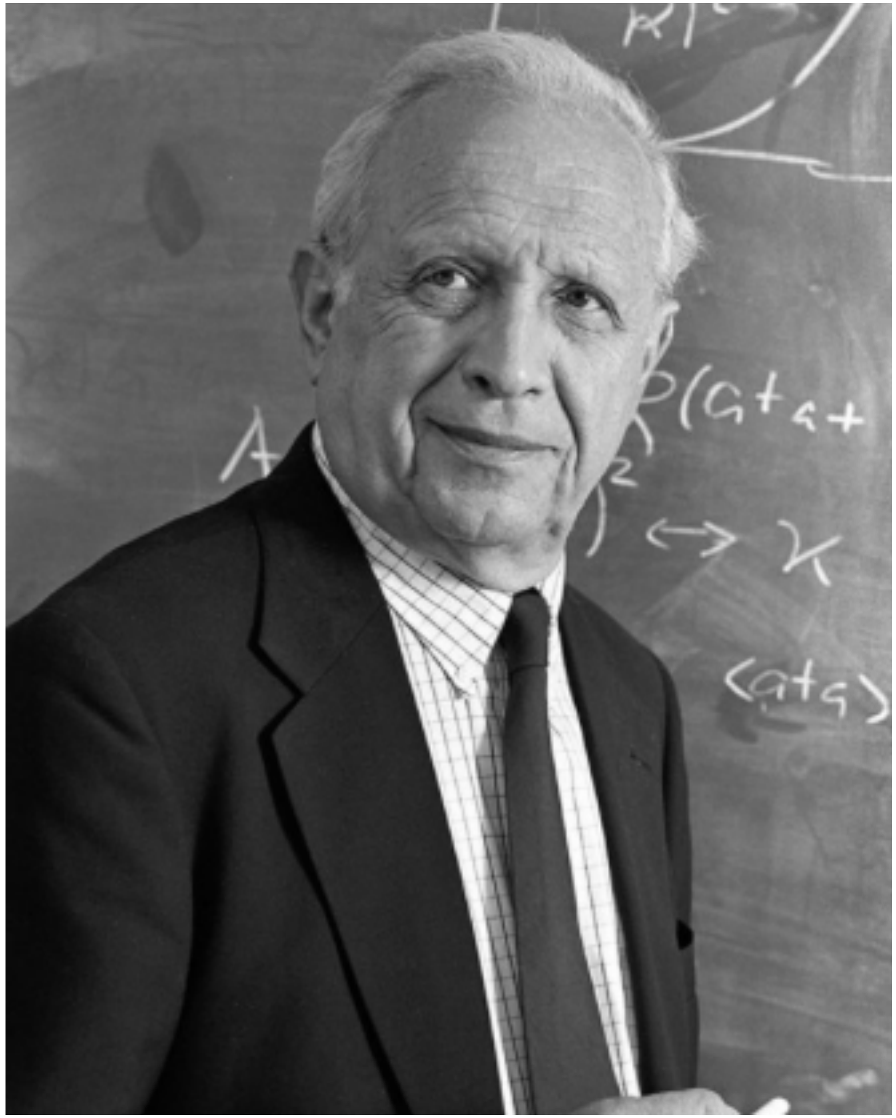
Controlling light and matter using cooperative radiation
Part I: Dicke states, cooperative effects, entanglement

Susanne Yelin

University of Connecticut
Harvard University

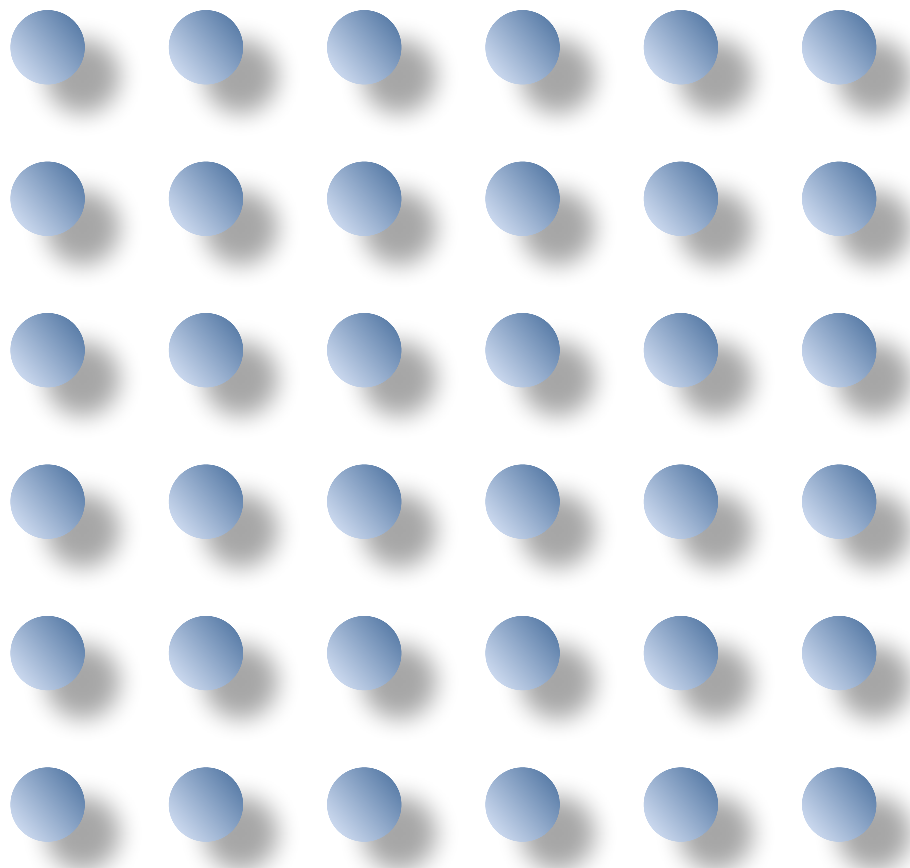
Herrsching, March 6, 2019



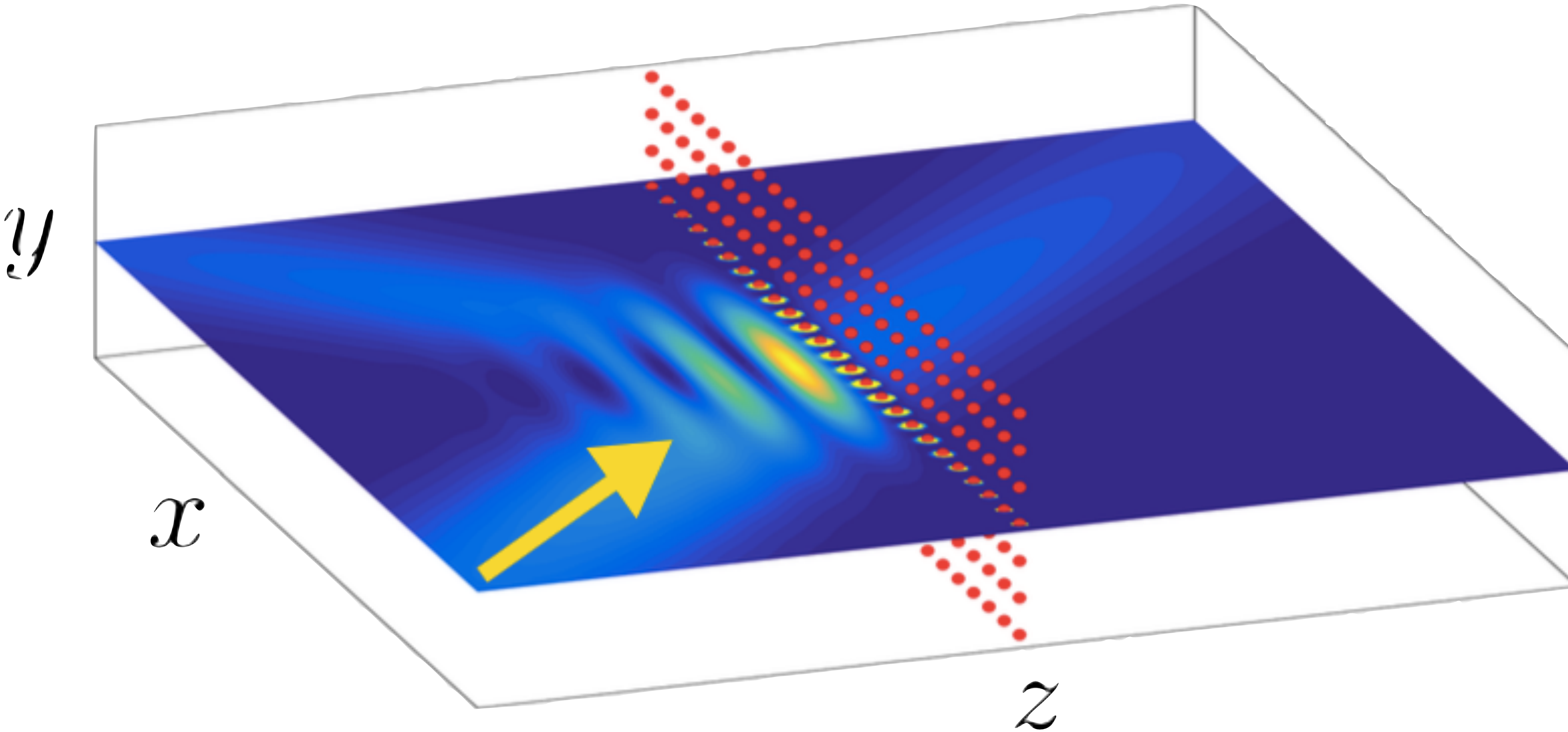
Roy Glauber, 1925 - 2018

Goals of the lectures

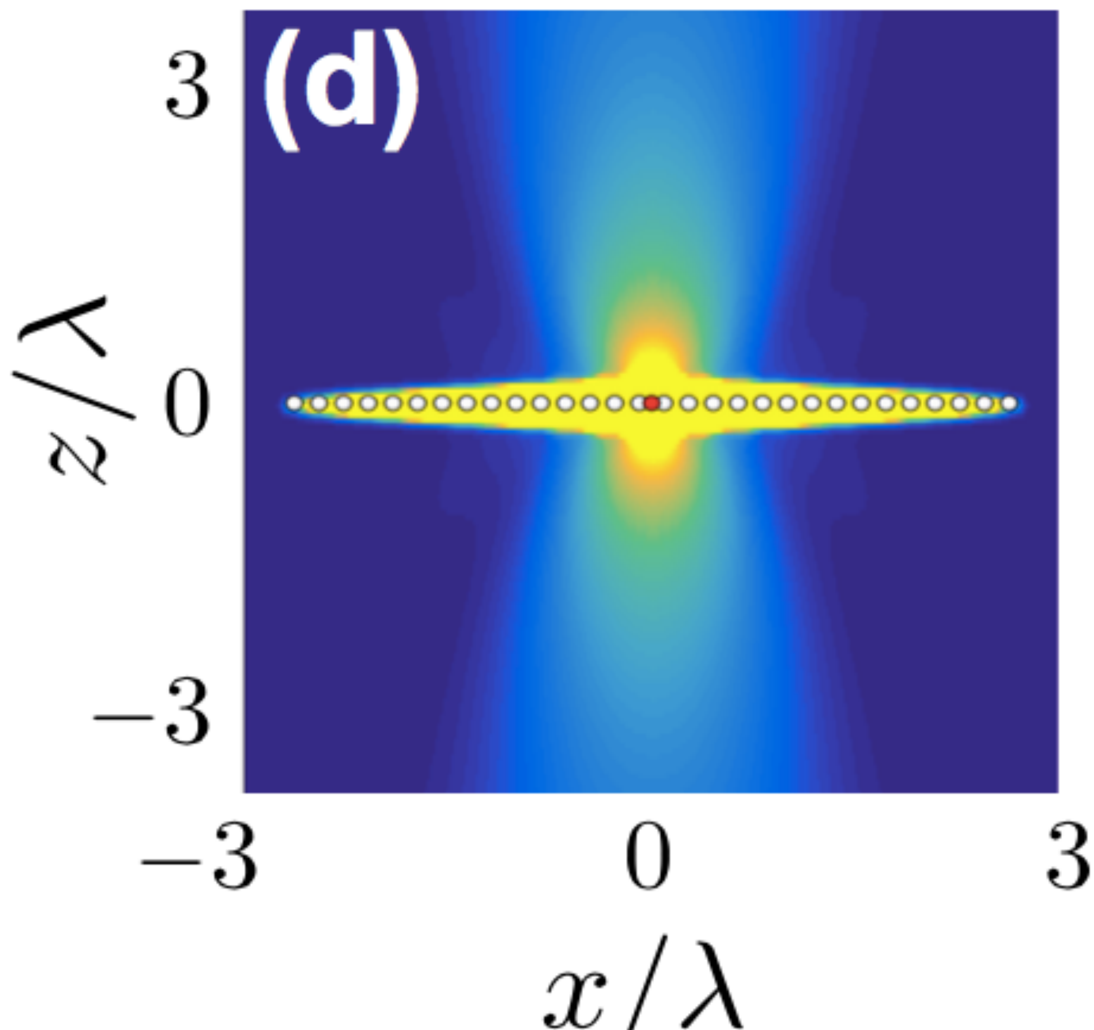
- Use 2D array…



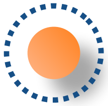
- … to reflect light …



- … emit in controlled way …



Increase (impurity) cross section?



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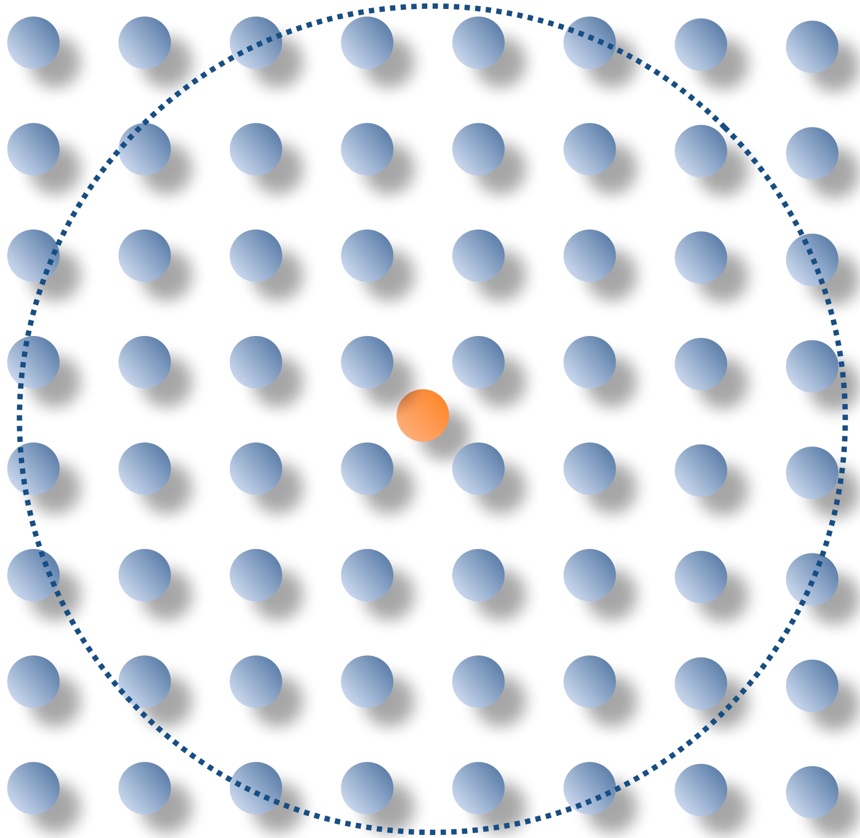
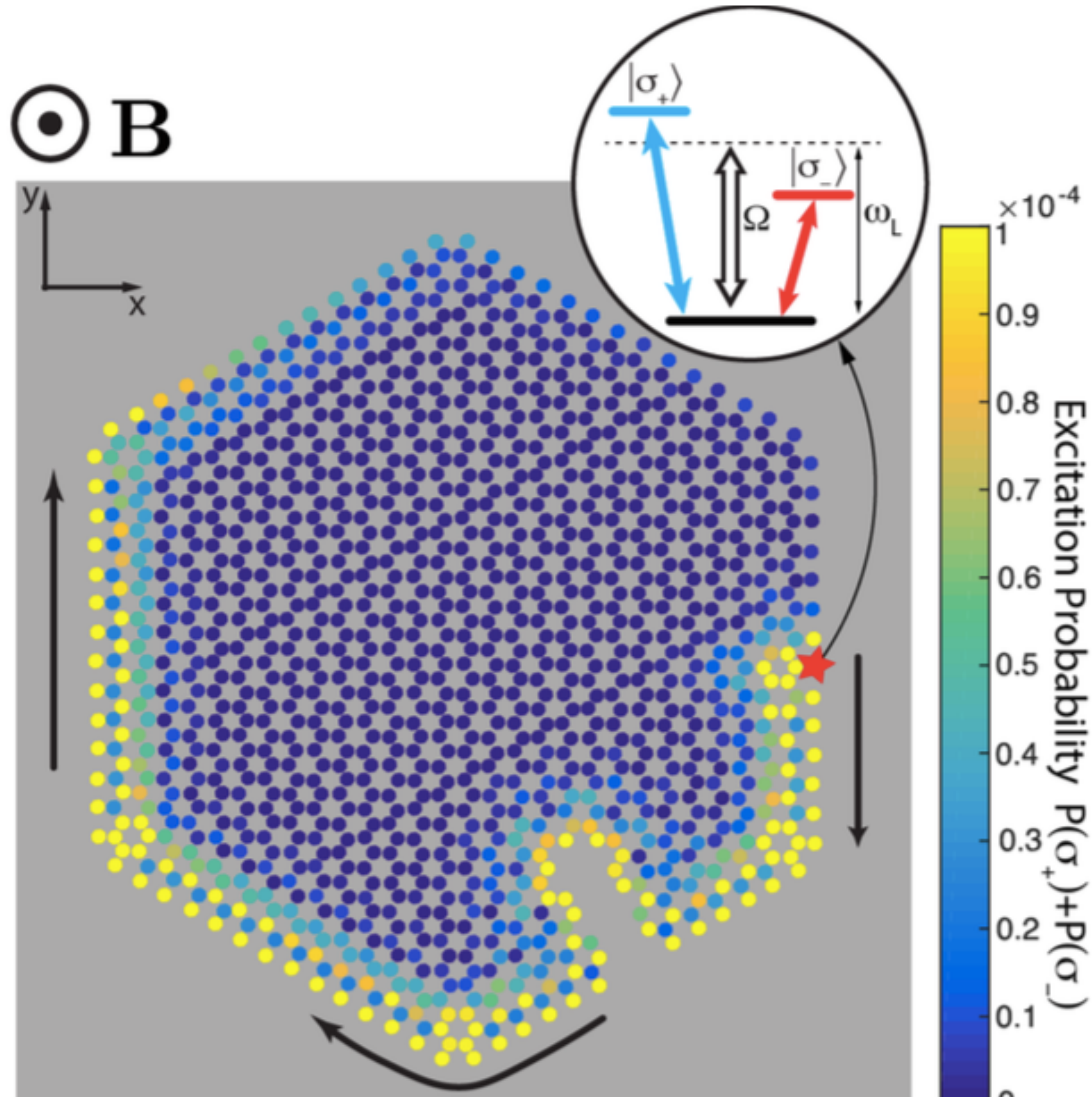
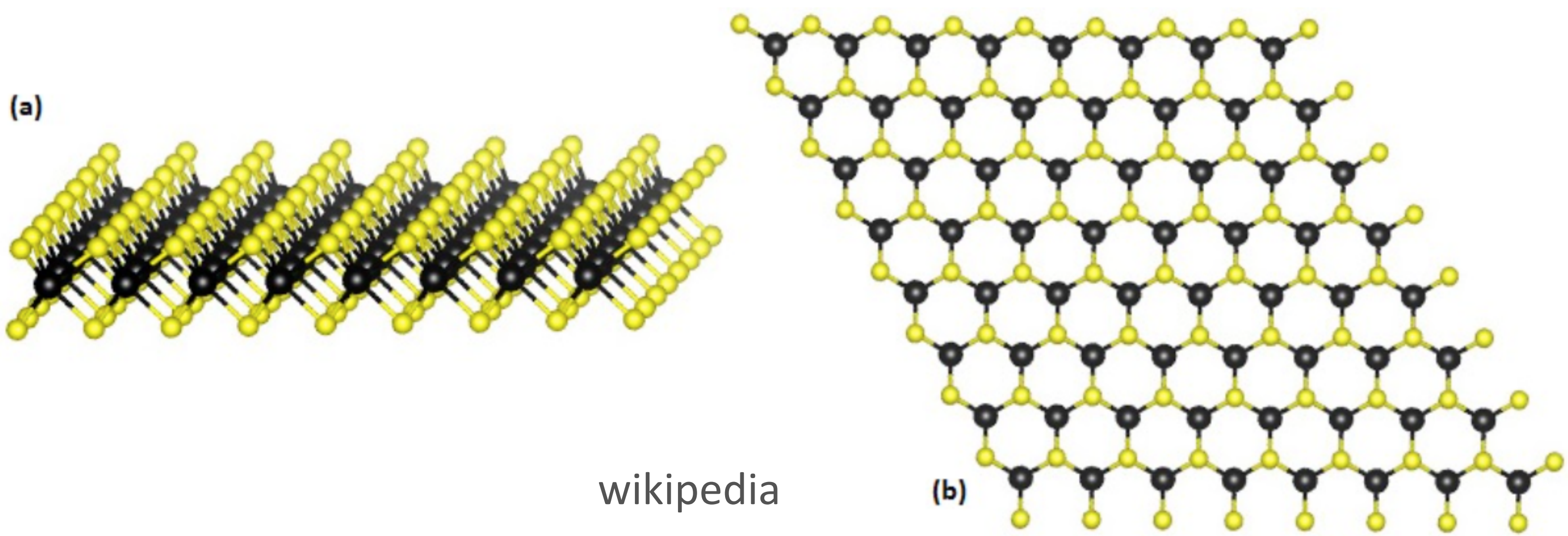


