

Quantum many-body phases of light
with matter-mediated interactions





Particles/spins coupled to few optical modes

- Particle number conserved
- Dissipative interactions mediated by photons

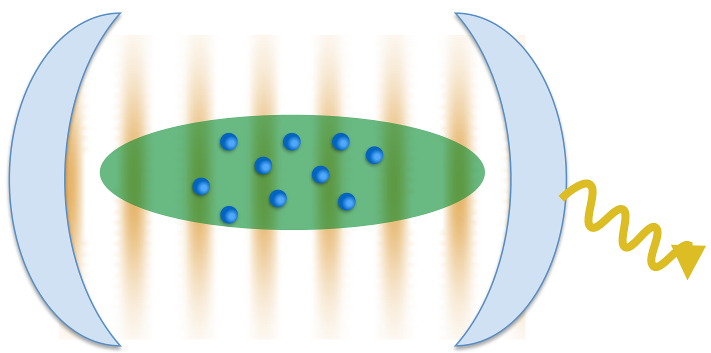
Propagating photons in a medium of dipoles

- Particle number and energy not conserved
- Interactions inherited from matter

Many-body nonlinear quantum optics

Two perspectives

Quantum many-body phases of matter
with light-mediated interactions

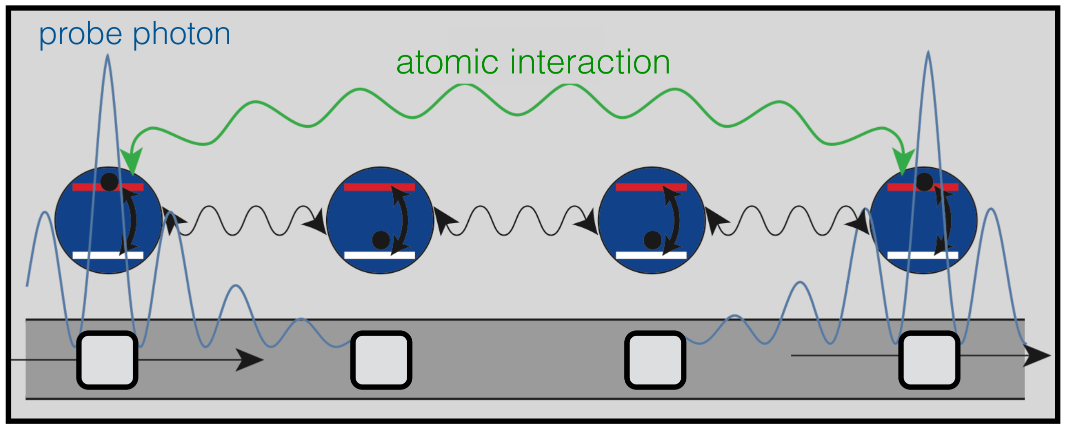


Particles/spins coupled to few optical modes

- Particle number conserved
- Dissipative interactions mediated by photons

THIS LECTURE
Matter made of cold atoms

Quantum many-body phases of light
with matter-mediated interactions



Propagating photons in a medium of dipoles

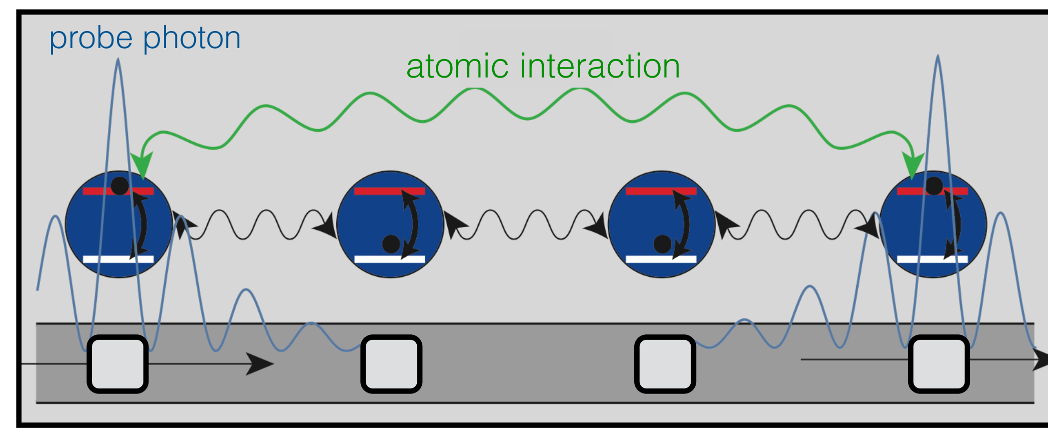
- Particle number and energy not conserved
- Interactions inherited from matter

Non-equilibrium diagrammatic approach to strongly interacting photons

Attempt at formulating
"a QED for optically dense media"

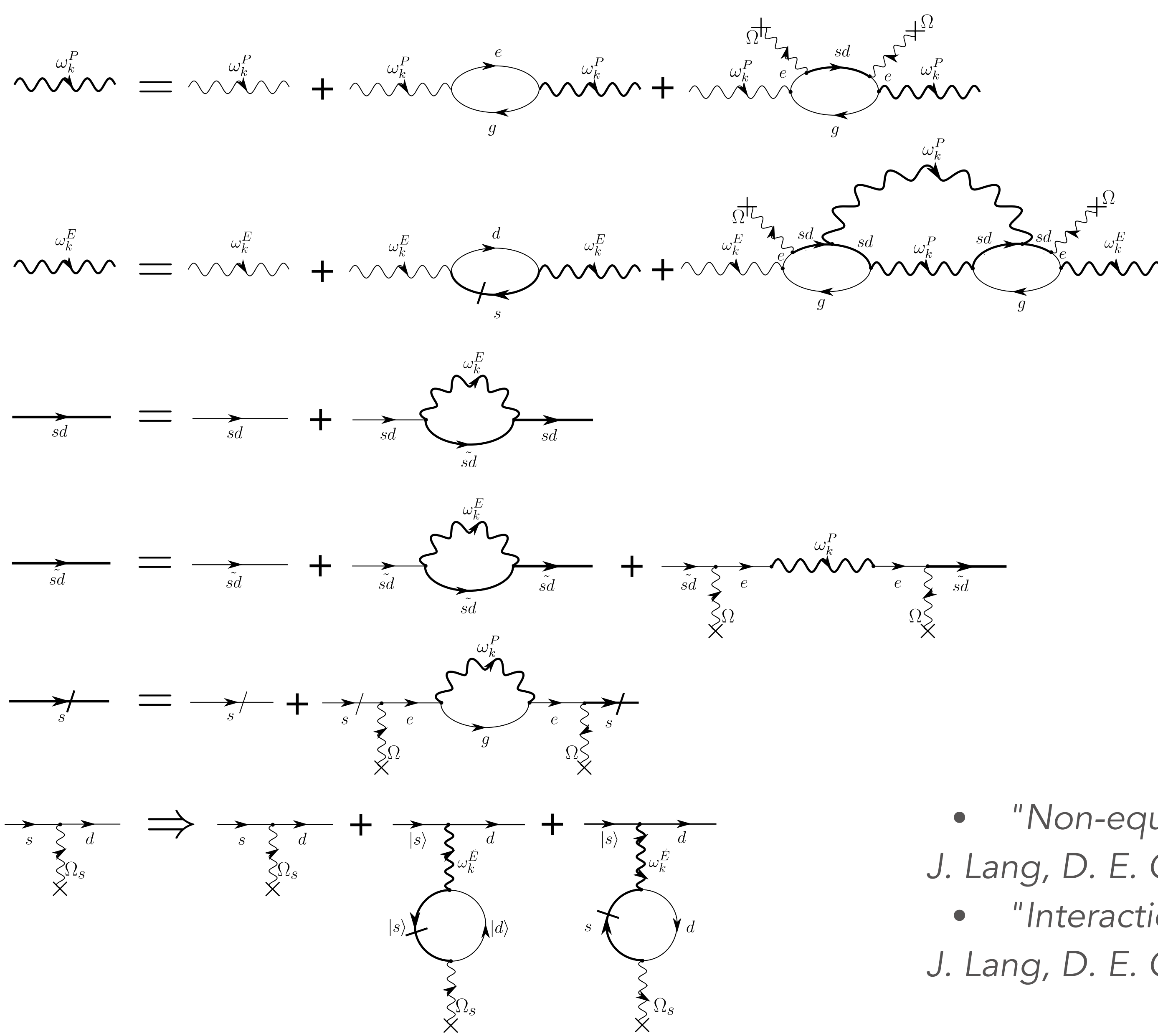
- no charges but static dipoles
- non-relativistic
- non perturbative regime

Quantum many-body phases of light
with matter-mediated interactions



Moving photons, medium of dipoles

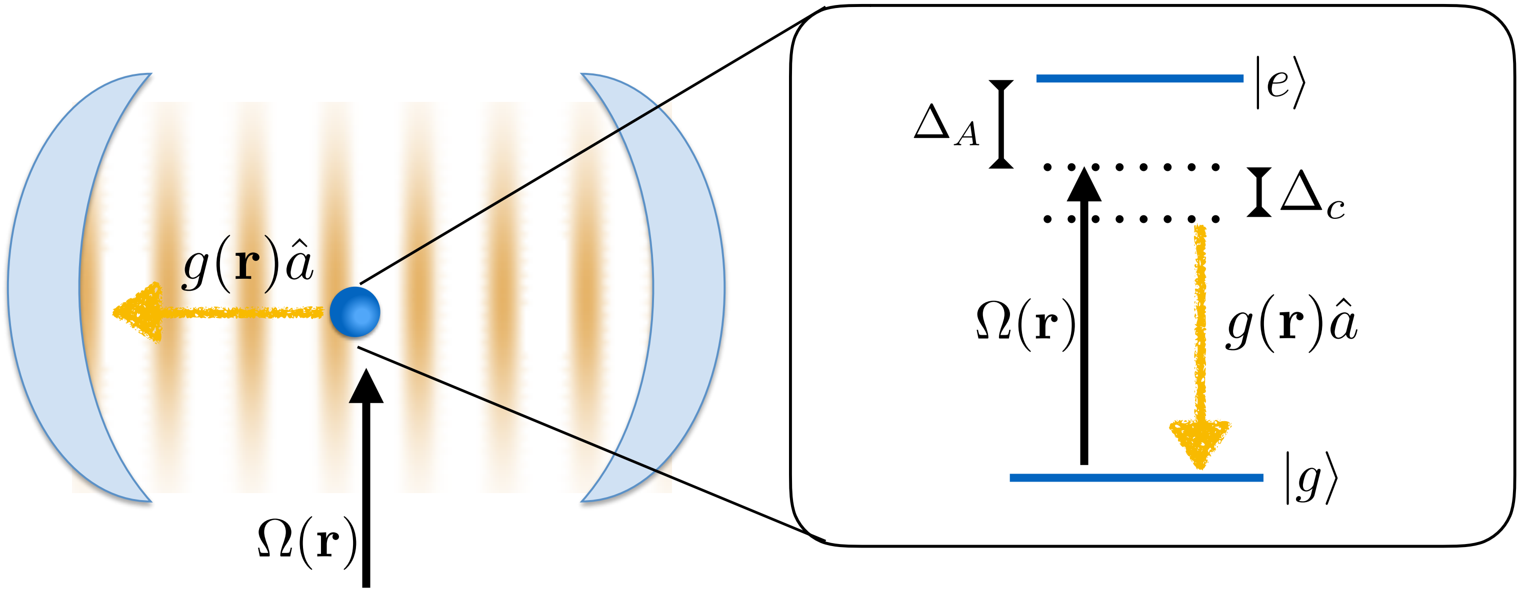
- Particle number and energy not conserved
- Interactions inherited from matter