- … edge states with photons?

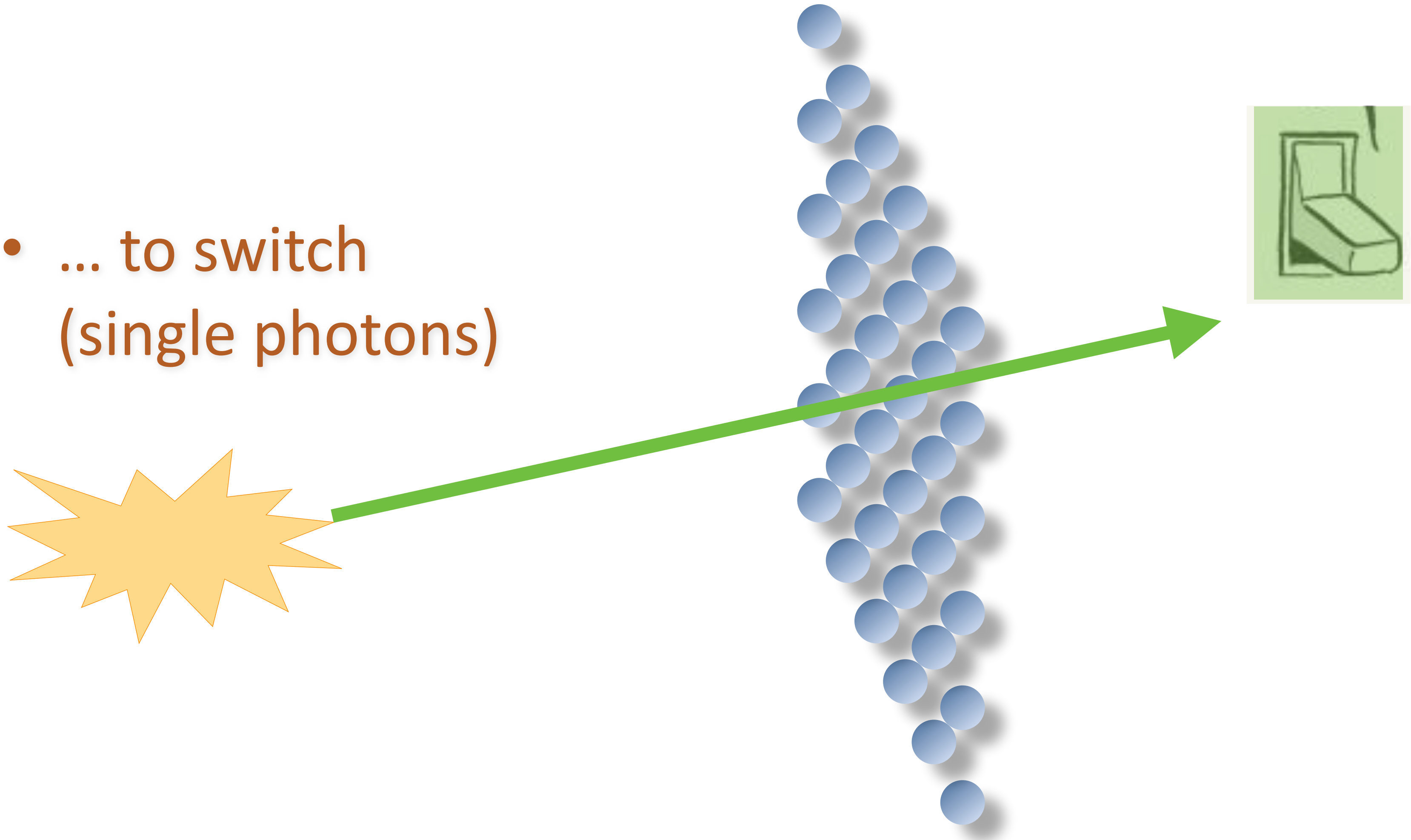


- 2D materials like graphene or TMDs



wikipedia

- … to switch (single photons)



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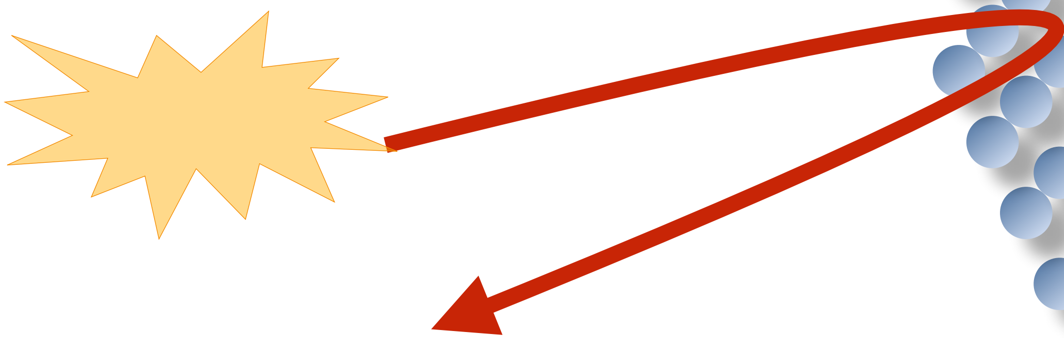
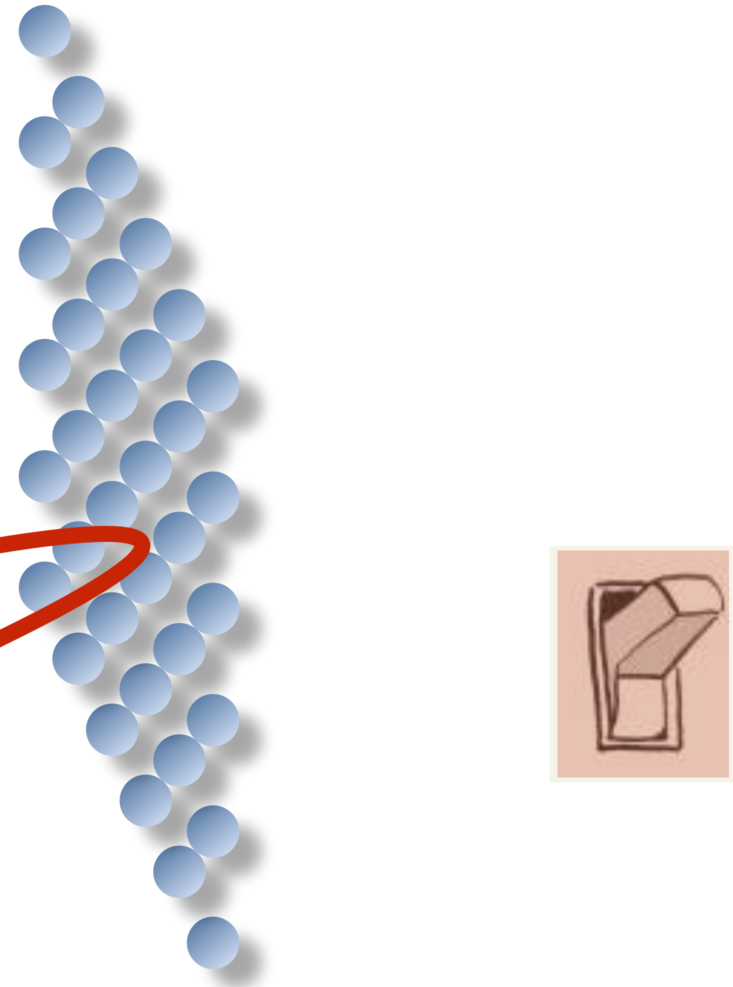




Quantum mirror: Refraction superposition

Why “cooperative effects”??

Cooperative radiation: superradiance

> For want of a better term, a gas which is radiating strongly because of coherence will be called "superradiant."
>
> — Robert H Dicke, 1954.

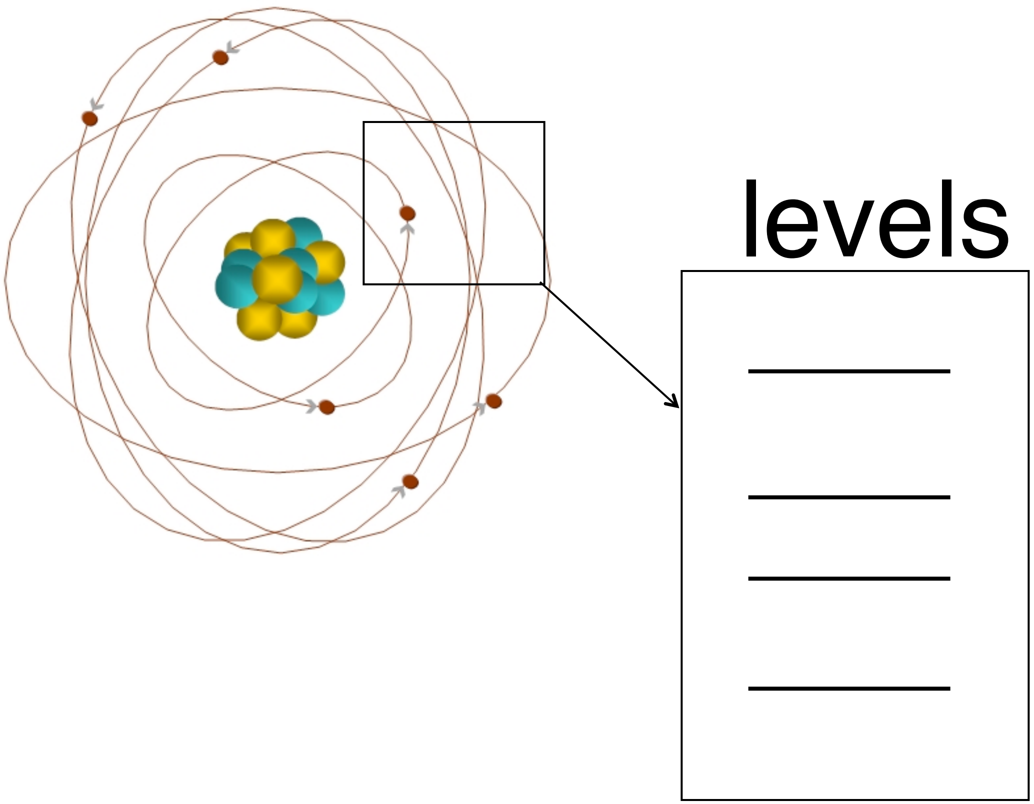
PHYSICAL REVIEW VOLUME 93, NUMBER 1 JANUARY 1, 1954

Coherence in Spontaneous Radiation Processes

R. H. DICKE
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received August 25, 1953)

By considering a radiating gas as a single quantum-mechanical system, energy levels corresponding to certain correlations between individual molecules are described. Spontaneous emission of radiation in a transition between two such levels leads to the emission of coherent radiation. The discussion is limited first to a gas of dimension small compared with a wavelength. Spontaneous radiation rates and natural line

What is an atom?



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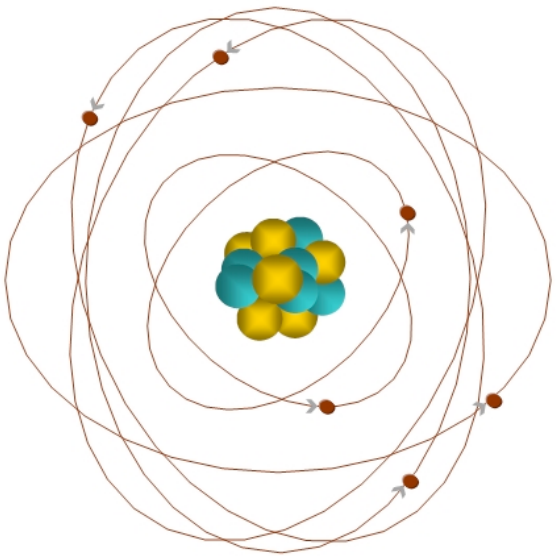
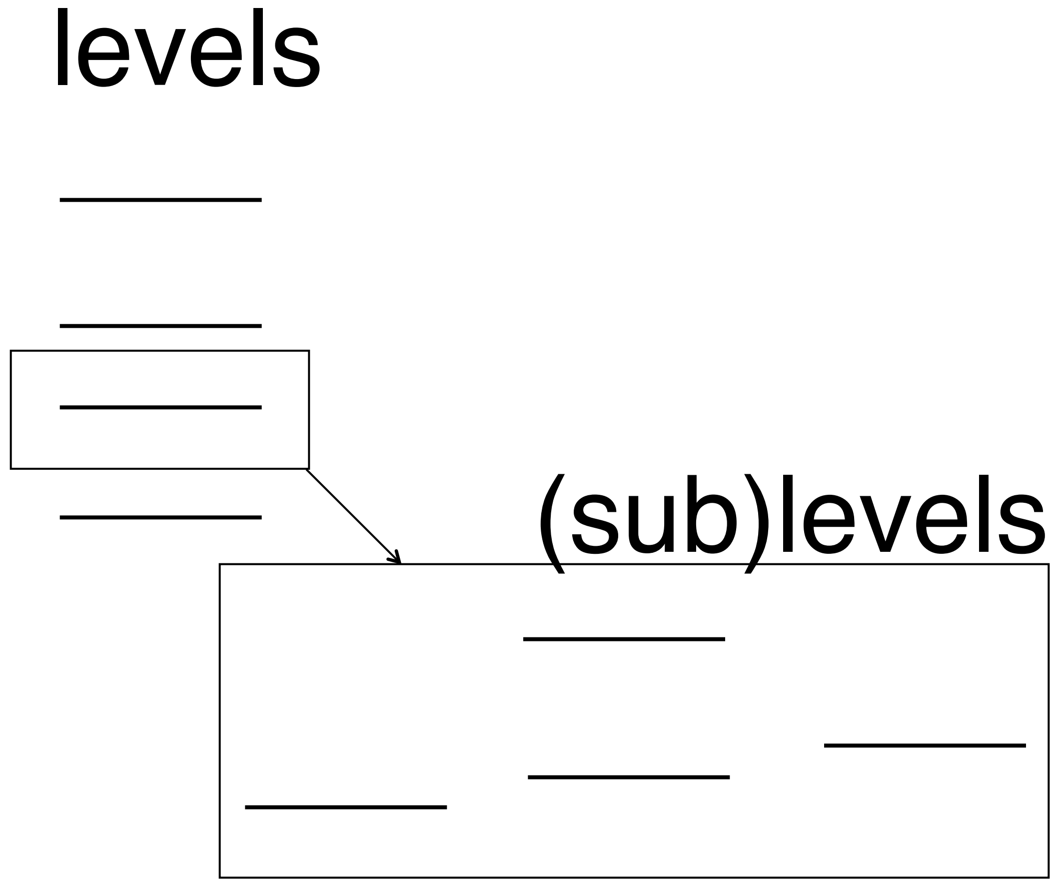
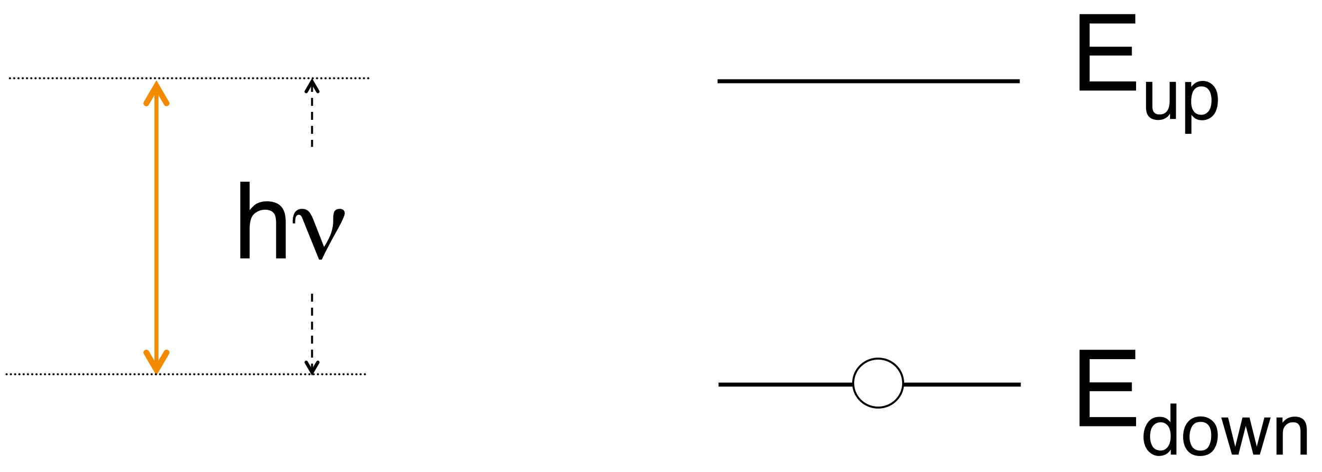