- *"Non-equilibrium diagrammatic approach to strongly interacting photons"* *J. Lang, D. E. Chang, FP, arXiv:1810.12921 (2018)*
- *"Interaction-induced transparency for strong-coupling polaritons"* *J. Lang, D. E. Chang, FP, arXiv:1810.12912 (2018)*

4a. Superradiant crystals and magnets

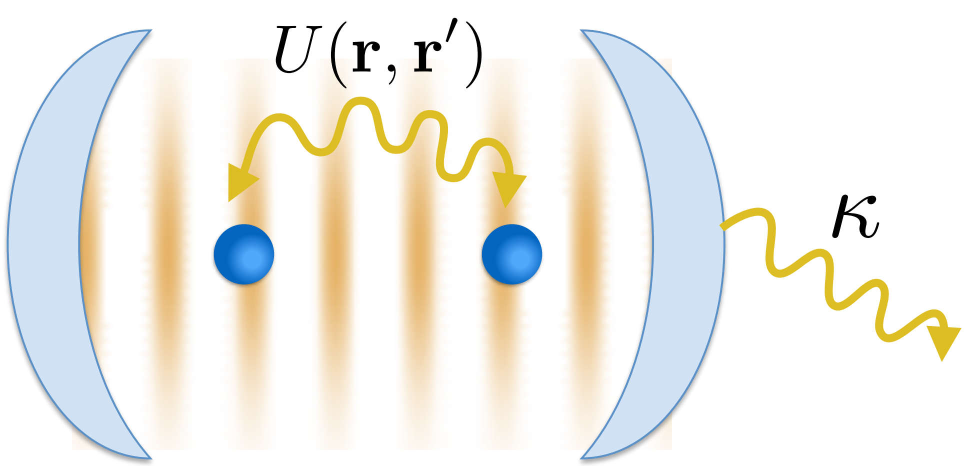
Nature of the interactions between driven atoms and confined photons



Coupling via two-photon transition:

$$\hat{H}_{\text{ca}} = \int_{\mathbf{r}} \frac{\Omega^*(\mathbf{r})g(\mathbf{r})}{\Delta_A}\hat{a}\hat{\psi}_g^\dagger(\mathbf{r})\hat{\psi}_g(\mathbf{r}) + \text{h.c.}$$

Photon-mediated interactions



Photon losses: $\mathcal{L}\hat{\rho} = \kappa\left(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}\right)$

Instantaneous density-density interaction

$$\hat{H}_{\text{int}} = \int_{\mathbf{r},\mathbf{r}'} U(\mathbf{r},\mathbf{r}')\hat{\psi}_g^\dagger(\mathbf{r})\hat{\psi}_g(\mathbf{r})\hat{\psi}_g^\dagger(\mathbf{r}')\hat{\psi}_g(\mathbf{r}')$$

$$U(\mathbf{r},\mathbf{r}') = -\frac{\Omega^*(\mathbf{r})g(\mathbf{r})\Omega(\mathbf{r}')g^*(\mathbf{r}')}{\Delta_A^2}\frac{|\Delta_c|}{\Delta_c^2+\kappa^2}$$

Interaction potential

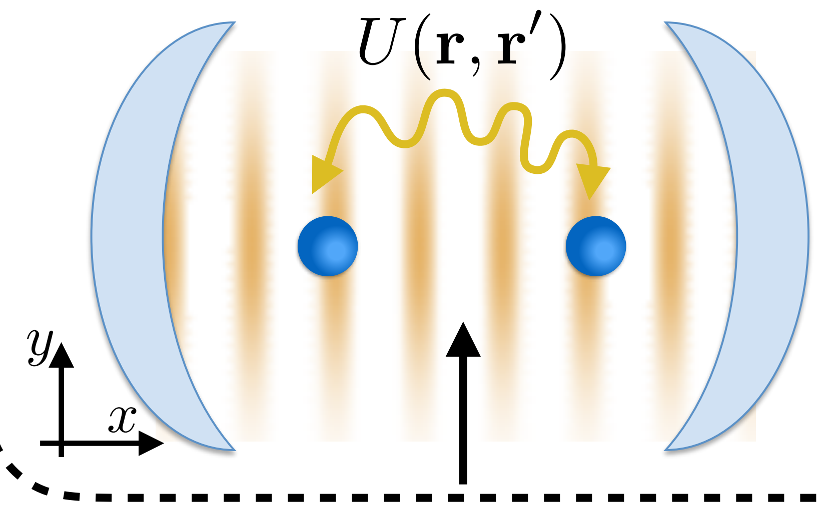
Shaping the photon mediated interactions

General case

A whole set of electromagnetic modes is available

$$U(\mathbf{r},\mathbf{r}') = -\sum_{\alpha} \frac{\Omega^*(\mathbf{r})\Omega(\mathbf{r}')g^*_{\alpha}(\mathbf{r}')g_{\alpha}(\mathbf{r})}{\Delta_A^2} \frac{|\Delta_\alpha|}{\Delta_\alpha^2+\kappa_\alpha^2}$$

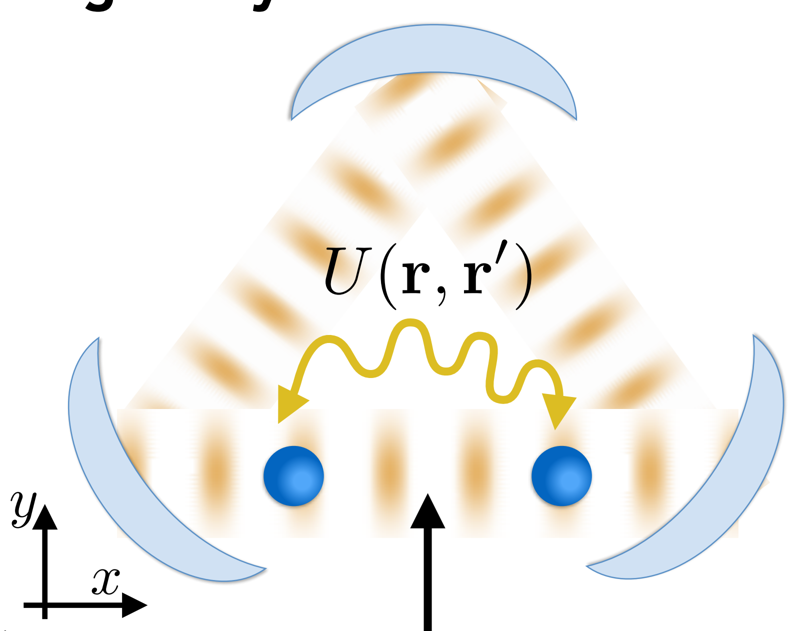
Near-planar cavity



$$U(\mathbf{r},\mathbf{r}') \propto -\cos(k_0 x)\cos(k_0 x')\cos(k_0 y)\cos(k_0 y')$$

Translation invariance is discrete: Z_2 even-odd sites of chequerboard

Ring cavity



$$U(\mathbf{r},\mathbf{r}') \propto -\cos(k_0(x-x'))\cos(k_0 y)\cos(k_0 y')$$

Translation invariance is continuous along x

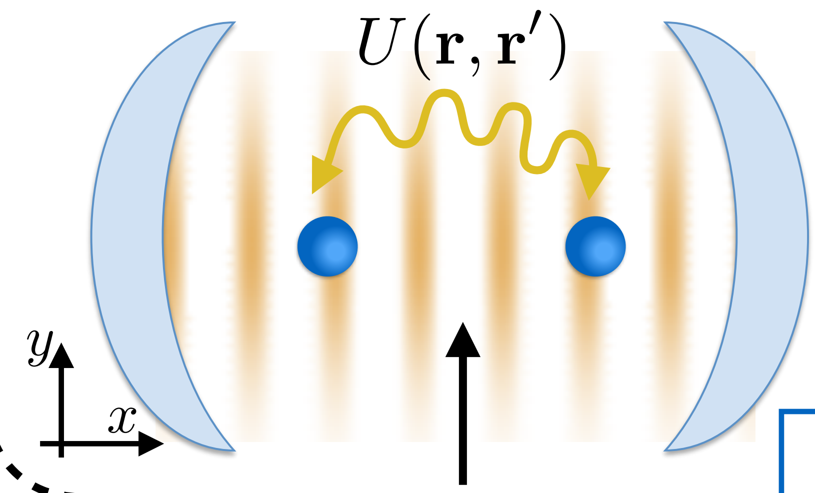
Shaping the photon mediated interactions

General case

A whole set of electromagnetic modes is available

$$U(\mathbf{r},\mathbf{r}') = -\sum_\alpha \frac{\Omega^*(\mathbf{r})\Omega(\mathbf{r}')g^*_\alpha(\mathbf{r}')g_\alpha(\mathbf{r})}{\Delta_A^2}\frac{|\Delta_\alpha|}{\Delta_\alpha^2+\kappa_\alpha^2}$$

Near-planar cavity



$$U(\mathbf{r},\mathbf{r}') \propto -\cos(k_0x)\cos(k_0x')\cos(k_0y)\cos(k_0y')$$