How does it couple to light?

photon energy: $h\nu$

atomic transition energy: E_{up}-E_{down}



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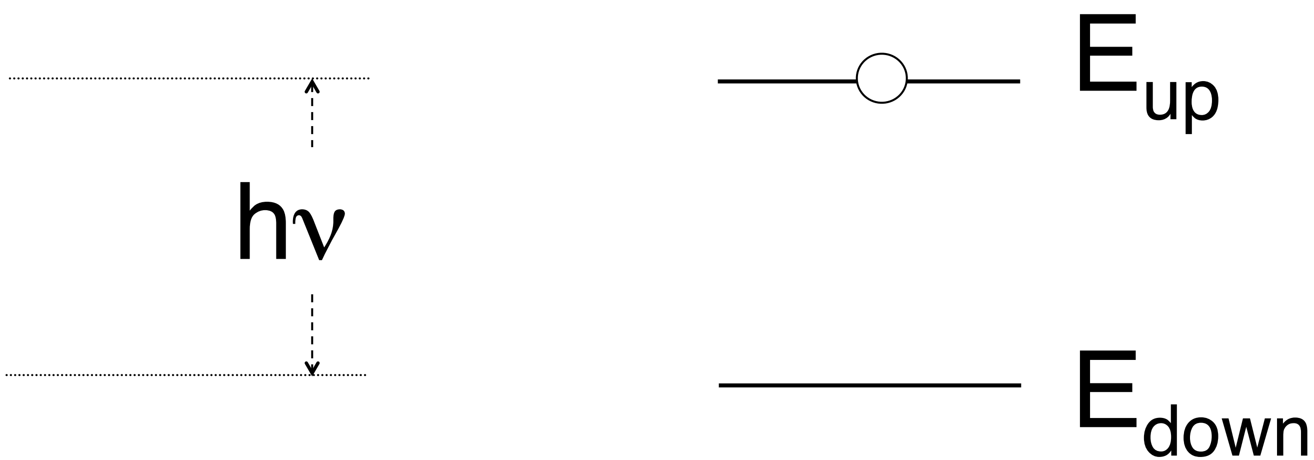
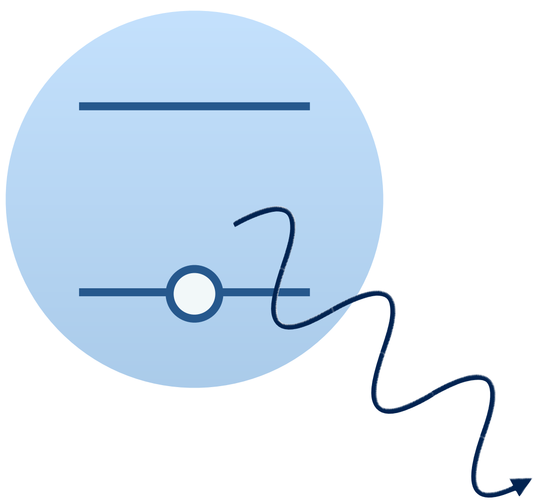
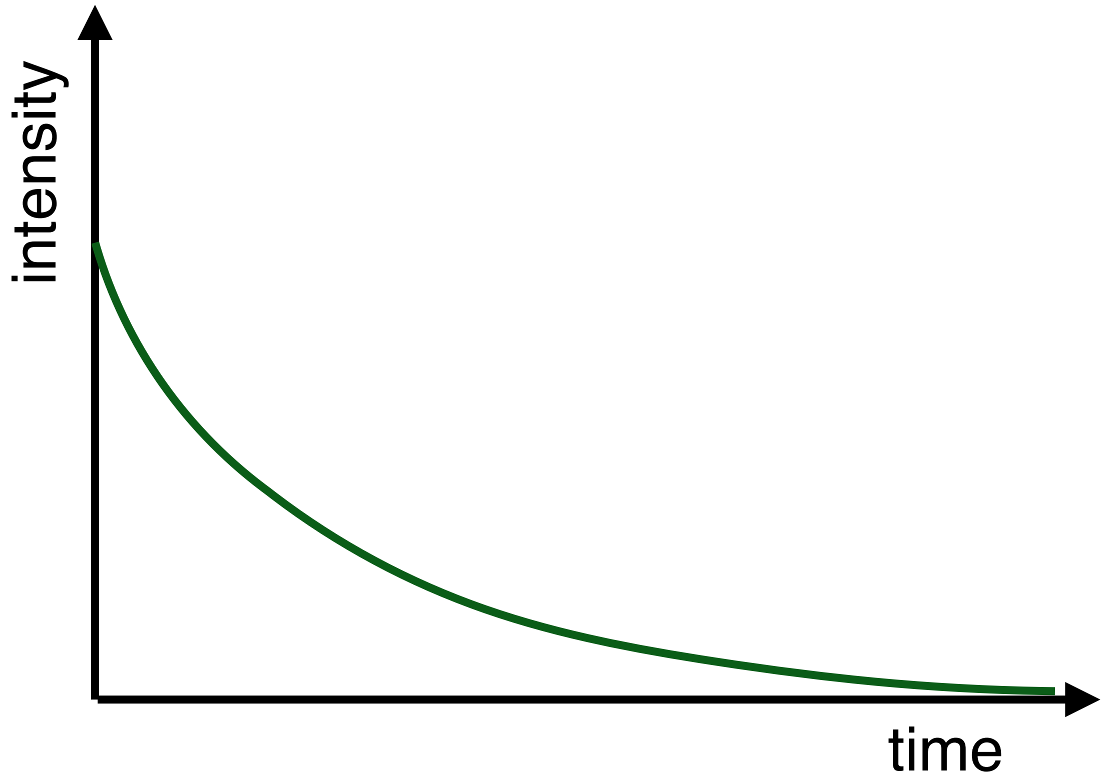


Cooperative effects in radiation

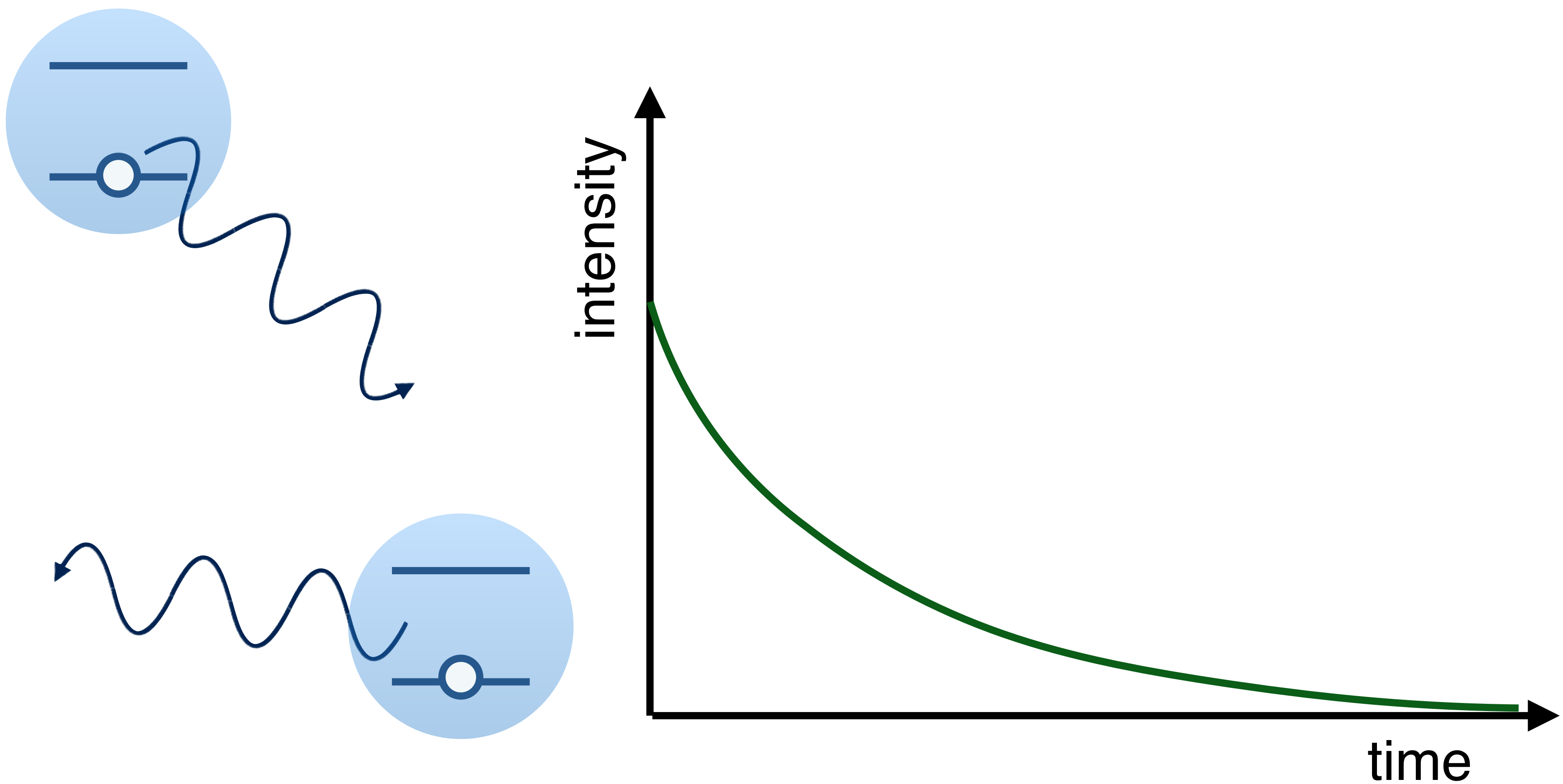
single atom





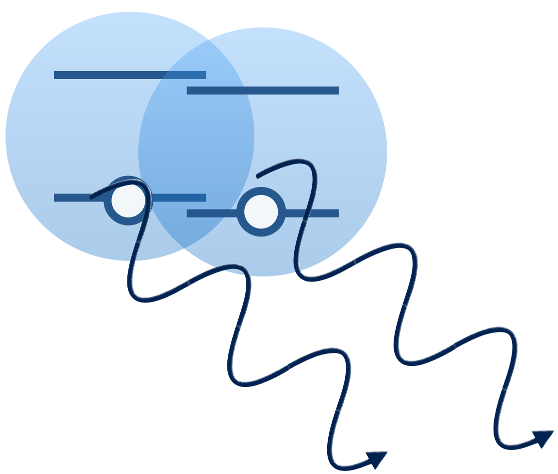
Cooperative effects in radiation

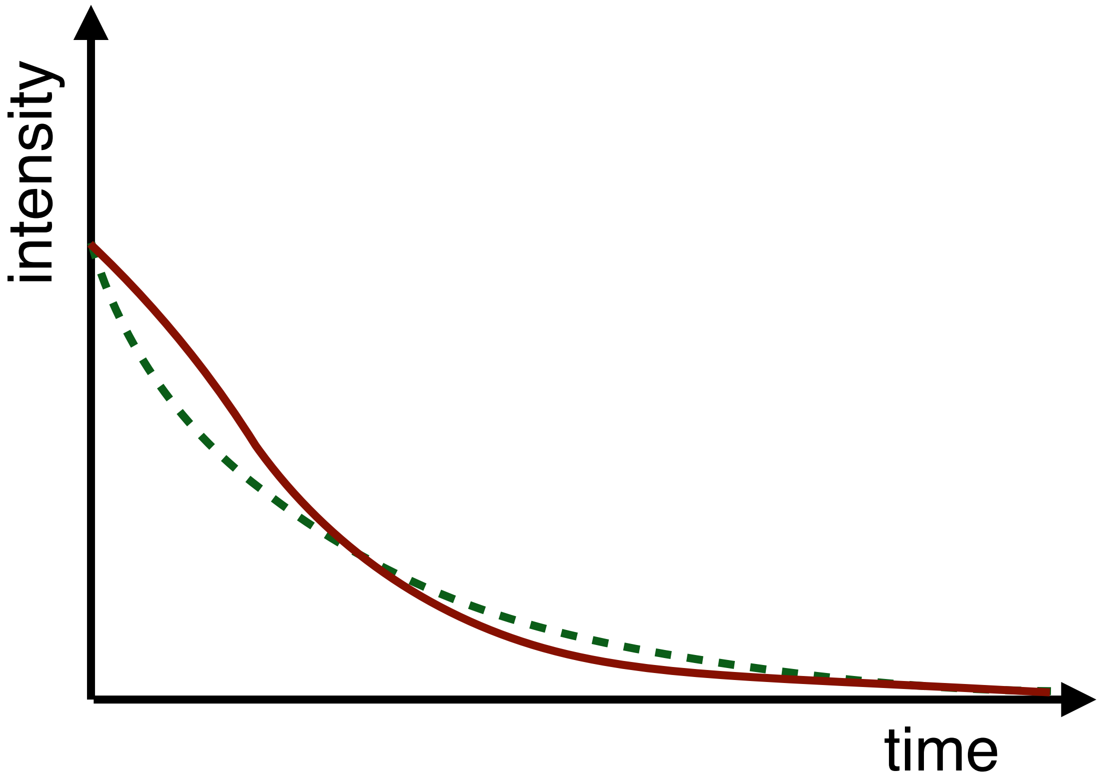
two far-away atoms



Cooperative effects in radiation

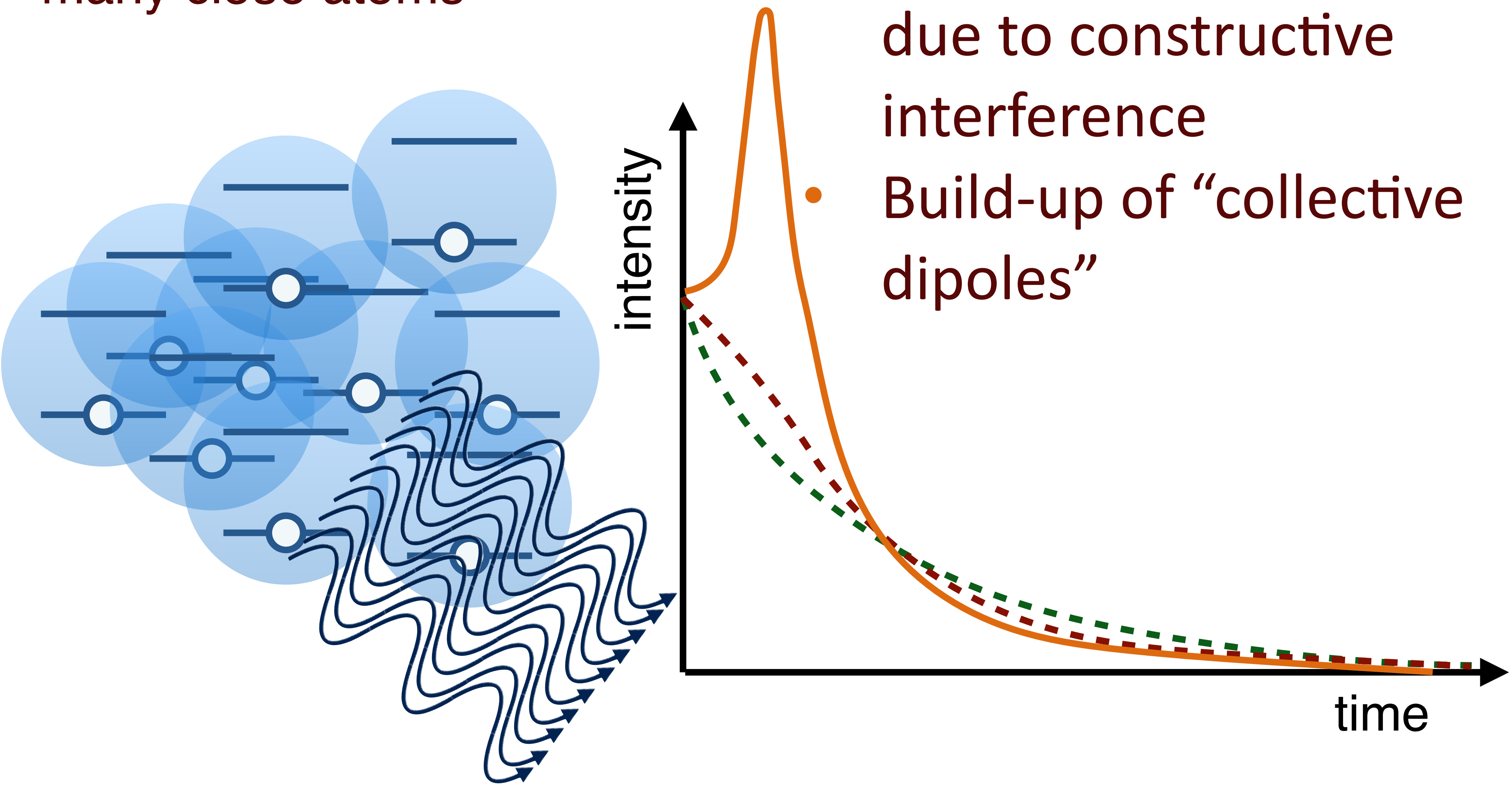
two close atoms





Cooperative effects in radiation

many close atoms



- Superradiance ∝ N^2 due to constructive interference
- Build-up of “collective dipoles”

Cooperative Effects

- Two important aspects:
  - collective effects (many particles)

Cooperative Effects

- Two important aspects:
  - collective effects (many particles)
    - e.g., much higher chance for photons to interact with many atoms than one
  - “exchange” due to dipole-dipole interaction
    - excitations are exchanged
- “Cooperative” is more than just “collective”!