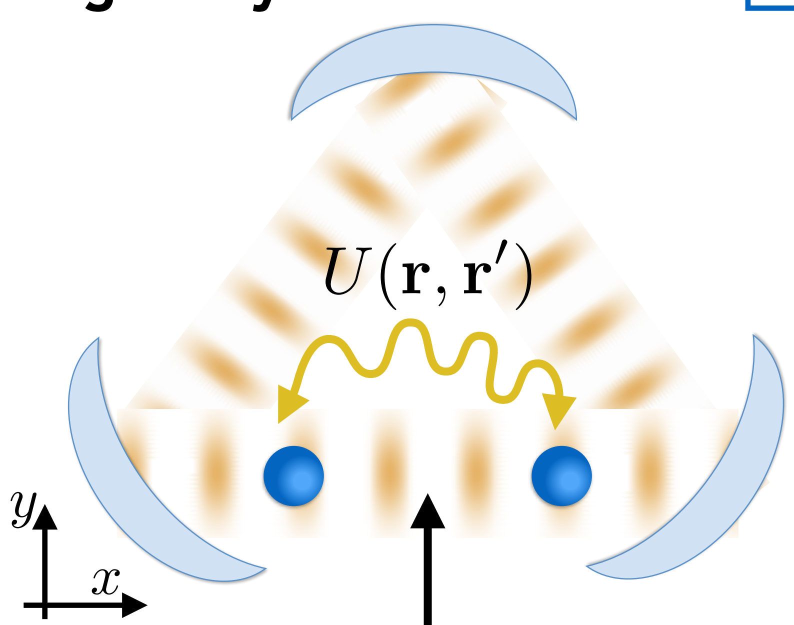
Translation invariance is discrete: Z_2 even-odd sites of chequerboard

Interesting physics:

Interactions are infinitely-long ranged and periodically sign-changing

Tendency toward crystallisation

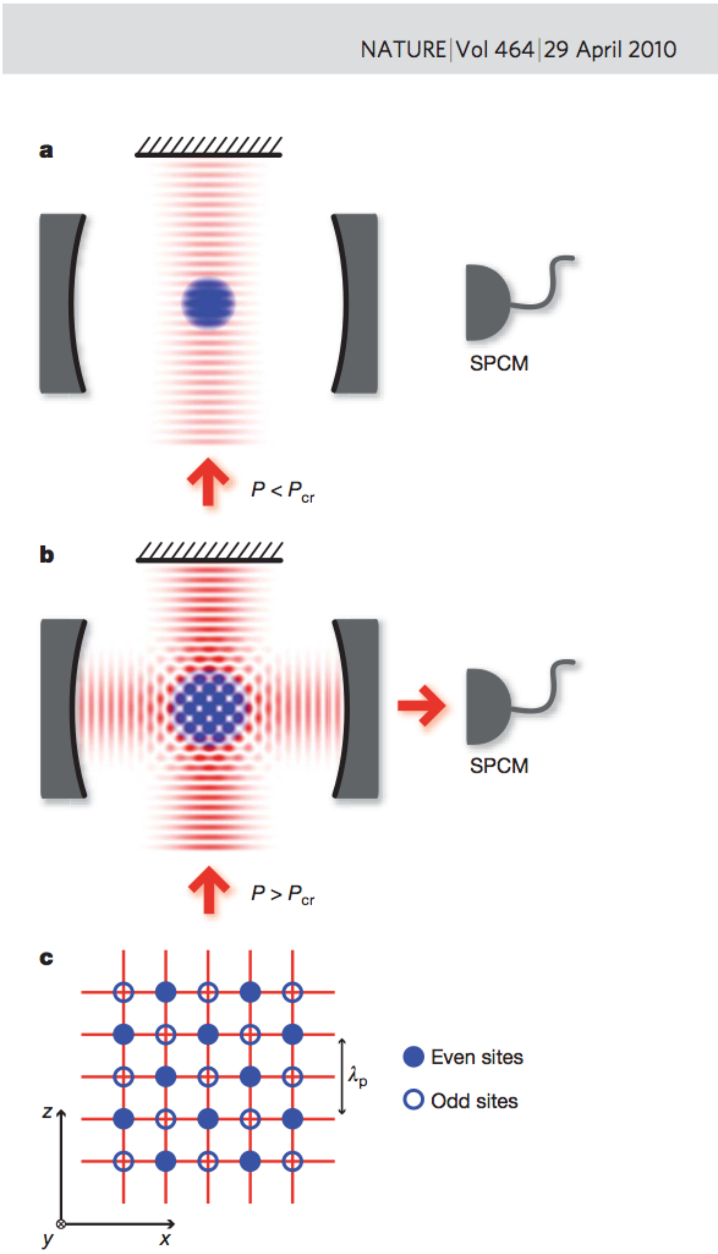
Ring cavity

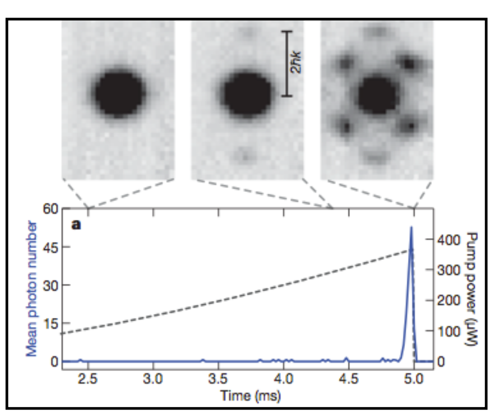


$$U(\mathbf{r},\mathbf{r}') \propto -\cos(k_0(x-x'))\cos(k_0y)\cos(k_0y')$$

Translation invariance is continuous along x

Crystallisation of quantum matter: experimental observation





- Laser-driven **Bose-Einstein condensate** inside an **near-planar cavity**

  Exp @ETH [Nature 464 (2010)], @Hamburg [PRL 113 (2014)]

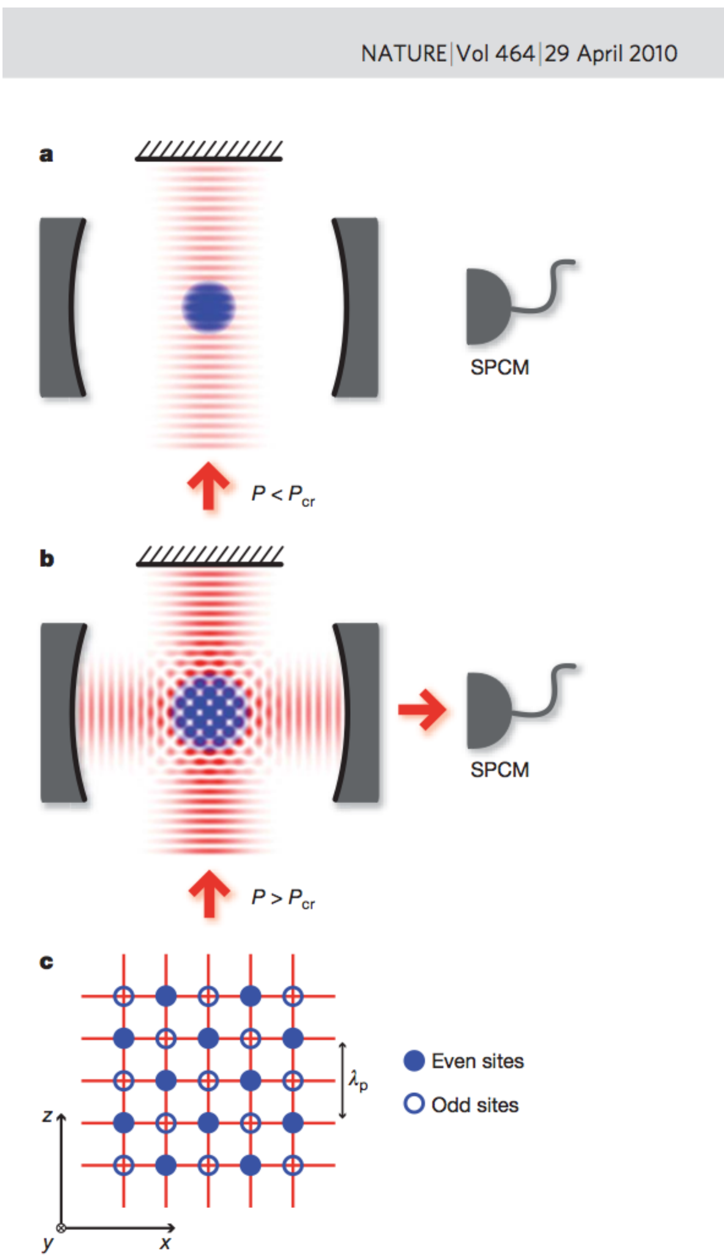
- Spatial ordering above a certain laser intensity i.e. interaction strength

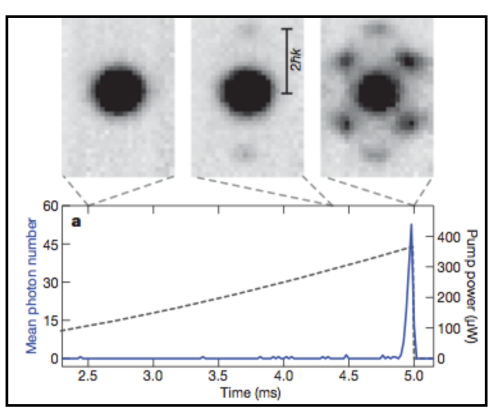
$$U(\mathbf{r},\mathbf{r}') \propto -\cos(k_0 x)\cos(k_0 x')\cos(k_0 y)\cos(k_0 y')$$

- **Simplest example of light-mediated crystallisation**

  Z_2 Translation invariance spontaneously broken:
  Choice between even-odd sites of chequerboard

Crystallisation of quantum matter: experimental observation





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$$U(\mathbf{r},\mathbf{r}') \propto -\cos(k_0 x)\cos(k_0 x')\cos(k_0 y)\cos(k_0 y')$$