Collective effects: Example of quantized field

- Interaction Hamiltonian with classical light (Rabi frequency Ω)

$$H/\hbar = \Omega\left(|e\rangle\langle g| + |g\rangle\langle e|\right)$$

- Interaction Hamiltonian with quantized light (coupling element g, annihilation operator a, number of atoms N)

$$H/\hbar = g\sqrt{N}\left(|e\rangle\langle g|\, a + a^{+}\, |g\rangle\langle e|\right)$$

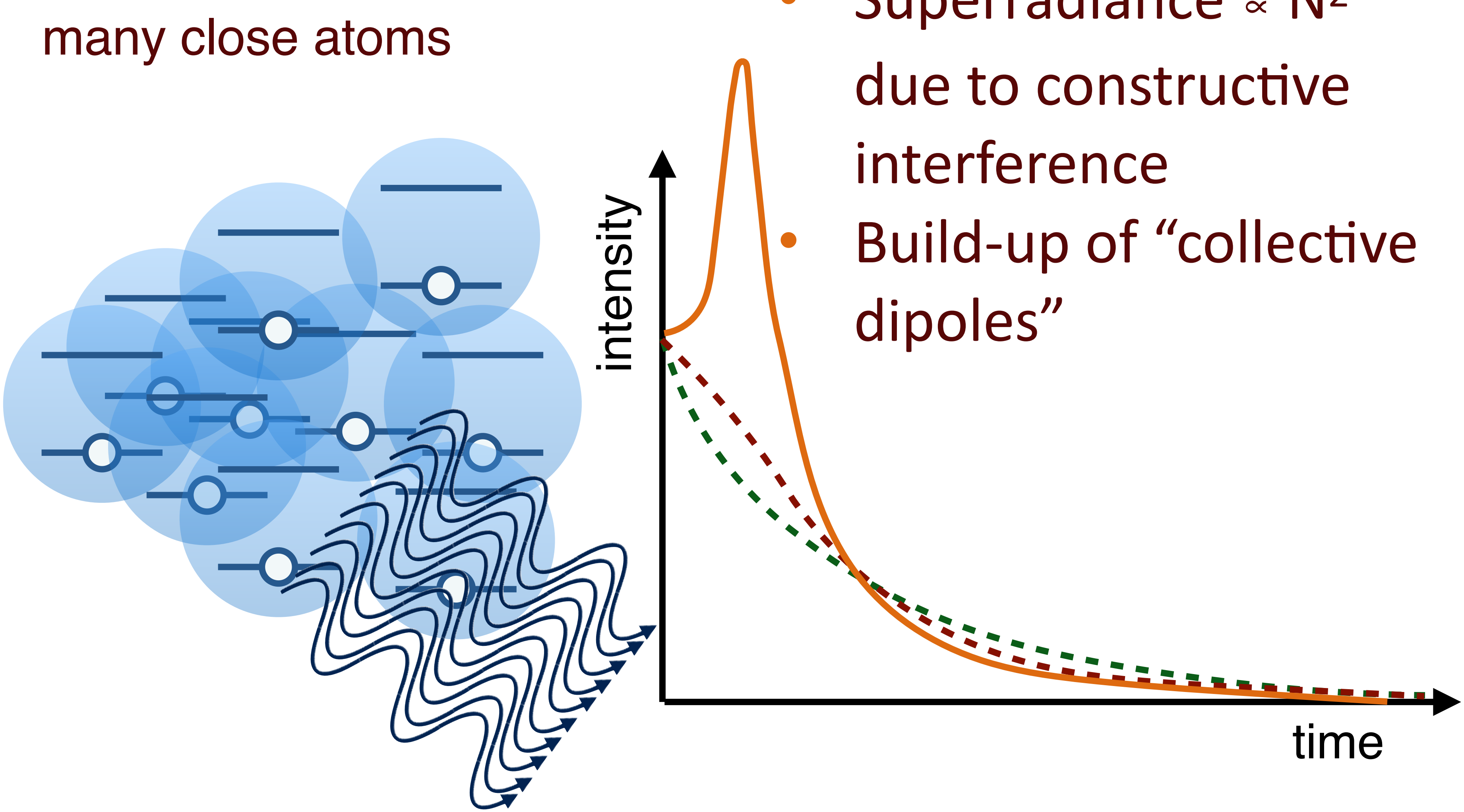
Cooperative effects

- simplest form of “exchange interaction”

Cooperative effects

- Traditional example: superradiance

Cooperative effects in radiation



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- Build-up of “collective dipoles”

What is super in superradiance?

man (Clark)

many men (Clarks)

What is super in superradiance?

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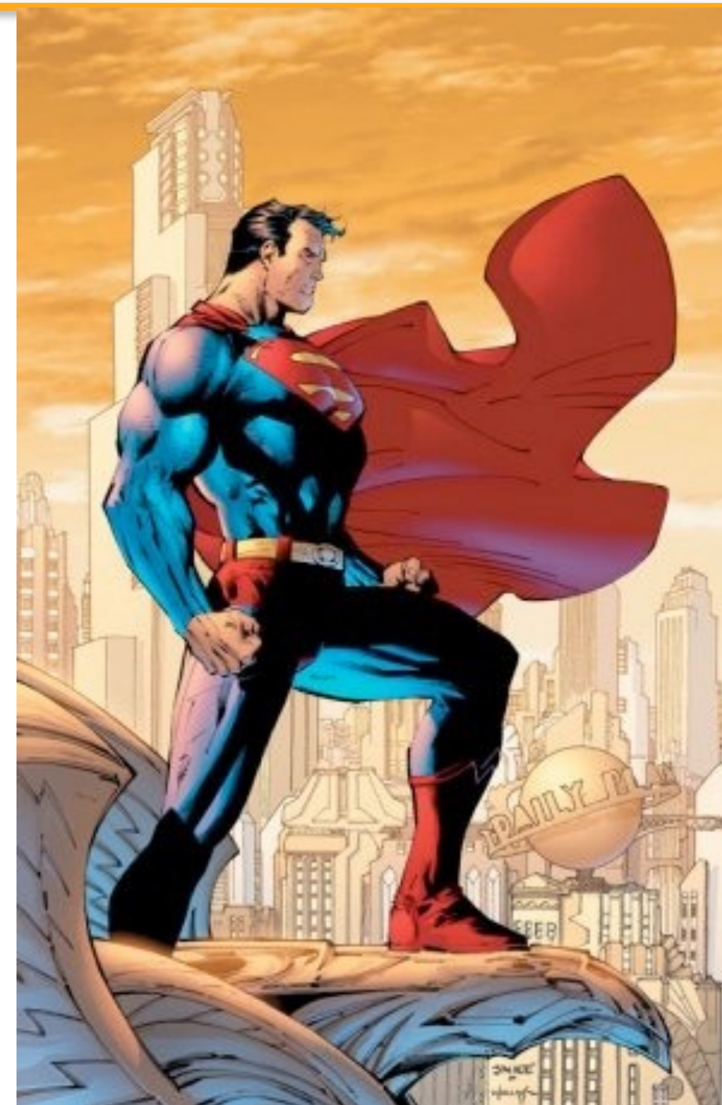


SUPERMAN

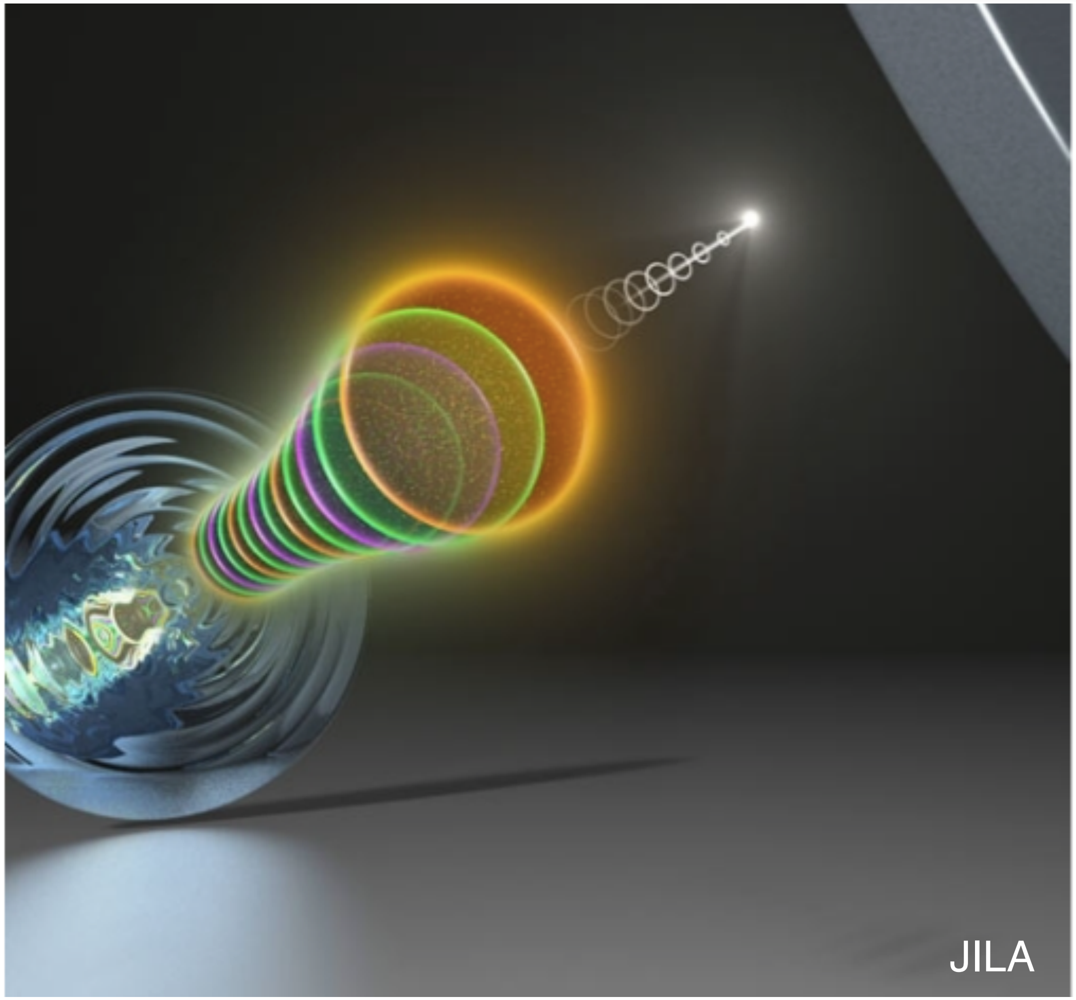
What is super in superradiance?

man (Clark)

many men (Clarks)

Examples

Superradiant laser



$\#_{\text{Atoms}} \gg \#_{\text{Photons}}$

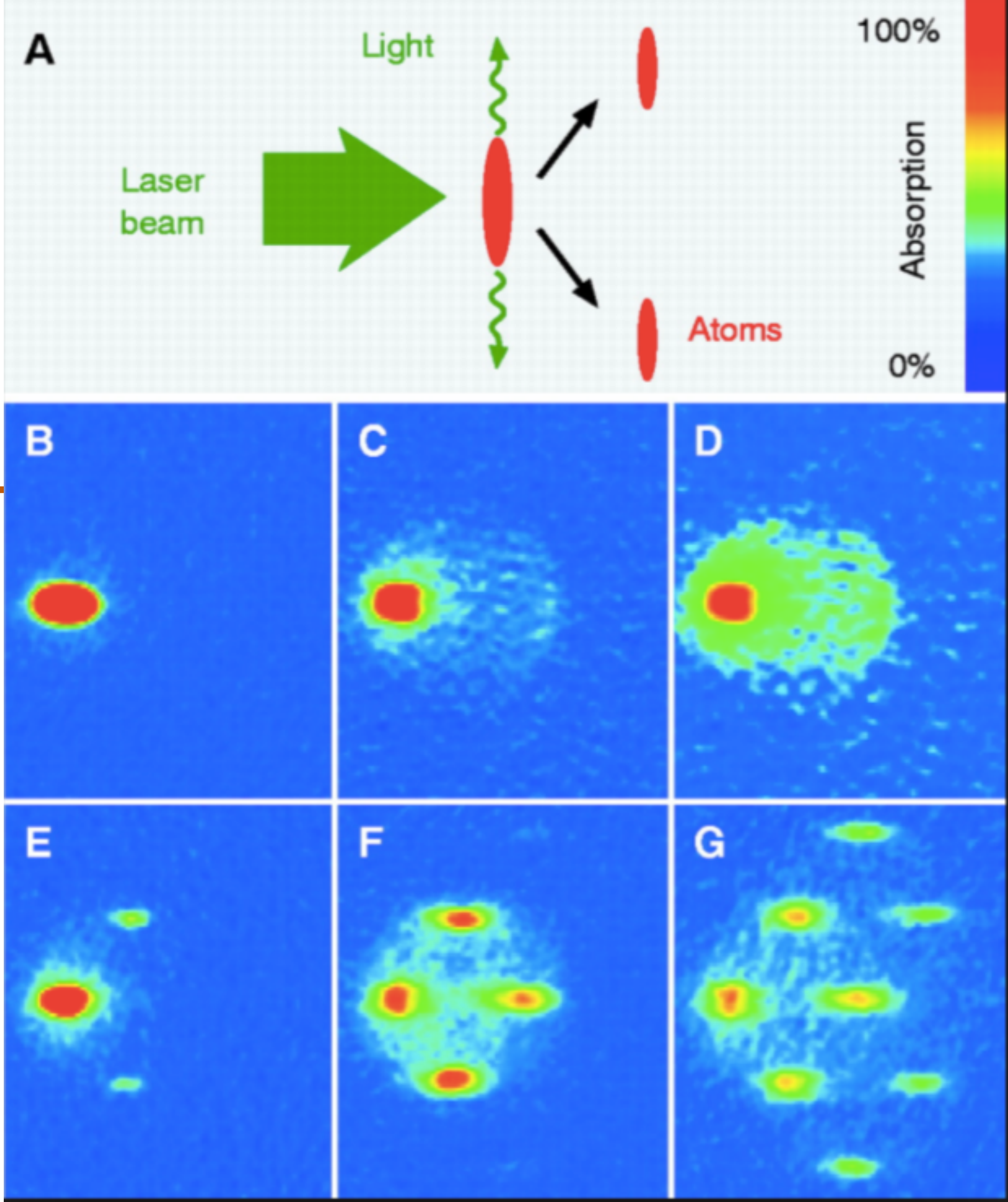
$\Downarrow$

much more coherent lasing

Bohnet, Chen, Weiner, Meiser, Holland, Thompson, Nature 484,78 (2012)

Super-radiance in BECs

momentum
conservation
+
interference of
matter waves

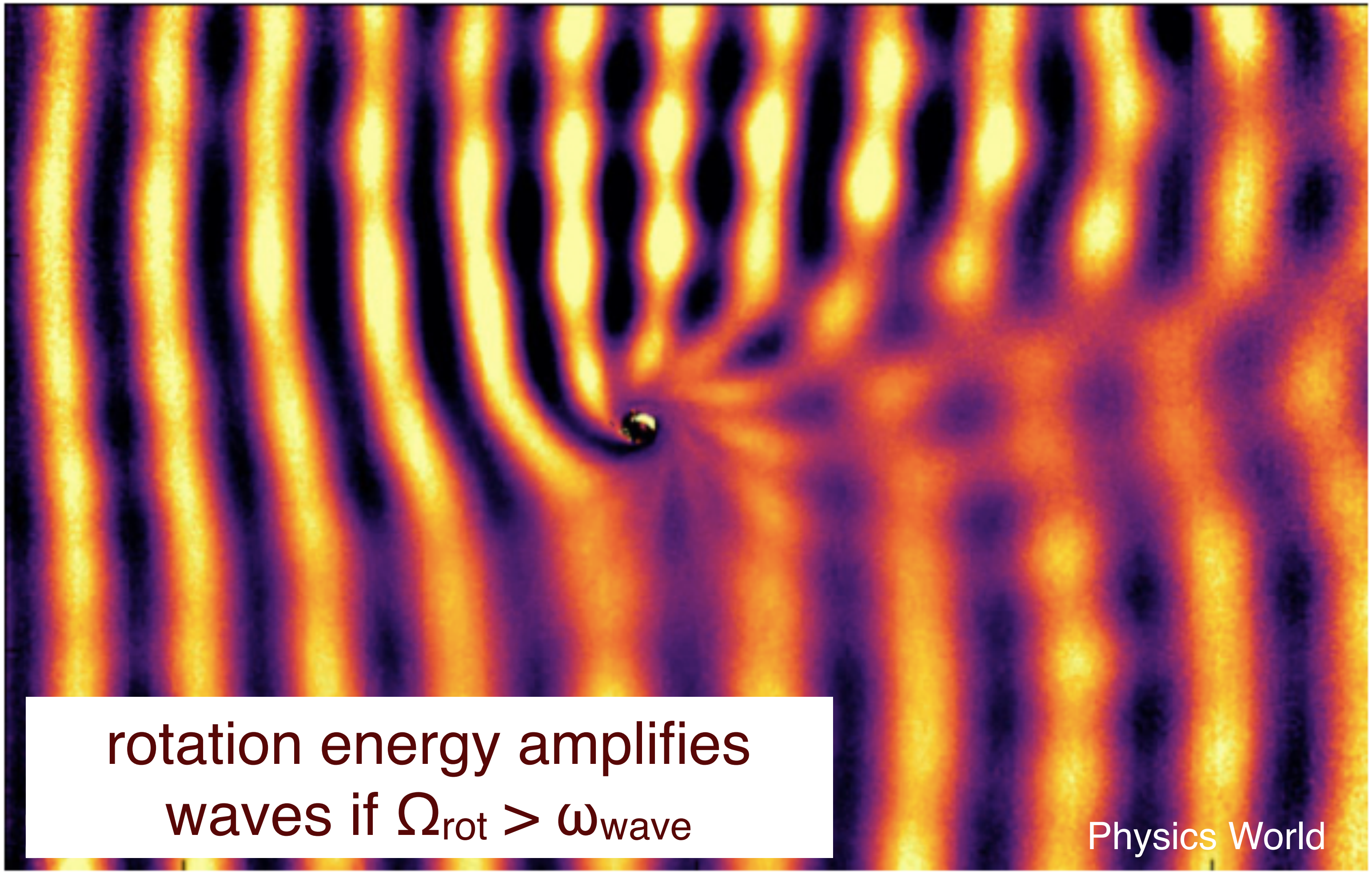


Inouye, Chikkatur, Stamper-Kurn, Stenger, Pritchard, Ketterle
Science 23 (1999)

SOFT MATTER AND LIQUIDS | RESEARCH UPDATE

Rotational superradiance spotted as water swirls down a drain

13 Jun 2017 Hamish Johnston

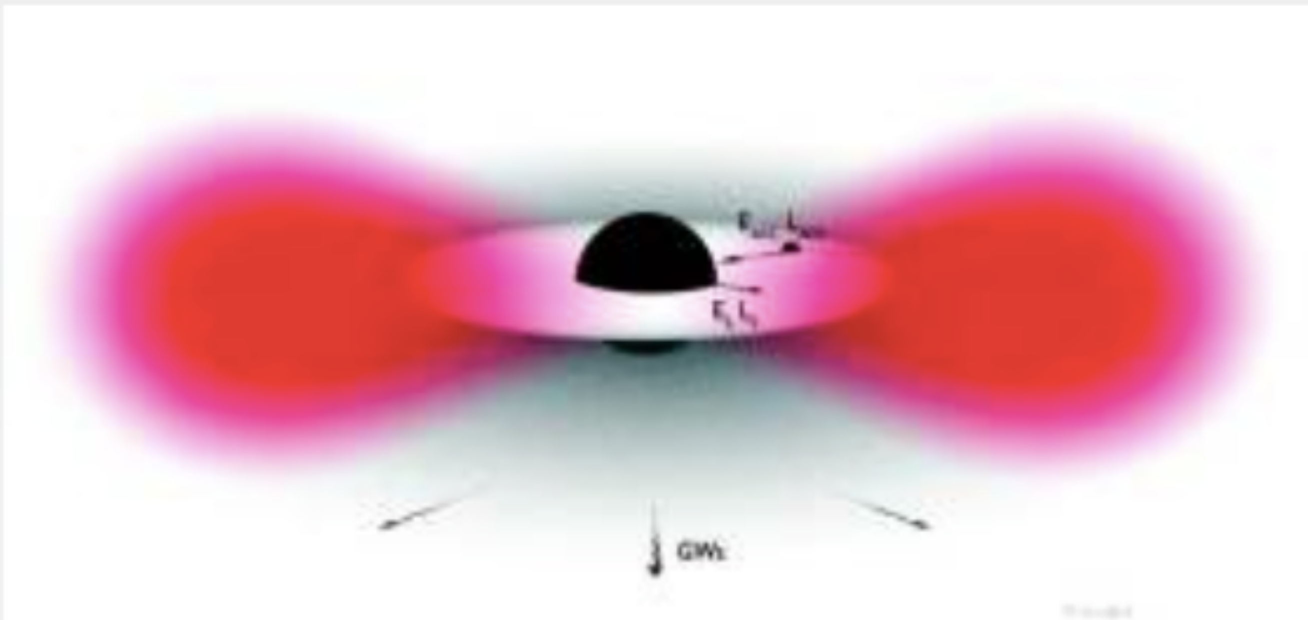


Whirling around: waves scatter from a vortex

Torres, Patrick, Coutant, Richartz, Tedford, Weinfurtner
Nature Physics 13, 833 (2017).

Black hole superradiance

spinning black hole….

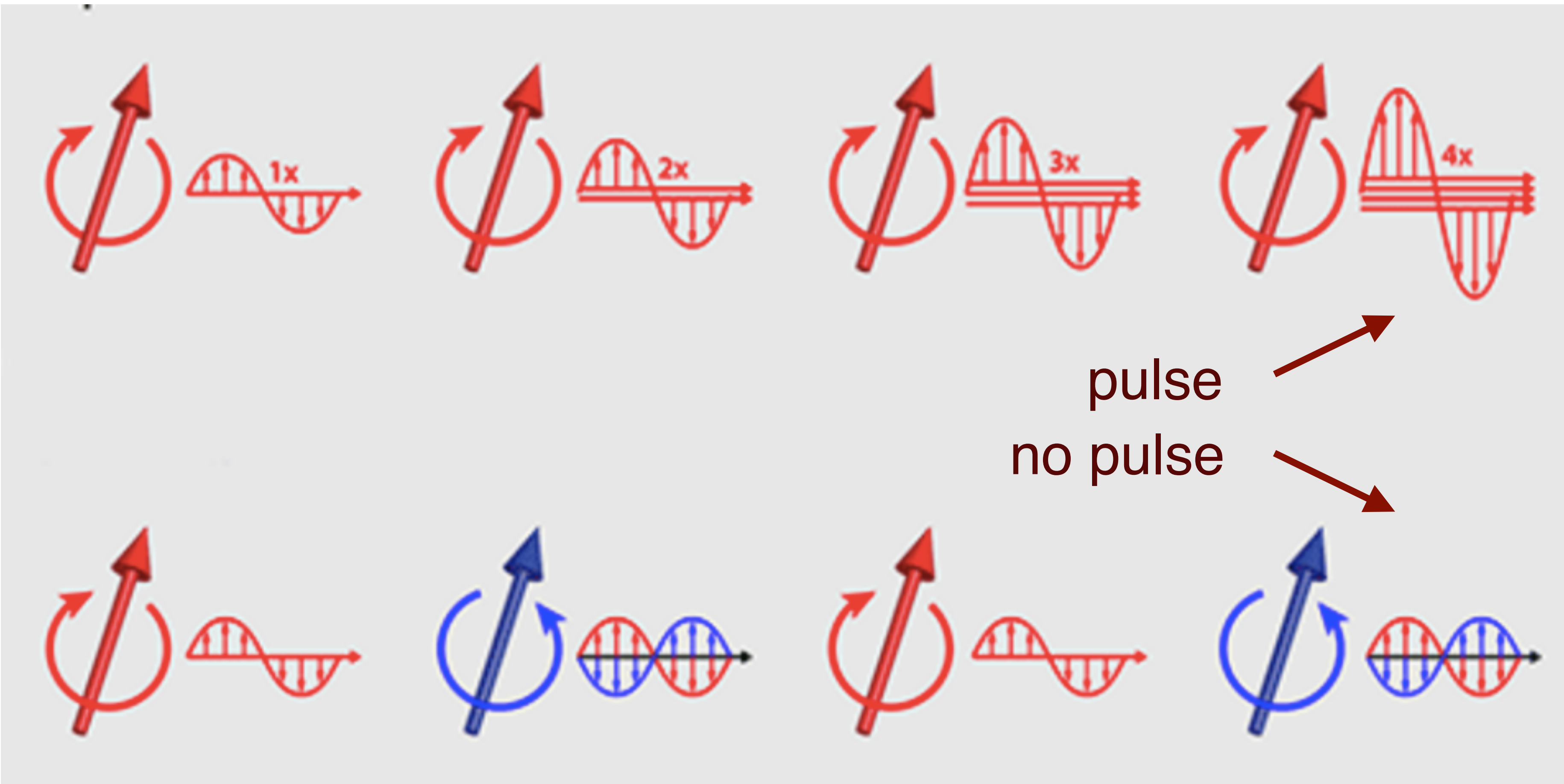


… amplifies graviational waves

(basically same as for water waves)

Pani, Brito, Cardoso, Class. Quantum Grav. **32** 134001 (2015)

Subradiance



Guerin, Araujo, Kaiser, PRL **116**, 083601 (2016)

These lectures

- Cooperative effects in complex systems
- New application: atomically thin mirrors

These lectures

- Cooperative effects in complex systems
  - Collective (Lamb) level shifts
  - Subradiance
  - Entanglement
- New application: atomically thin mirrors

These lectures

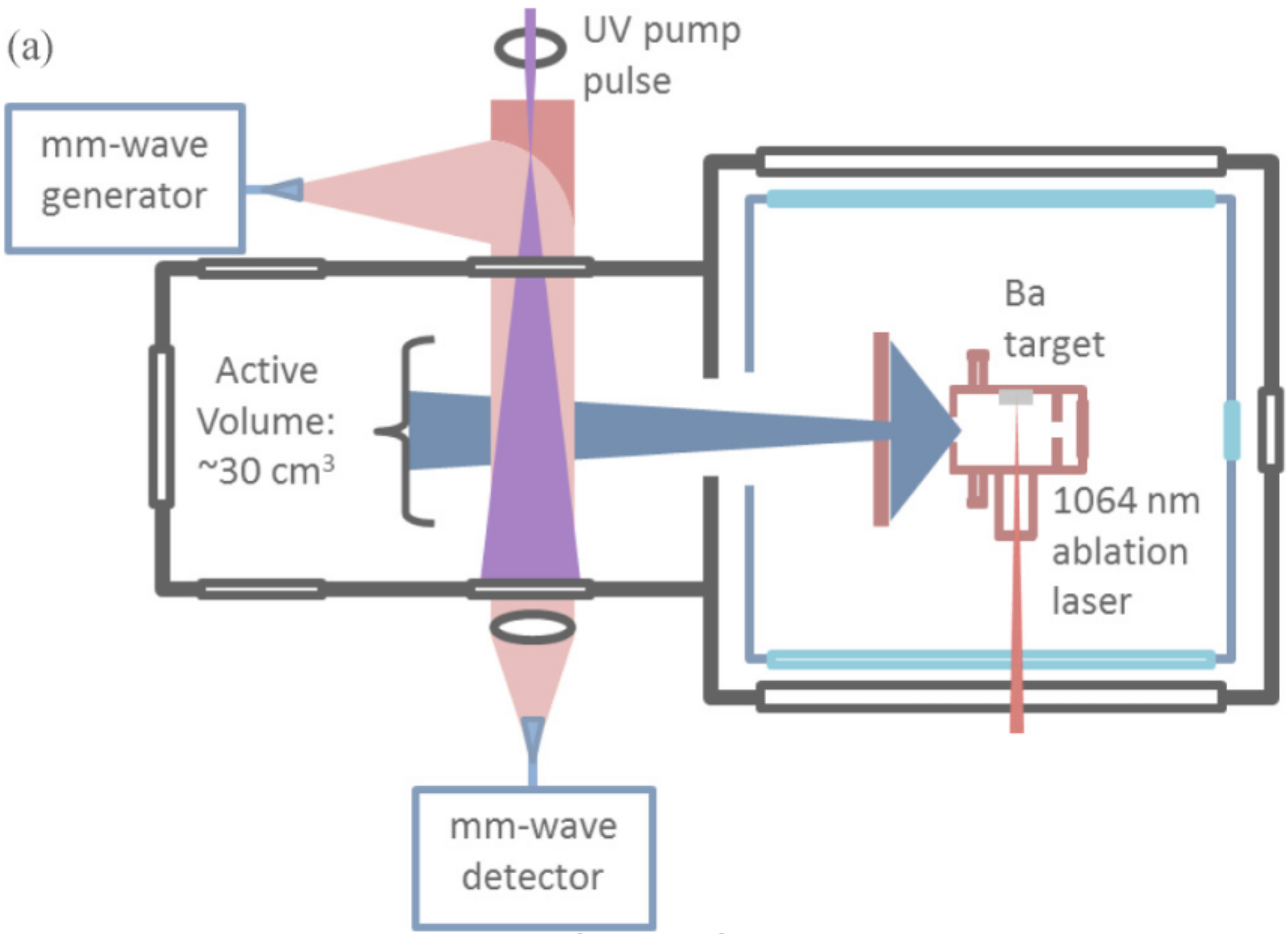
- Cooperative effects in complex systems
- New application: atomically thin mirrors
  - Cooperative resonances
  - Applications:

These lectures

- Cooperative effects in complex systems

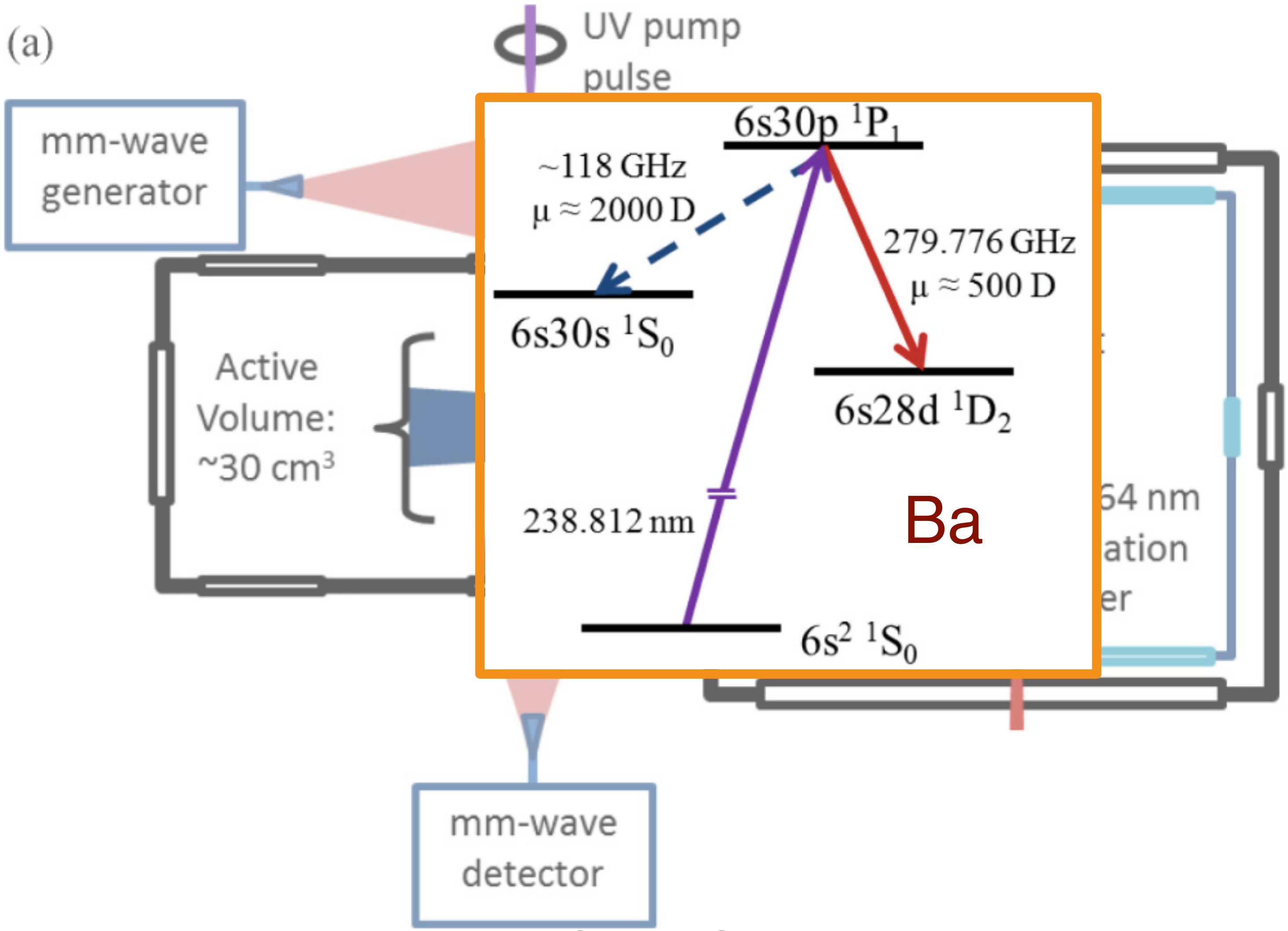
- New application: atomically thin mirrors
  - Cooperative resonances
  - Applications:
    - topology with photons
    - nonlinear quantum optics
    - Quantum metasurfaces