- **Simplest example of light-mediated crystallisation**

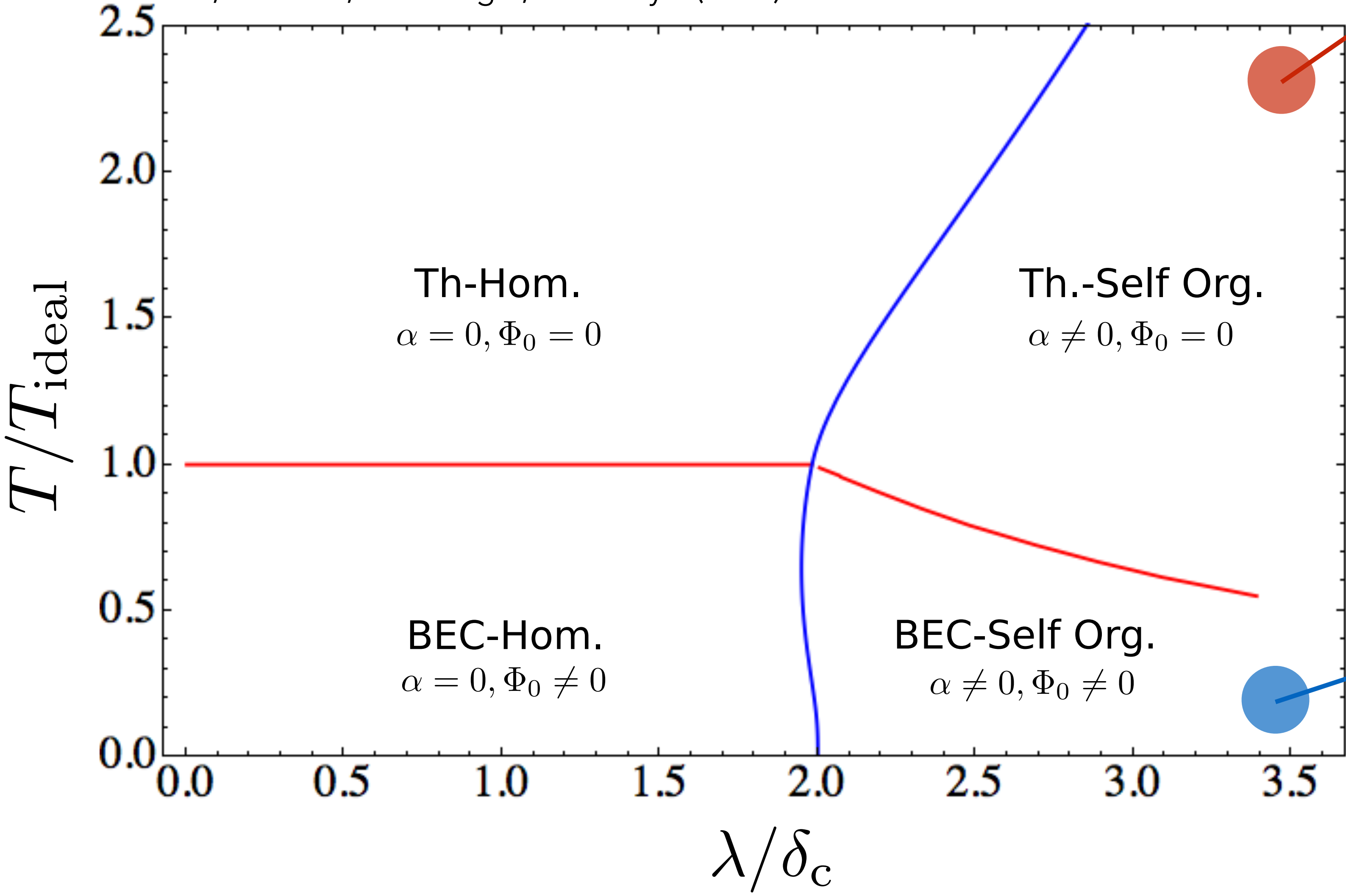
  Z_2 Translation invariance spontaneously broken:
  Choice between even-odd sites of chequerboard

Extension to continuous translation symmetry:
Provided the first unambiguous experimental realisation of a supersolid!

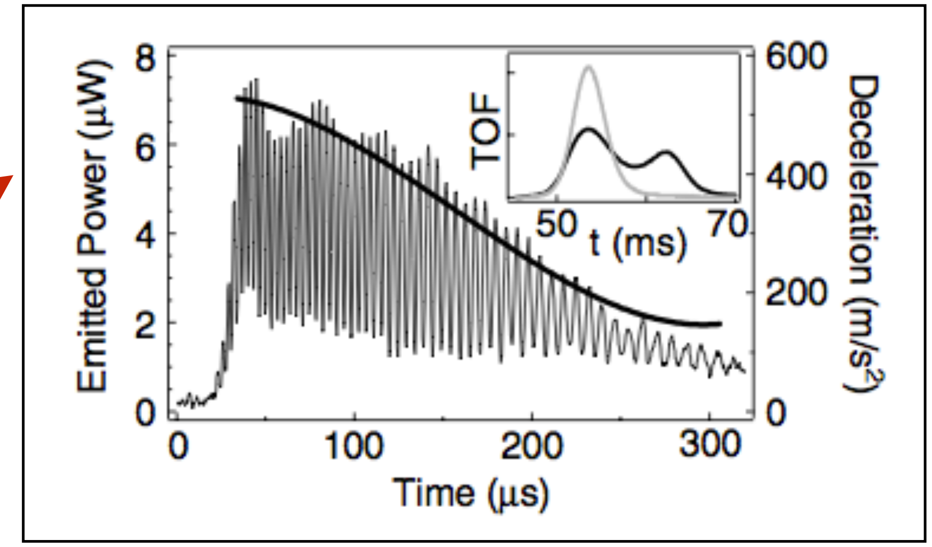
Experimental observation with bosonic atoms

Atoms: complex quantum system with **intrinsic correlations.**
Example: transition to Bose-Einstein condensation

FP, P. Strack, W. Zwerger, Ann. Phys. (2013)



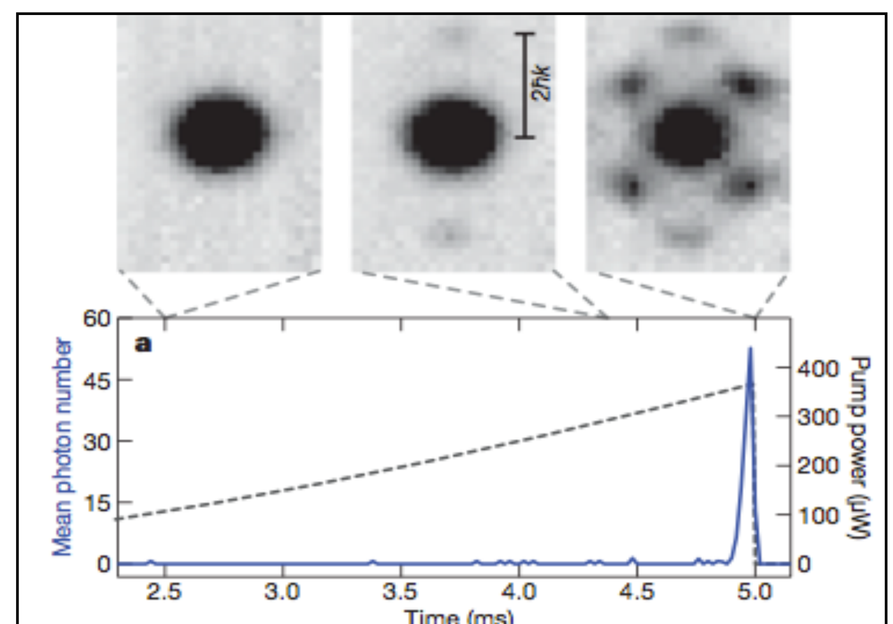
Cold Atoms



MIT [PRL 91 (2003)]

Singapore [PRL 109 (2012)]

Bose-Einstein condensate



ETH [Nature 464 (2010)]

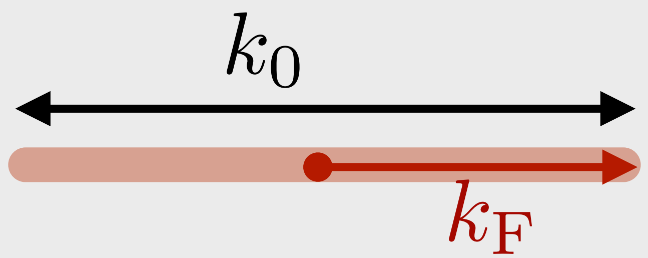
Hamburg [PRL 113 (2014)]

We studied:

Interplay between **condensation** and **self-ordering**

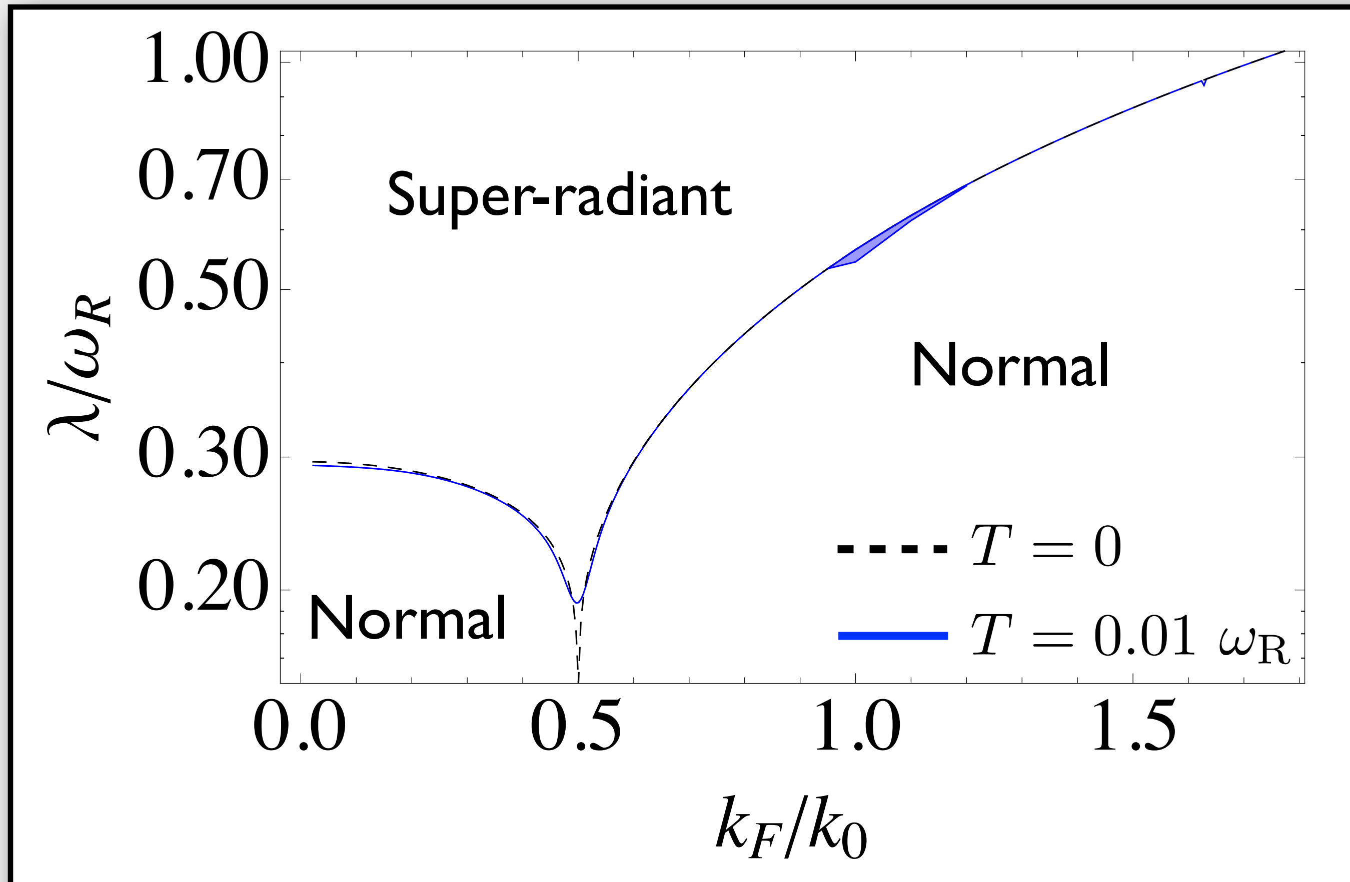
Dicke-Peierls super-radiance in 1D

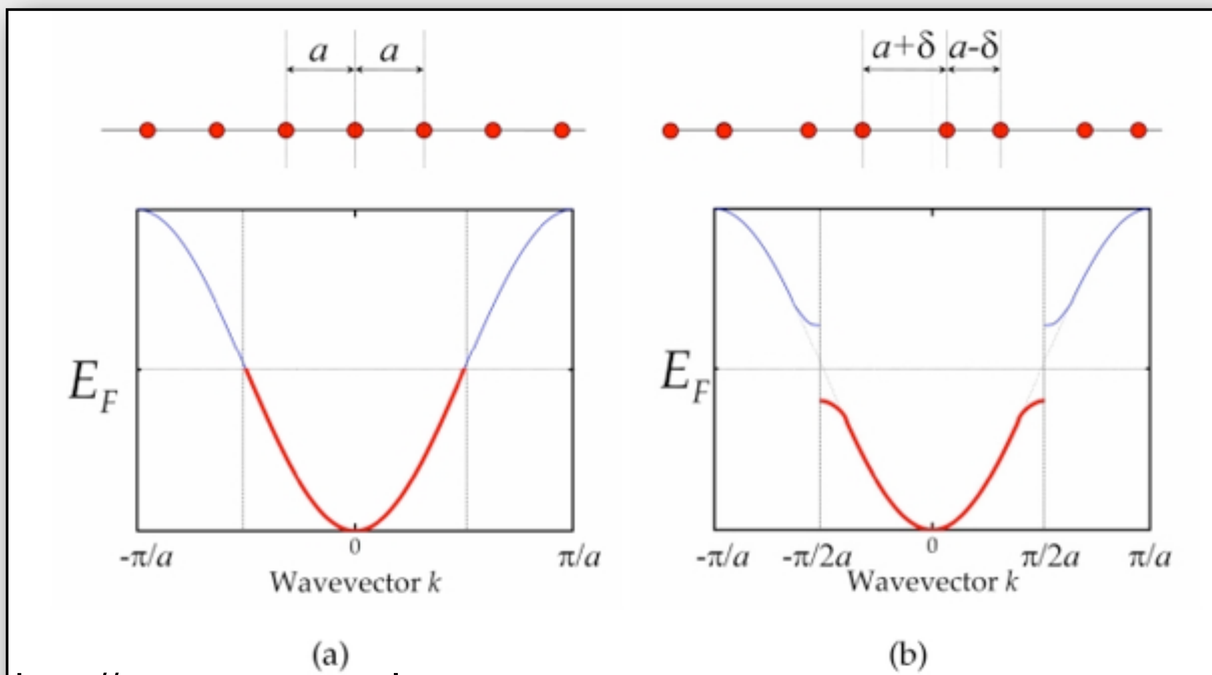
1d spinless fermions in a transv. driven cavity



At k0=2kF superradiant with infinitesimal pump

Perfect Nesting
Self-organized system is insulating



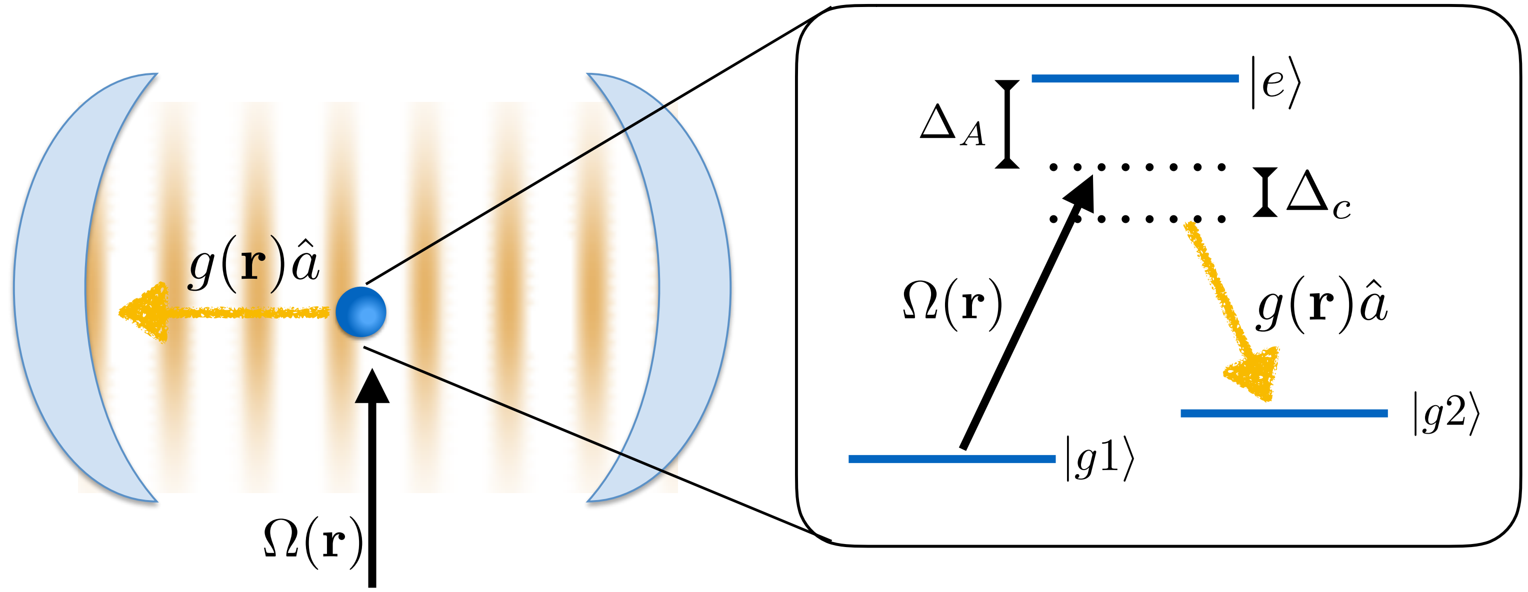


http://www.utwente.nl

Analog to Peierls instability in 1D metals

Difference:
the resulting lattice is dynamical
in contrast to the heavy ions of the metal

Magnetic models with photon-mediated interactions



Coupling via two-photon transition:

$$\hat{H}_{\text{ca}} = \int d\mathbf{r}\, \frac{\Omega^*(\mathbf{r})g(\mathbf{r})}{\Delta_A}\hat{a}\psi^\dagger_{g1}(\mathbf{r})\psi_{g2}(\mathbf{r}) + \text{h.c.}$$

Several types of spin-spin interactions [*F.Mivehvar, H.Ritsch, FP, arXiv:1809.09129*]

$$\hat{H}_{\text{spin}} = \int \Big\{ J^x_{\text{Heis}}(\mathbf{r}',\mathbf{r})\hat{s}_x(\mathbf{r}')\hat{s}_x(\mathbf{r}) + J^y_{\text{Heis}}(\mathbf{r}',\mathbf{r})\hat{s}_y(\mathbf{r}')\hat{s}_y(\mathbf{r}) + J^z_{\text{DM}}(\mathbf{r}',\mathbf{r})\left[\hat{s}_x(\mathbf{r}')\hat{s}_y(\mathbf{r}) - \hat{s}_y(\mathbf{r}')\hat{s}_x(\mathbf{r})\right]$$
$$+ J^{xy}_{\text{c}}(\mathbf{r}',\mathbf{r})\left[\hat{s}_x(\mathbf{r}')\hat{s}_y(\mathbf{r}) + \hat{s}_y(\mathbf{r}')\hat{s}_x(\mathbf{r})\right]\Big\} d\mathbf{r}d\mathbf{r}' + \int B_z(\mathbf{r})\hat{s}_z(\mathbf{r})d\mathbf{r},$$

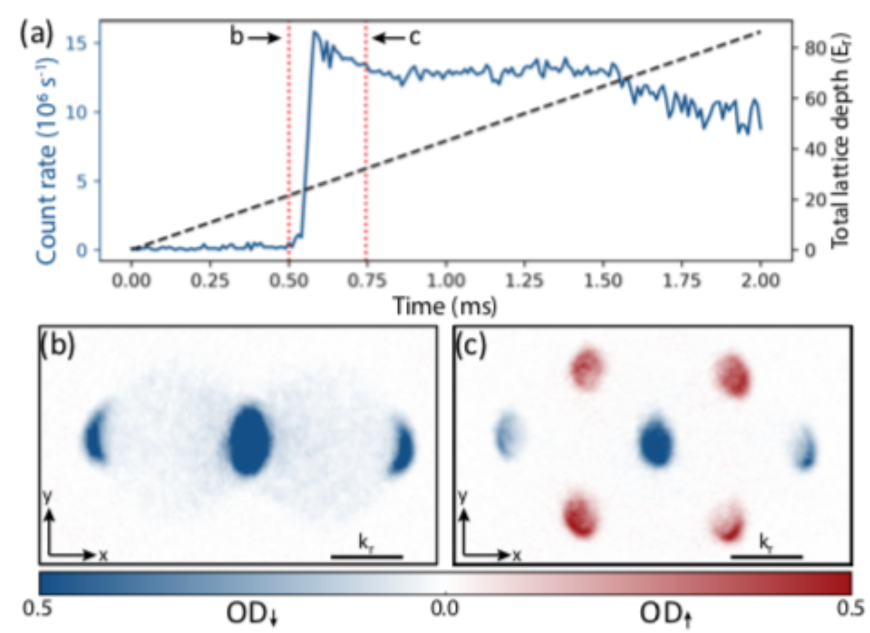
Pseudospin: $\vec{s} = \hat{\Psi}^\dagger \cdot \vec{\sigma} \cdot \hat{\Psi}$

AFM phase predicted

[*F.Mivehvar, FP, H.Ritsch PRL 119, 063602 (2017)*]

Recently observed in experiment @ Stanford

[*R.M. Kroeze, et al., PRL 121, 163601 (2018)*]



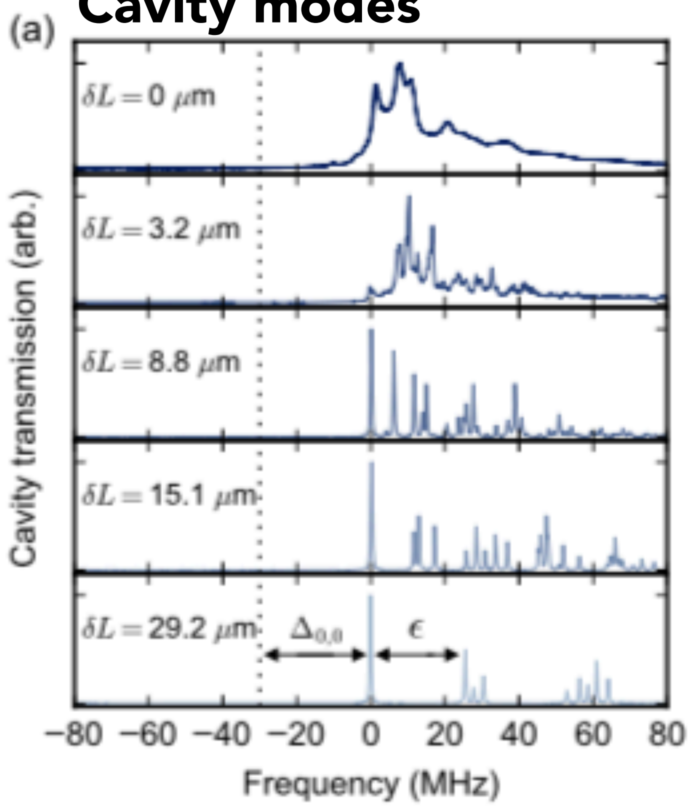
Shaping the photon mediated interactions

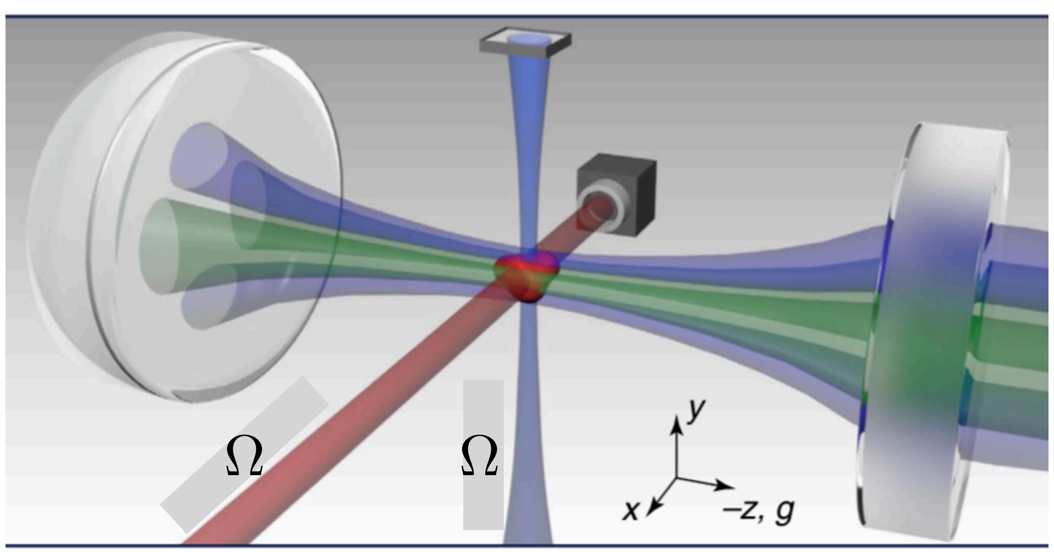
General case

A whole set of electromagnetic modes is available

$$U(\mathbf{r},\mathbf{r}') = -\sum_{\alpha} \frac{\Omega^*(\mathbf{r})\Omega(\mathbf{r}')g_\alpha^*(\mathbf{r}')g_\alpha(\mathbf{r})}{\Delta_A^2} \frac{|\Delta_\alpha|}{\Delta_\alpha^2+\kappa_\alpha^2}$$

Cavity modes





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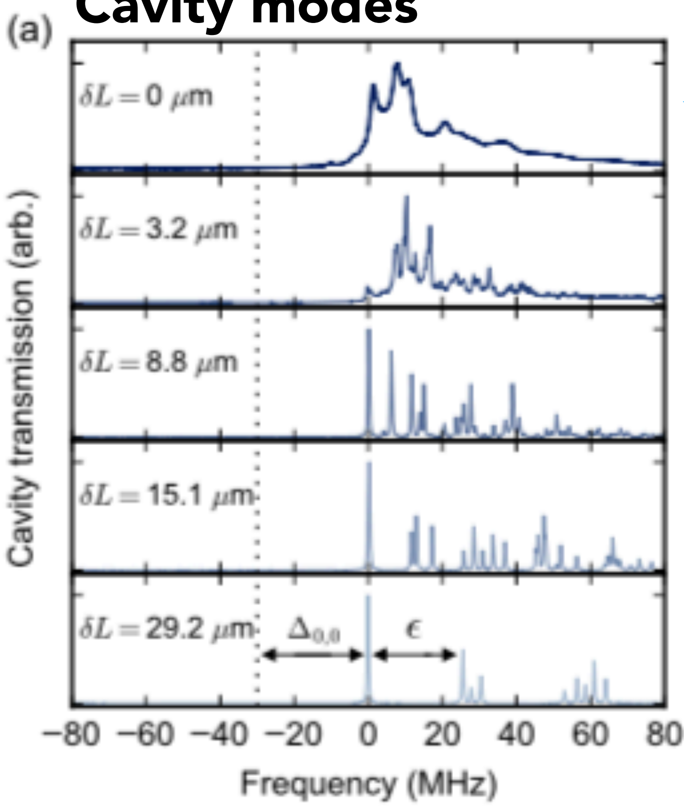
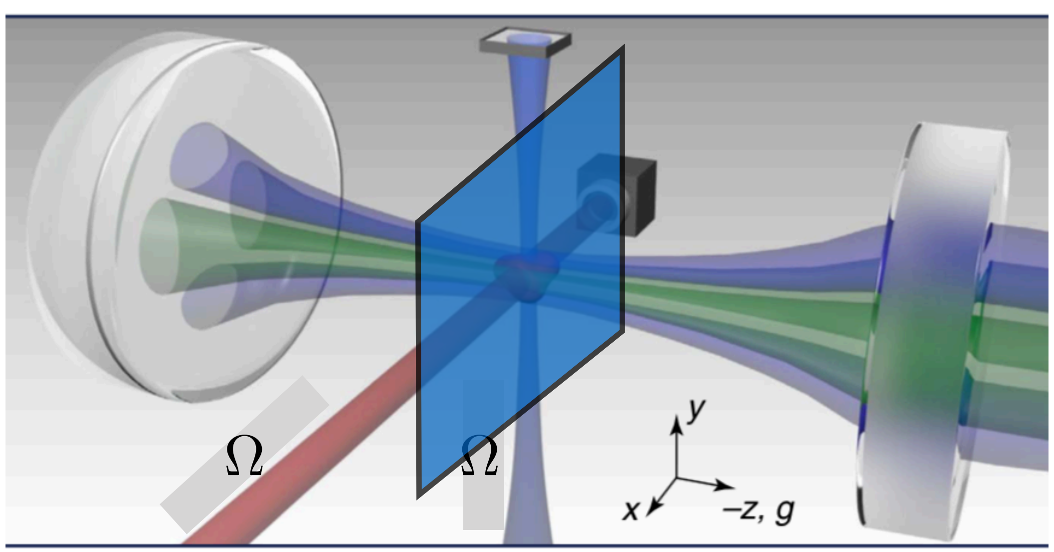
Cavity modes





Interaction potential (transverse plane)

$$U(\mathbf{x},\mathbf{x}') = -\frac{g_0^2}{\Delta_a^2}\Omega^*(\mathbf{x})\Omega(\mathbf{x}') \times \frac{1}{4\pi\tilde{\epsilon}} K_0\left(\sqrt{\frac{2}{\tilde{\epsilon}}}\left|\frac{\mathbf{x}-\mathbf{x}'}{w_0}\right|\right)$$

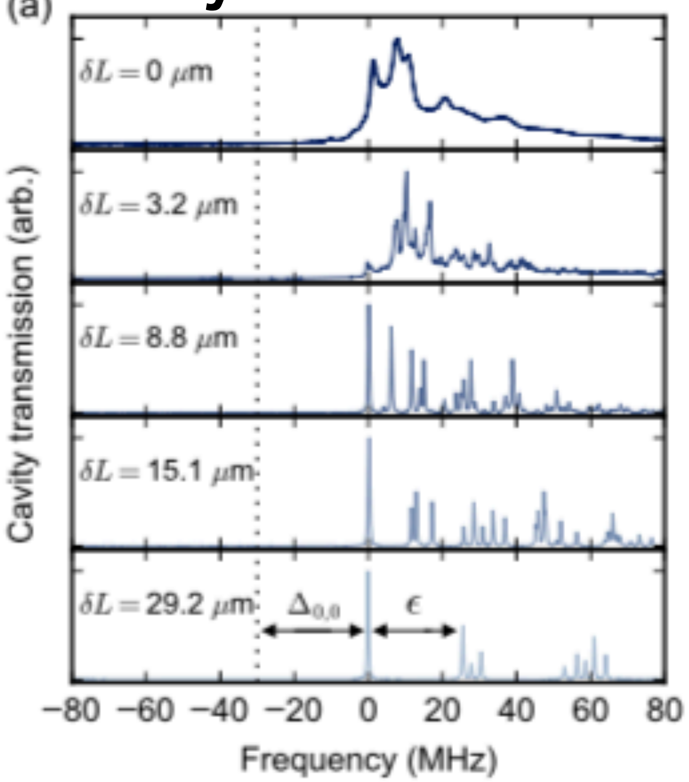
Shaping the photon mediated interactions: Confocal Cavity