Superradiance: recent experiment



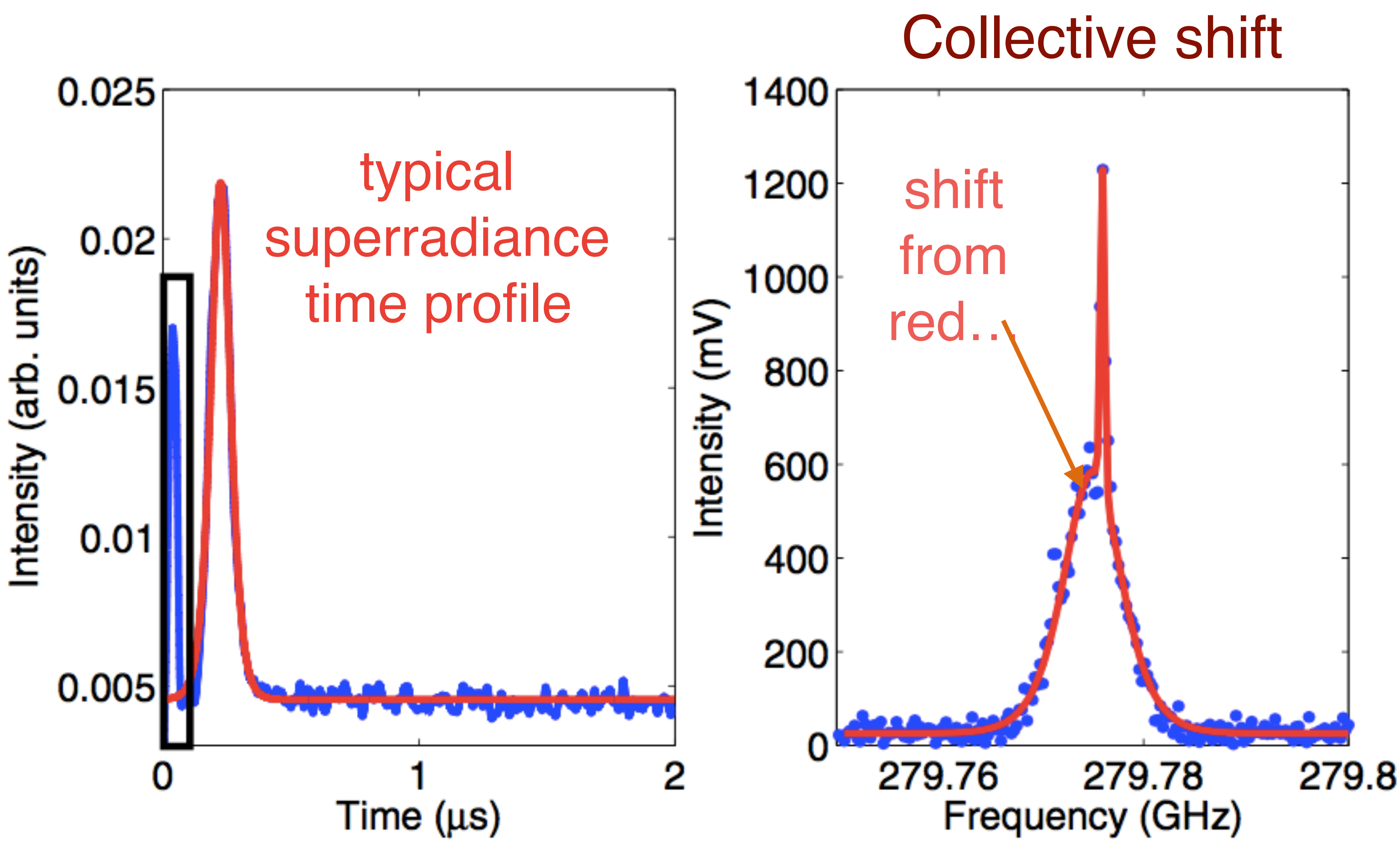
Grimes, Coy, Barnum, Zhou, Yelin, Field, PRA 95, 043818 (2017)

Superradiance: recent experiment



Grimes, Coy, Barnum, Zhou, Yelin, Field, PRA 95, 043818 (2017)

Superradiance in Ba Rydbergs



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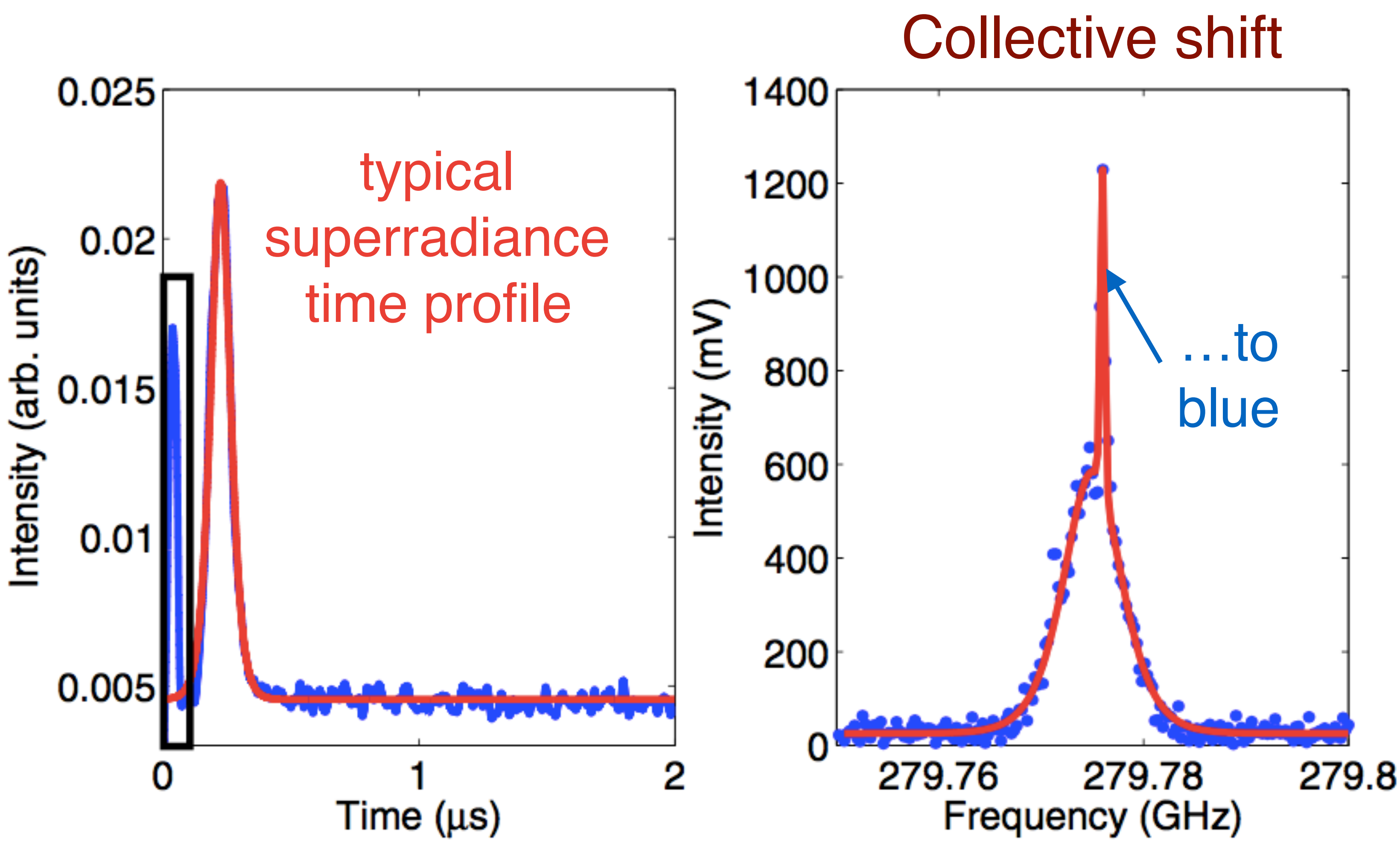
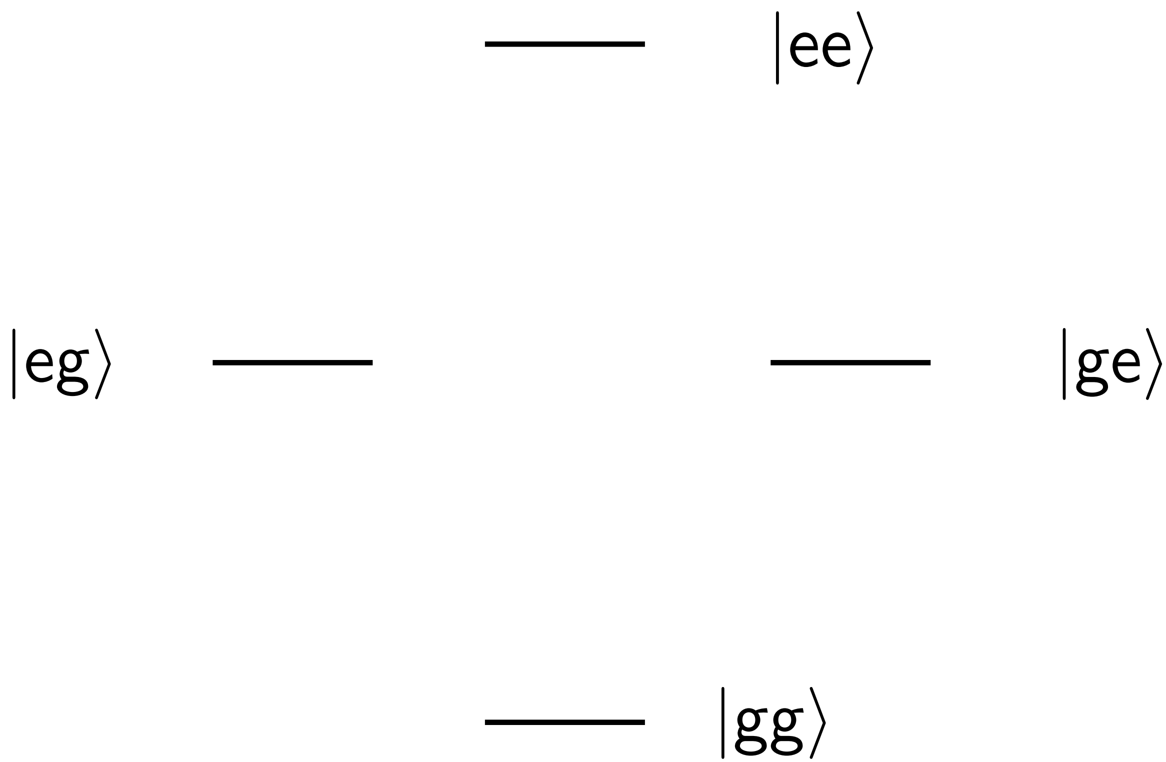


Cooperative radiation: two atoms

atoms distinguishable



Cooperative radiation: two atoms

atoms indistinguishable

$$|ee\rangle \equiv |1,1\rangle$$

$$\frac{1}{\sqrt{2}}(|eg\rangle + |ge\rangle) \equiv |1,0\rangle$$

$$\frac{1}{\sqrt{2}}(|eg\rangle - |ge\rangle) \equiv |0,0\rangle$$

$$|gg\rangle \equiv |1,-1\rangle$$

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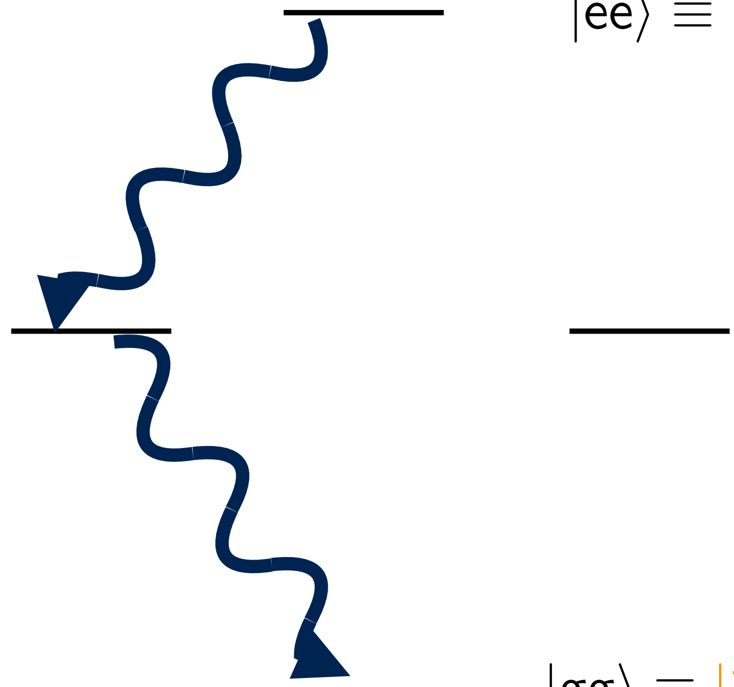

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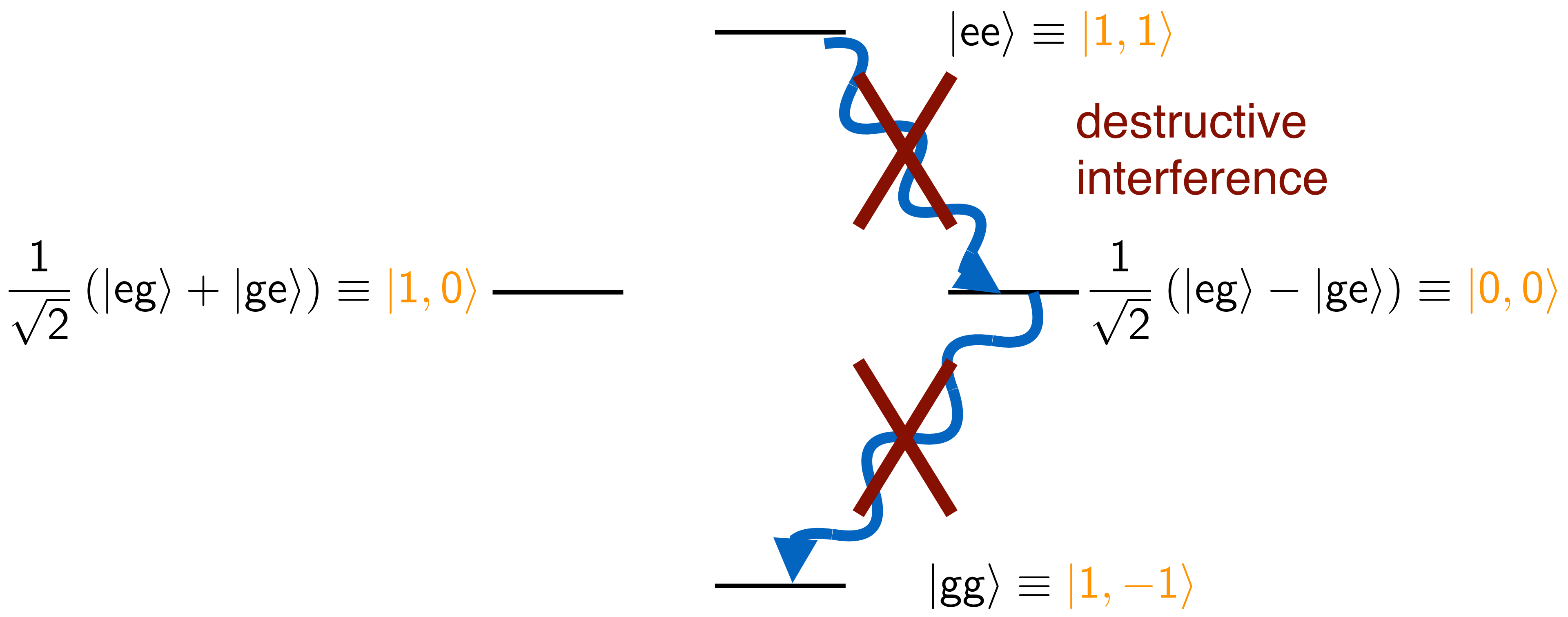

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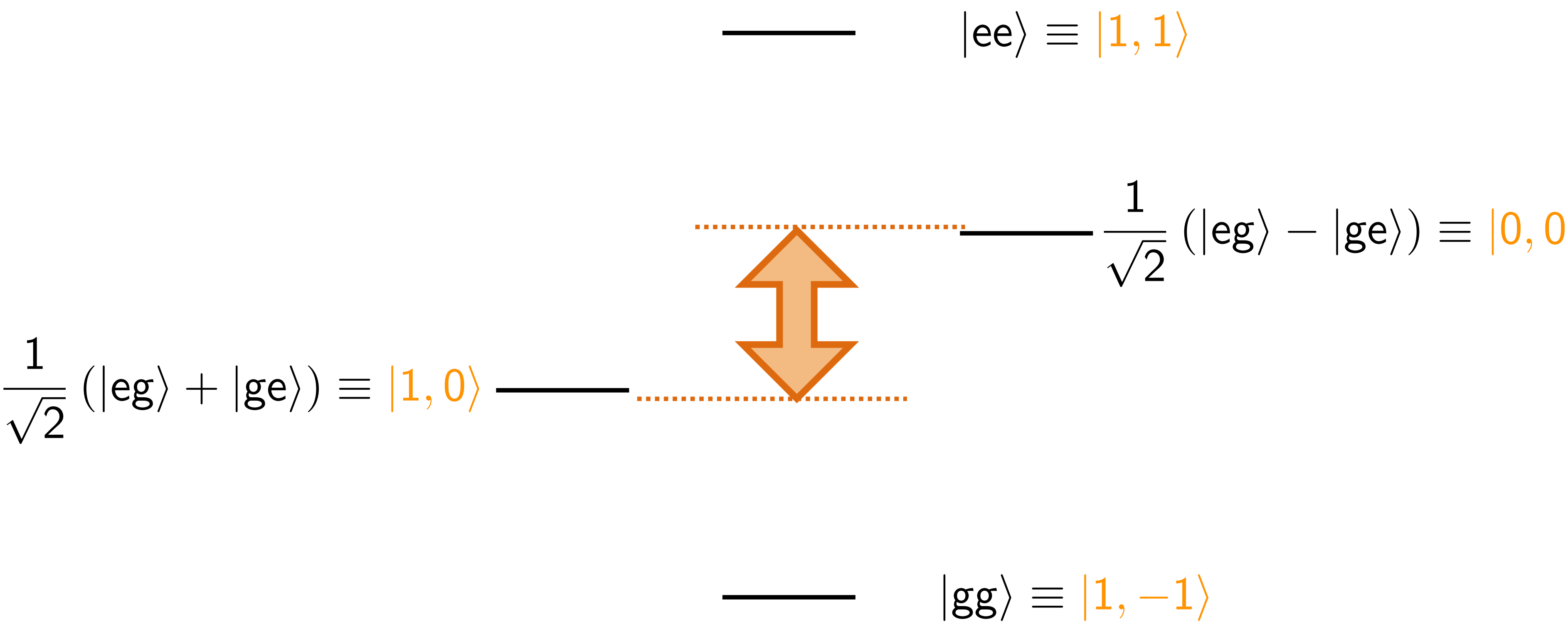

dipole-dipole exchange interaction

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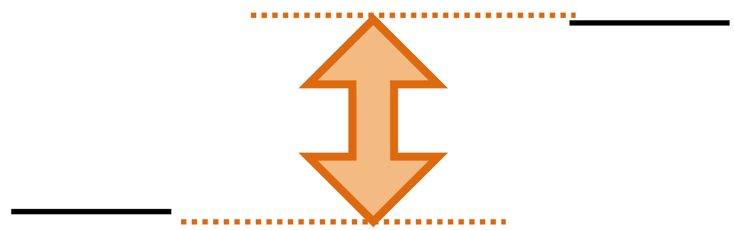

$\frac{1}{\sqrt{2}}(|eg\rangle + |ge\rangle) \equiv |1,0\rangle$

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exchange interaction:

- usually dipole-dipole mediated
- creates shift and broadening (Kramers-Kronig)

dipo

What is “superradiance”?