General case

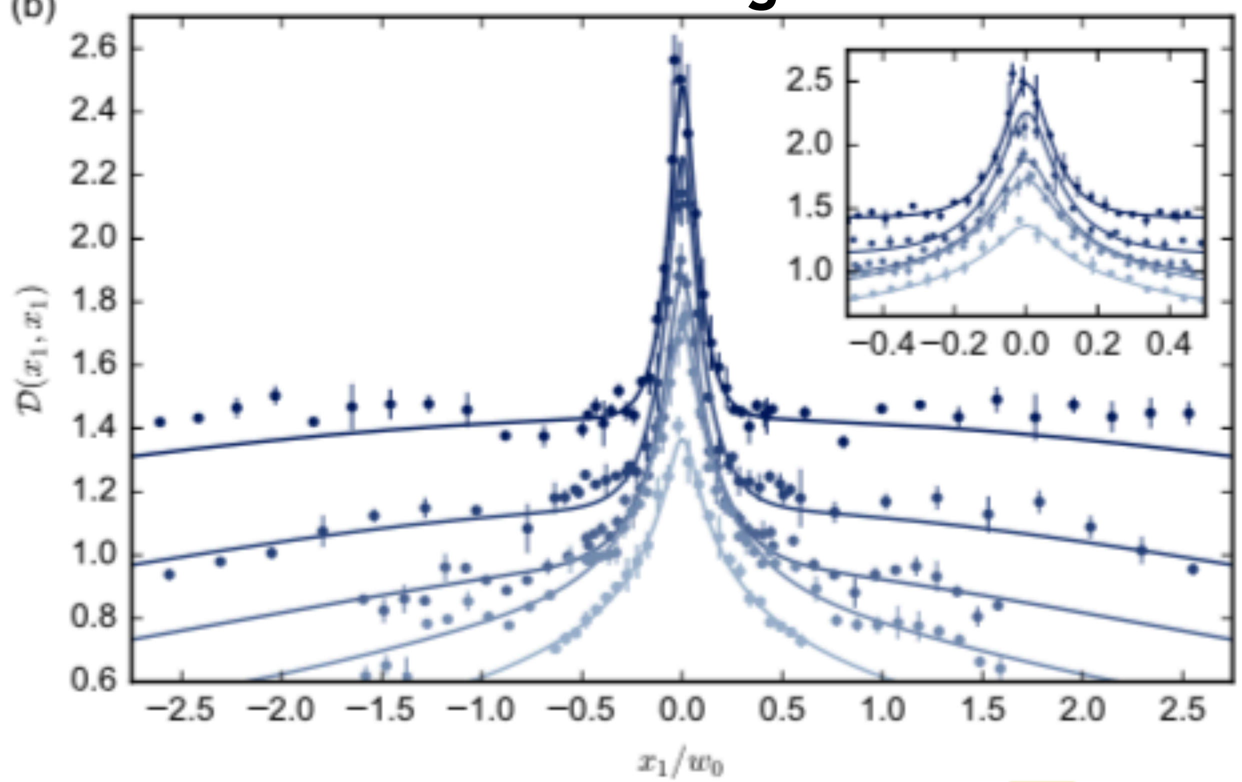
A whole set of electromagnetic modes is available

$$U(\mathbf{r},\mathbf{r}') = -\sum_{\alpha} \frac{\Omega^*(\mathbf{r})\Omega(\mathbf{r}')g_\alpha^*(\mathbf{r}')g_\alpha(\mathbf{r})}{\Delta_A^2} \frac{|\Delta_\alpha|}{\Delta_\alpha^2+\kappa_\alpha^2}$$

Cavity modes



Tuneable interaction range

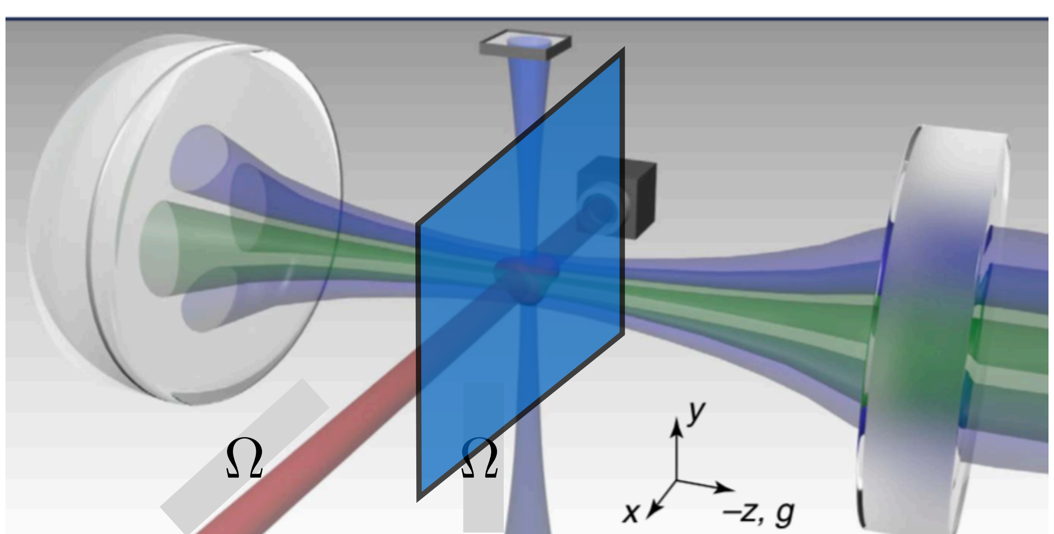


Confocal cavity @ Stanford

[V.D.Vaidya et al. PRX 8, 011002 (2018)]

Tuneable parameter

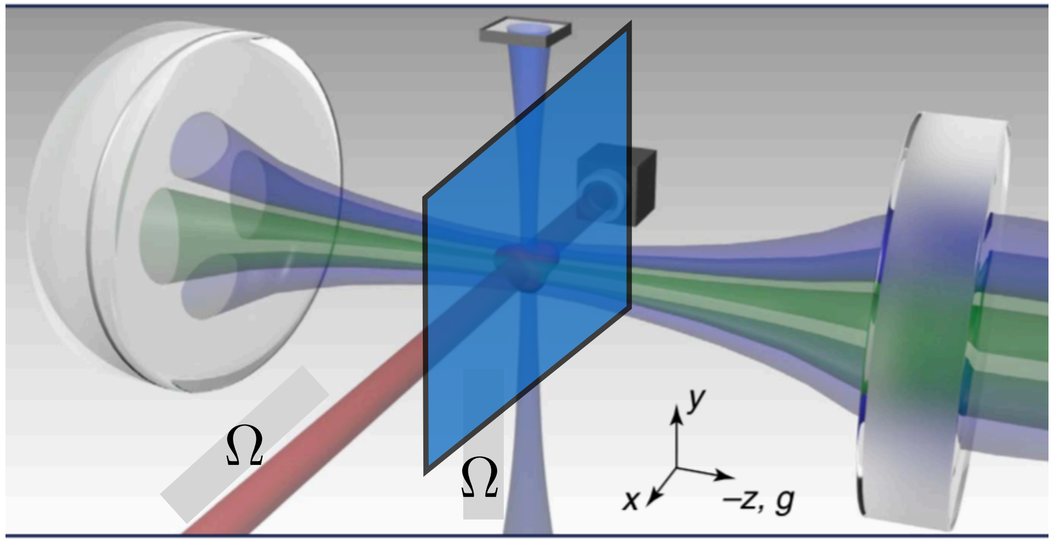
$$\tilde{\epsilon} = \epsilon/\Delta_{00}$$



Interaction potential (transverse plane)

$$U(\mathbf{x},\mathbf{x}') = -\frac{g_0^2}{\Delta_a^2}\Omega^*(\mathbf{x})\Omega(\mathbf{x}') \times \frac{1}{4\pi\tilde{\epsilon}} K_0\left(\sqrt{\frac{2}{\tilde{\epsilon}}}\left|\frac{\mathbf{x}-\mathbf{x}'}{w_0}\right|\right)$$

Finite-range cavity-mediated interactions



Example:

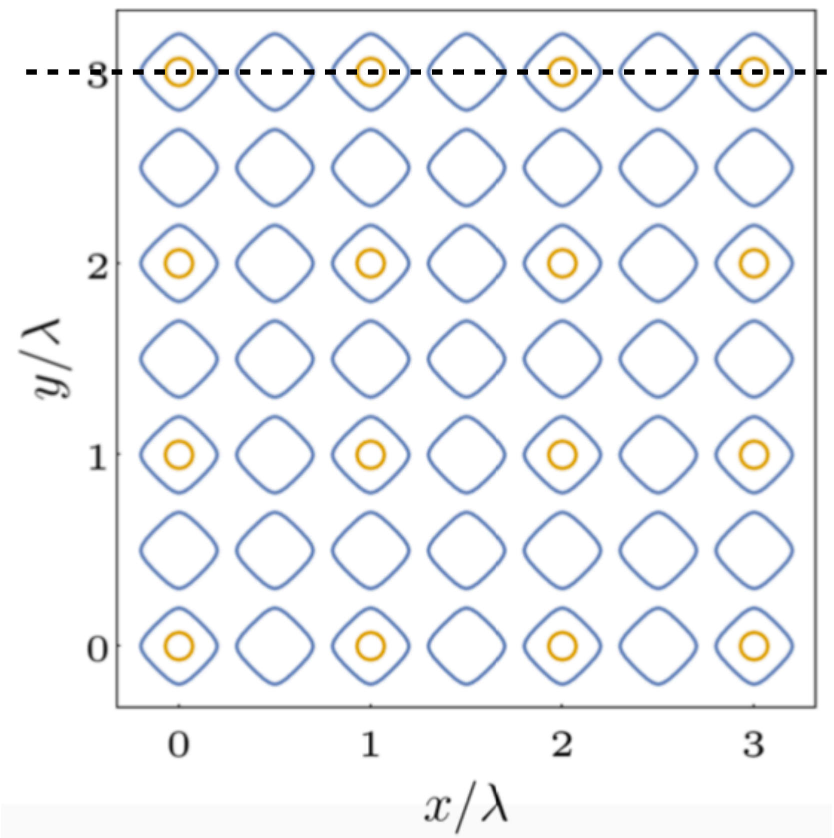
Two standing-wave pumps with orthogonal polarisation

$$\Omega(x, y) = \vec{e}_1 \cos(2\pi x/\lambda) + \vec{e}_2 \cos(2\pi y/\lambda)$$

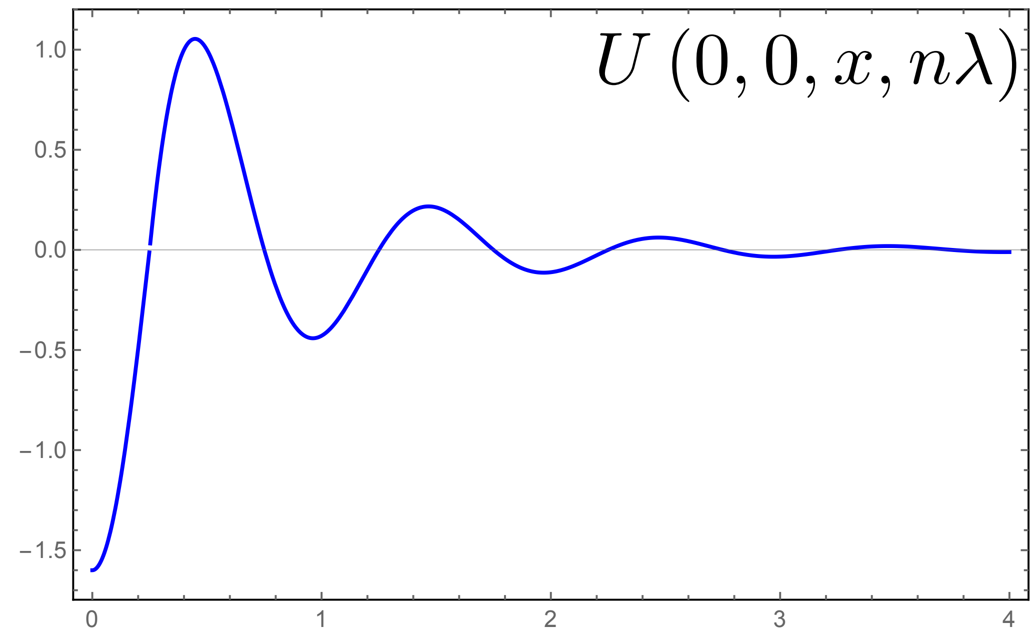
$$\vec{e}_1 \perp \vec{e}_2$$

Geometry (Z4 Symmetry)

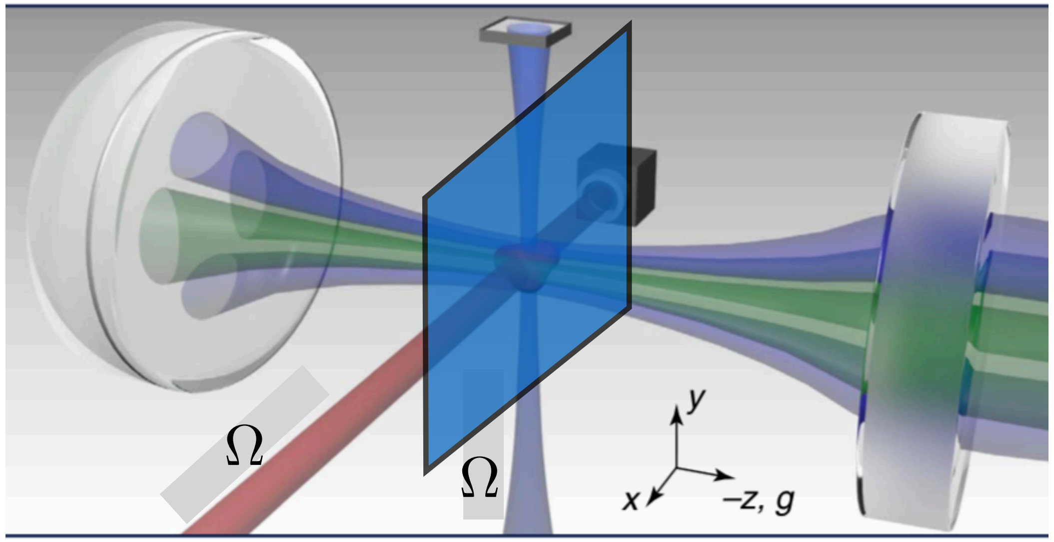
- External potential contour (Blue)
- Interaction potential minima (Yellow)



Interaction potential



Finite-range cavity-mediated interactions



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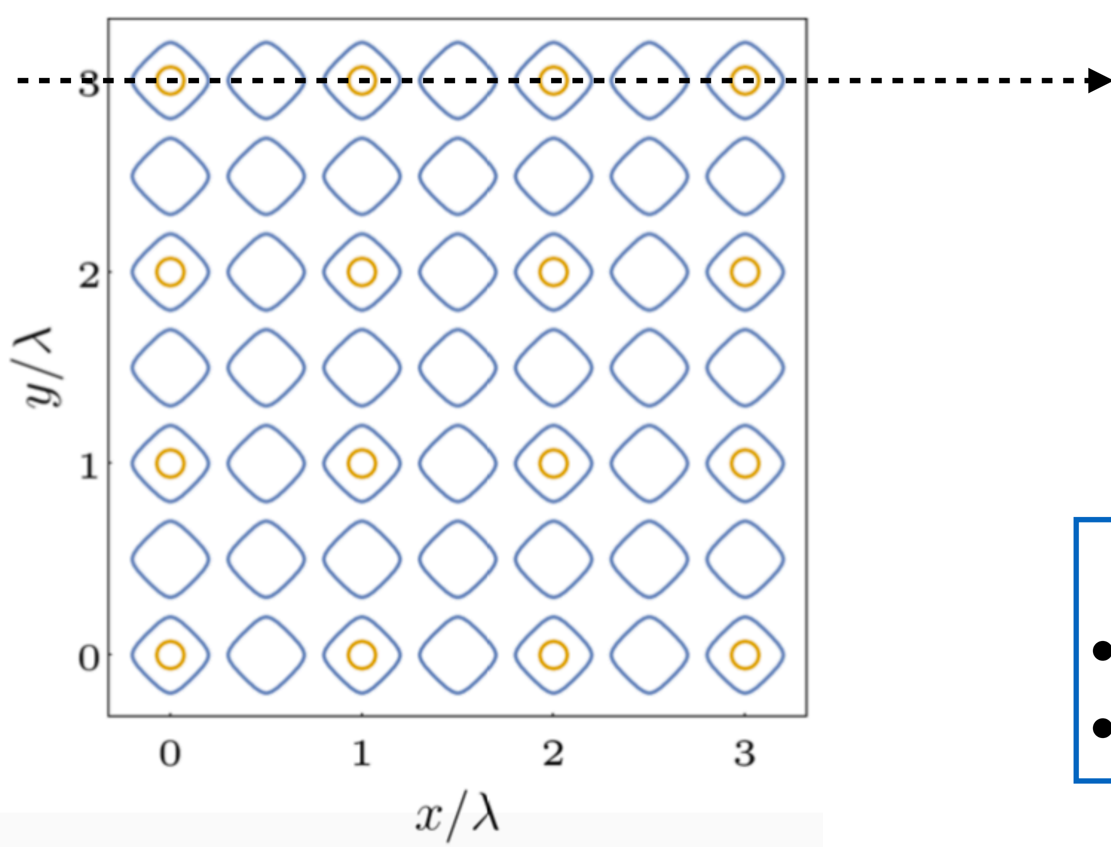
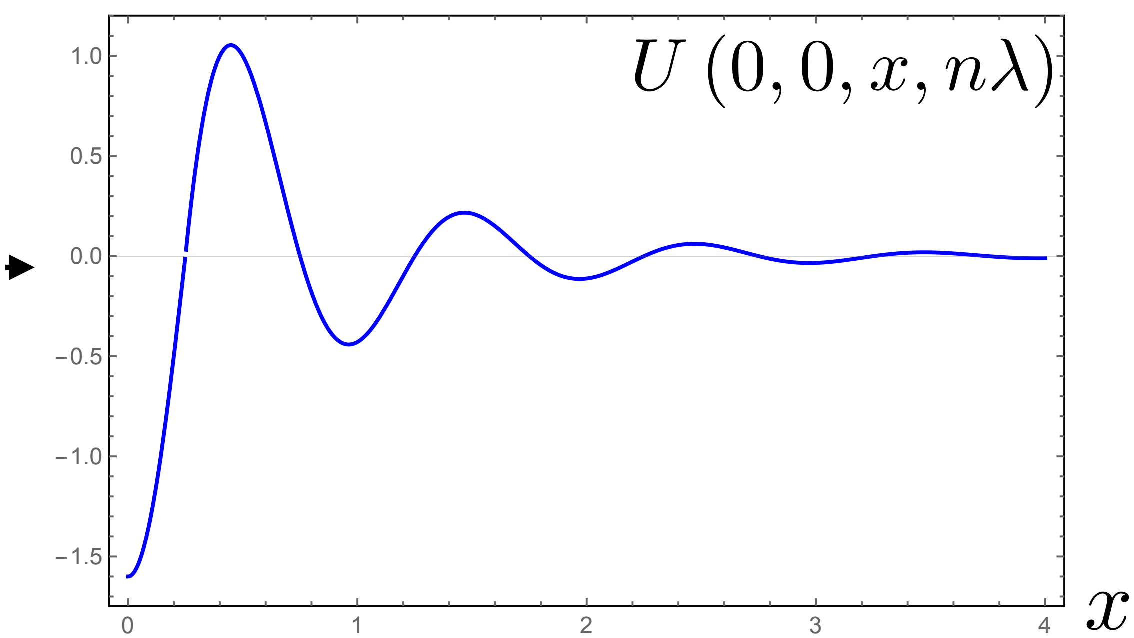


Interaction potential



Generic properties

- **Sign-changing potential**
- **Absolute minimum always at zero distance**