1. Everything that involves Dicke states
   - (e.g., collective √N effects,
   - bad-cavity limit,
   - …)

Dicke states

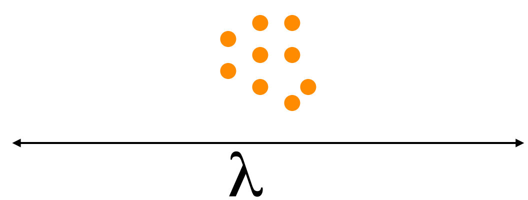
Fully symmetric state of n excitations in N particles, for example

$$|\mathbf{2}\rangle_4 = \frac{1}{\sqrt{6}}\left(|1100\rangle + |1010\rangle + |1001\rangle + |0110\rangle + |0101\rangle + |0011\rangle\right)$$

N-particle Dicke states decay with up to N^2 speedup

Dicke states

- Question: When do we have a system that consists only of Dicke states?
- Answer 1: When there exists no mechanism to distinguish atoms. Example:



- Problem: interactions (dip-dip) drive system out of purely symmetric state! How to deal?
- (Answer 2: When the exchange interaction is infinitely high. In this case, other states cannot be reached. Example: "many-body protected manifolds")

Describing superradiance

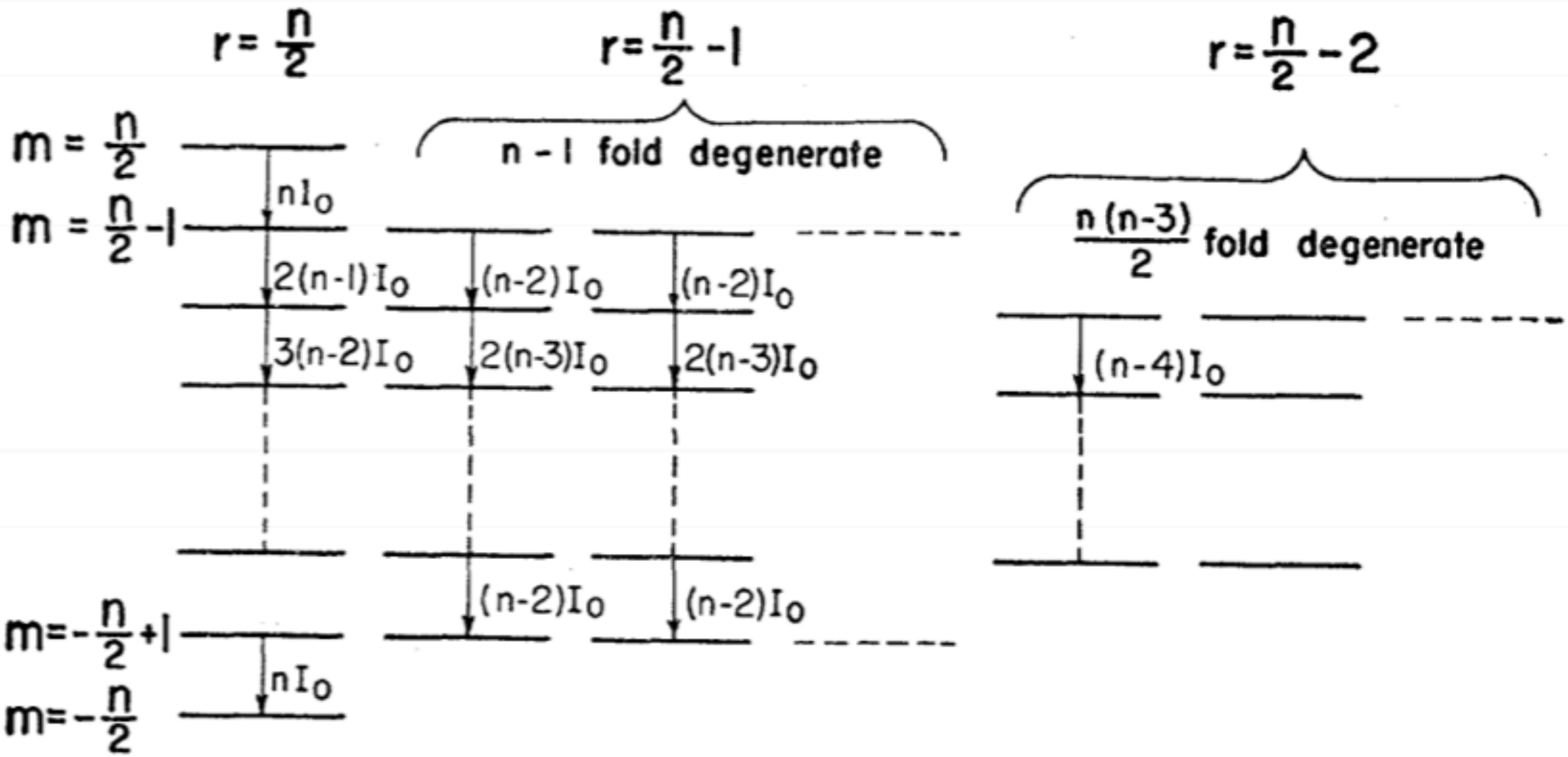
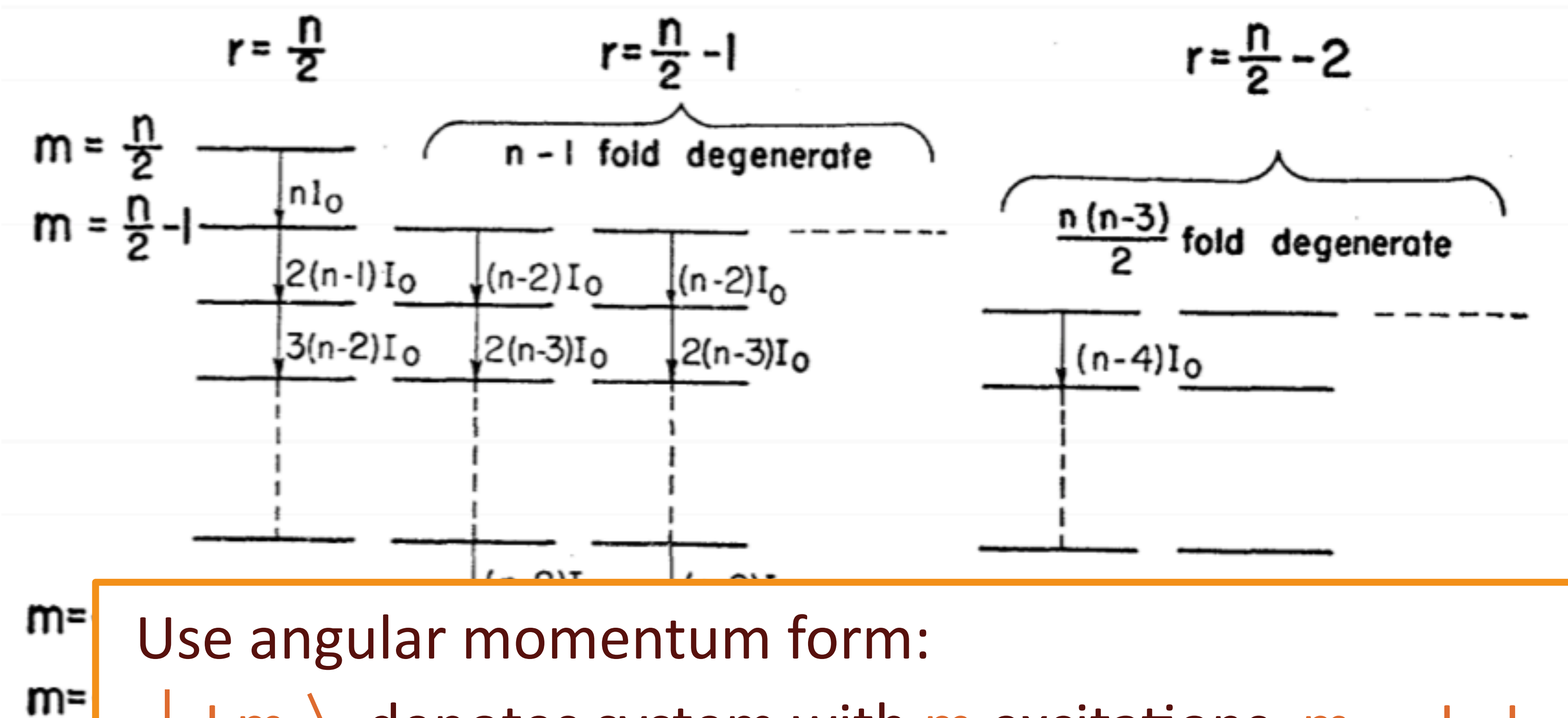


FIG. 1. Energy level diagram of an n-molecule gas, each molecule having 2 nondegenerate energy levels. Spontaneous radiation rates are indicated. $E_m = mE$.

Describing superradiance



Use angular momentum form:

| J,m 〉 denotes system with m excitations, m = -J…J.

FIG. 1. Energy level diagram of an n-molecule gas, each molecule having 2 nondegenerate energy levels. Spontaneous radiation rates are indicated. $E_m = mE$.

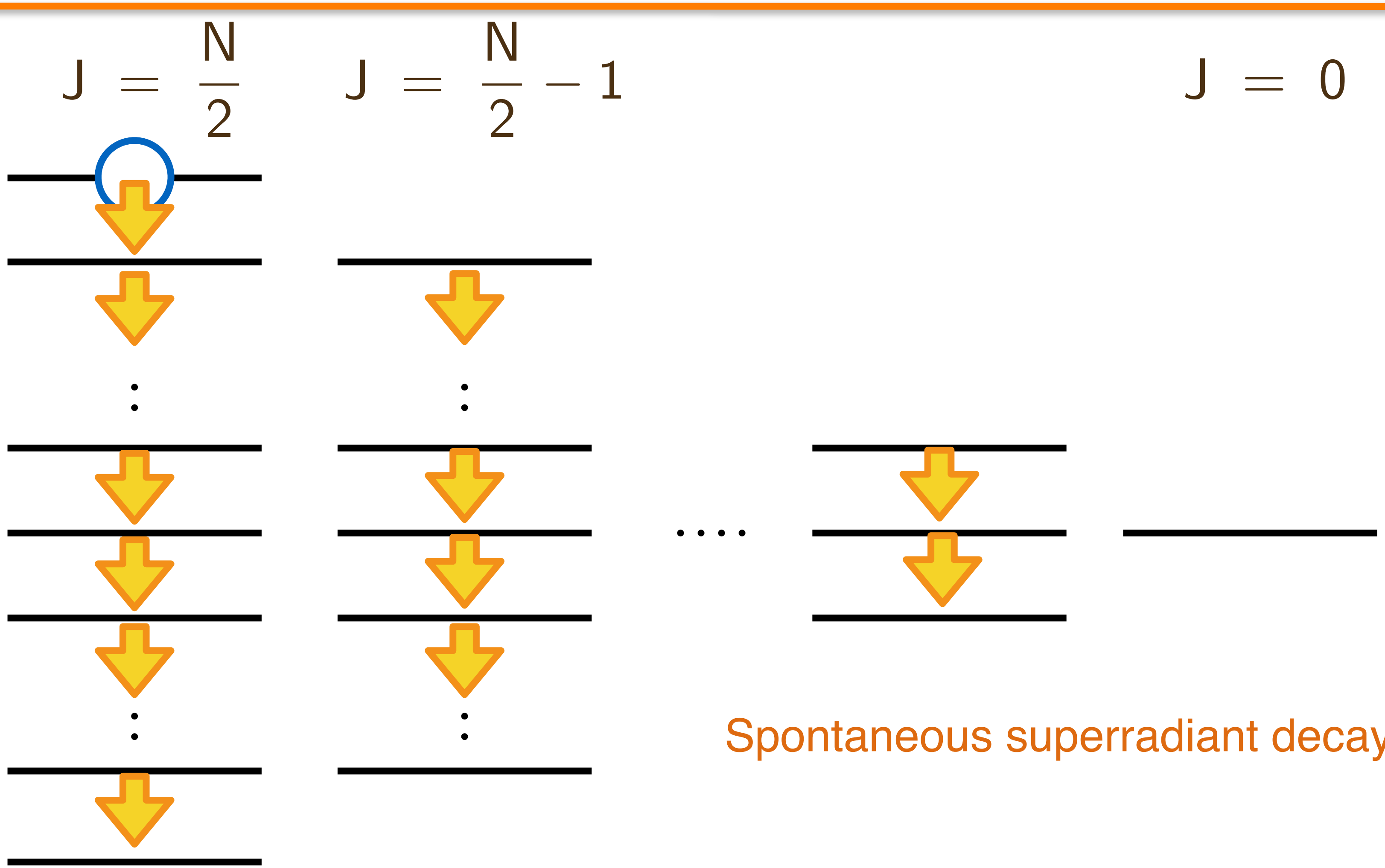
Angular momentum states

- Form of 3-atom Dicke state?
- What are the other states?
- 4 atoms?
- 40?

Population of symmetric states

- What possible path could a system starting in |11111…> take,
- with only decay?
- when there is dipole-dipole interaction?

State connections



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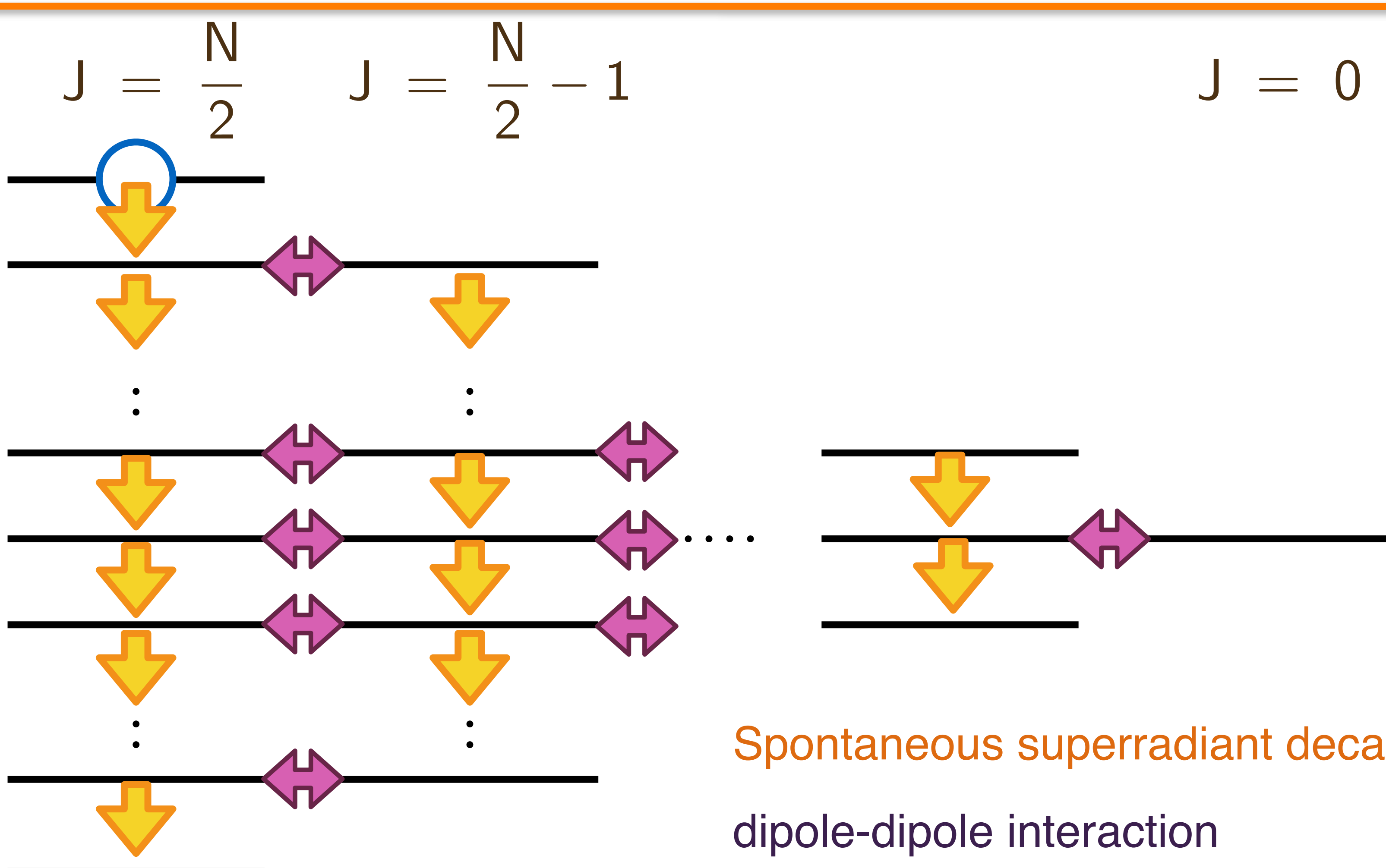

Spontaneous superradiant decay

dipole-dipole interaction

Form of dipole-dipole interaction

$$V_{\text{dip-dip}} \propto \sum_{i \neq j} \frac{e^{i\theta_{ij}}}{x_{ij}^3} \left[\left(1 - \cos^2\theta_{ij}\right) x_{ij}^2 + \left(1 - 3\cos^2\theta_{ij}\right)\left(i x_{ij} - 1\right) \right]$$

$$H_{\text{dip-dip}} = \sum_{i \neq j} V_{ij}\, \sigma_i^+ \sigma_j^-$$

“Exchange”term

- How would this show up in a master equation?

Form of dipole-dipole interaction

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dipole-dipole interaction:

- distance dependence
- angle dependence
- real + imaginary part

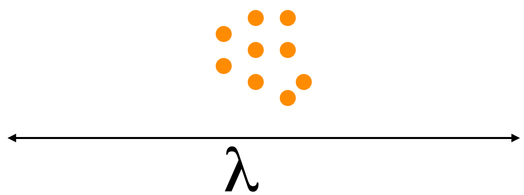
virtual real

photon exchange

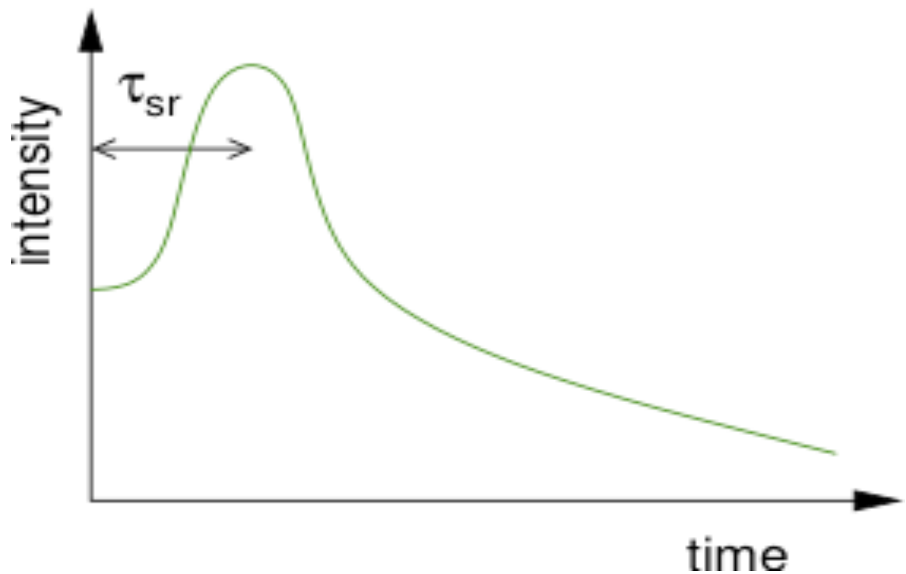
flip-flop "exchange"

shift/ dephasing

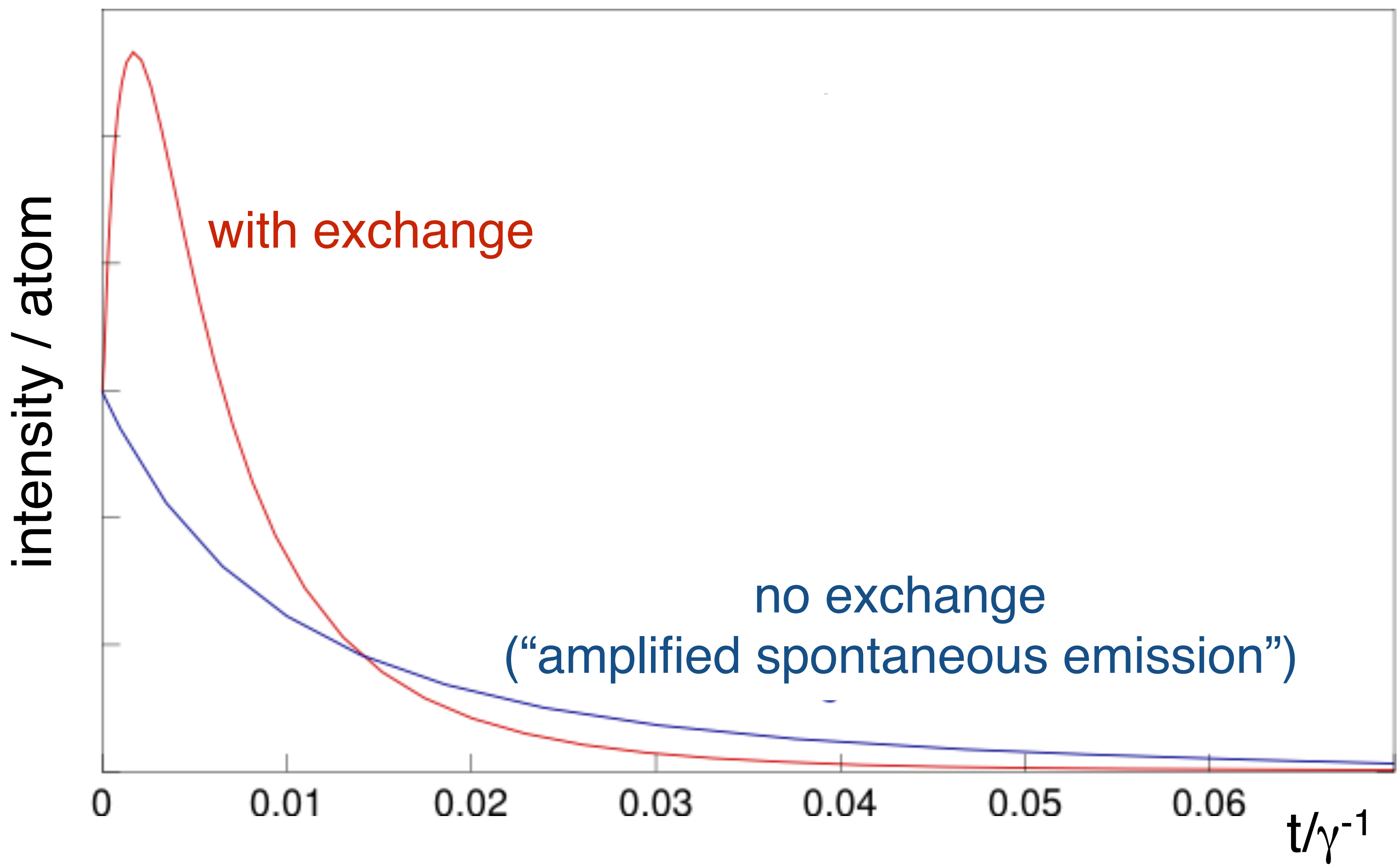
Atom-atom correlations in superradiance: Classic example



- Superradiance



Gross, Haroche, Phys. Rep. **93**, 301 ('82)



Fleischhauer, Yelin, *PRA 59, 2427 (99)*; Lin, Yelin, *Adv. Atom. Mol. Opt. Phys. 61, 295 (2012)*

What is “superradiance”?

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   - (e.g., collective $\sqrt{N}$ effects,
   - bad-cavity limit,
   - …)
2. Only systems involving cooperative (and nonlinear) effects
   - i.e., effect of exchange interaction
   - more than single excitation

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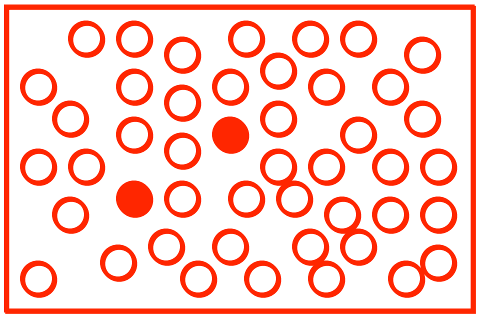
only for purists

Dynamics of atoms in dense media - Schwinger-Keldysh & Dyson Eq.

Full dynamics (all degrees of freedom of atoms, fields)



two probe atoms
+
surrounding atoms

Two atoms + field

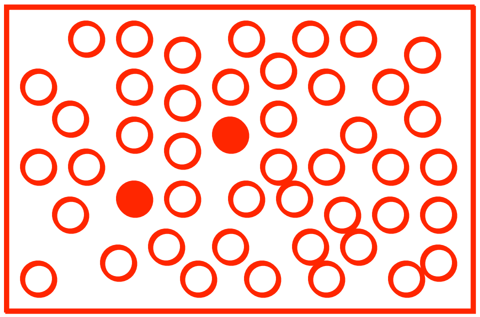
effective two-atom description

Fleischhauer, Yelin, *PRA 59, 2427 (99)*; Lin, Yelin, *Adv. Atom. Mol. Opt. Phys. 61, 295 (2012)*

Dynamics of atoms in dense media - Schwinger-Keldysh & Dyson Eq.

Full dynamics (all degrees of freedom of atoms, fields)

$$H = H_{\text{atoms}} + H_{\text{field}} - \sum \mathbf{p}_i \mathbf{E}_i$$



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+
surrounding atoms

Two atoms + field

$$V_{\text{probe}} = \sum_{i=1,2} \mathbf{p}_i \mathbf{E}_{iH} \Rightarrow S = Te^{-\frac{i}{\hbar}\int d\tau V_{\text{probe}}(\tau)}$$

$$\Big\langle \quad \dots \quad \Big\rangle$$

field degrees of freedom

- $\langle e^{sX} \rangle = e^{\sum \frac{s^m}{m!} \langle\langle X^m \rangle\rangle}$
- Gauss: $m \leq 2$

effective two-atom description

Two atom Master equation

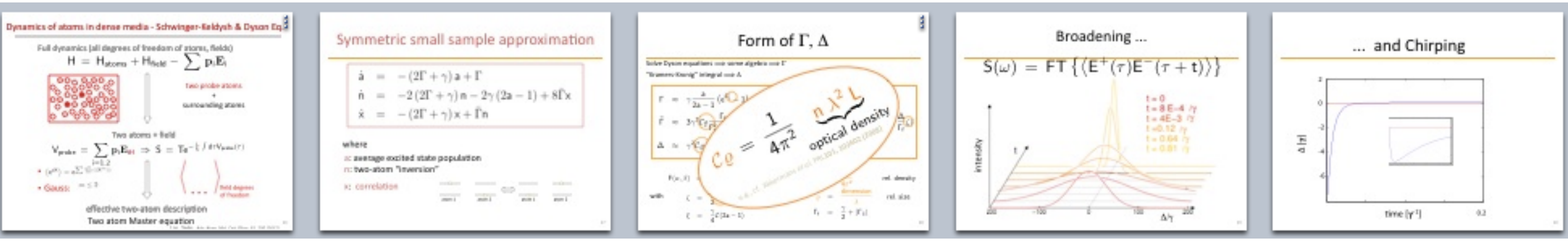
Fleischhauer, Yelin, *PRA 59, 2427 (99)*; Lin, Yelin, *Adv. Atom. Mol. Opt. Phys. 61, 295 (2012)*

$$\int \left\langle\!\left\langle \hat{E}_i(t_1)\hat{E}_j(t_2) \right\rangle\!\right\rangle$$

trace out field degrees of freedom

$$V_{\text{dip-dip}} \propto \sum_{i\neq j} \frac{e^{i\theta_{ij}}}{x_{ij}^3}\left[\left(1-\cos^2\theta_{ij}\right)x_{ij}^2 + \left(1-3\cos^2\theta_{ij}\right)\left(ix_{ij}-1\right)\right]$$

Can one expect superradiance?



The important parameter is

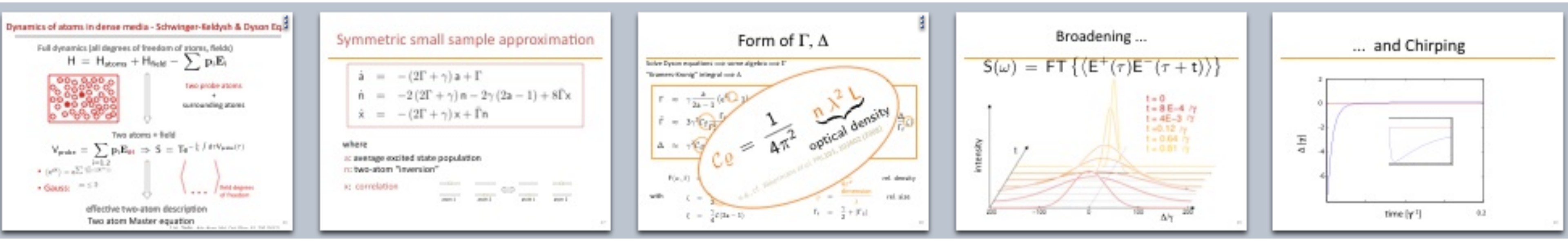
$$n\lambda^2 r$$

optical depth

n: density, λ: wavelength, r: system size

Lin, Yelin, Adv. Atom. Mol. Opt. Phys. 61, 295 (2012)

Can one expect superradiance?



The important parameter is

or $n\lambda^3$?

$$n\lambda^2 r$$

optical depth

n: density, λ: wavelength, r: system size

Lin, Yelin, Adv. Atom. Mol. Opt. Phys. 61, 295 (2012)

Master Equation

$$\dot{\rho}_{i,j} = \frac{\mathrm{i}}{\hbar}\sum_{\mu}\wp_{\mu}\sum_{k=i,j}[\sigma_{k\mu}\mathcal{E}^{-}_{L,\mu}(\vec{r}_k) + \sigma^{\dagger}_{k\mu}\mathcal{E}^{+}_{L\mu}(\vec{r}_k), \rho]$$

$$+ \frac{\mathrm{i}}{\hbar}\sum_{\mu,\nu}\sum_{k=i,j} H_{k\mu,k\nu}[[\sigma_{k\mu}, \sigma^{\dagger}_{k\nu}], \rho]$$

$$-\sum_{\mu,\nu}\sum_{k,l=i,j}\frac{\Gamma_{k\mu,l\nu}}{2}\left([\rho\sigma_{k\mu}, \sigma^{\dagger}_{l\nu}] + [\sigma_{k\mu}, \sigma^{\dagger}_{l\nu}\rho]\right)$$

$$-\sum_{\mu,\nu}\sum_{k,l=i,j}\frac{\Gamma_{k\mu,l\nu} + \gamma_{k\mu,l\nu}}{2}\left([\rho\sigma^{\dagger}_{l\nu}, \sigma_{k\mu}] + [\sigma^{\dagger}_{l\nu}, \sigma_{k\mu}\rho]\right)$$

Master Equation

$$\dot{\rho}_{i,j} = \frac{\mathrm{i}}{\hbar}\sum_{\mu}\wp_{\mu}\sum_{k=i,j}[\sigma_{k\mu}\mathcal{E}^{-}_{L,\mu}(\vec{r}_k) + \sigma^{\dagger}_{k\mu}\mathcal{E}^{+}_{L\mu}(\vec{r}_k), \rho]$$

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$$- \sum_{\mu,\nu}\sum_{k,l=i,j}\frac{\Gamma_{k\mu,l\nu} + \gamma_{k\mu,l\nu}}{2}\Big([\rho\sigma^{\dagger}_{l\nu}, \sigma_{k\mu}] + [\sigma^{\dagger}_{l\nu}, \sigma_{k\mu}\rho]\Big)$$

$$\left\langle\left\langle \hat{E}(t) \right\rangle\right\rangle$$

Mast

$$\langle\langle \hat{E}_1(t_1)\hat{E}_2(t_2) \rangle\rangle$$

$$\dot{\rho}_{i,j} = \frac{\mathrm{i}}{\hbar}\sum_{\mu} \wp_{\mu} \sum_{k=i,j} [\sigma_{k\mu}\mathcal{E}^{-}_{L,\mu}(\vec{r}_k) + \sigma^{\dagger}_{k\mu}\mathcal{E}^{+}_{L\mu}(\vec{r}_k), \rho]$$

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$$- \sum_{\mu,\nu}\sum_{k,l=i,j} \frac{\Gamma_{k\mu,l\nu}}{2}\Big([\rho\sigma_{k\mu}, \sigma^{\dagger}_{l\nu}$$

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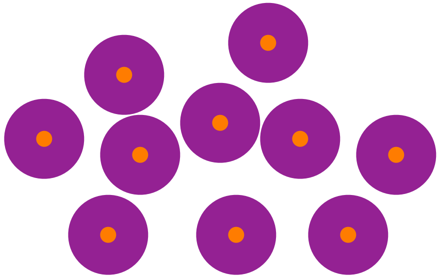
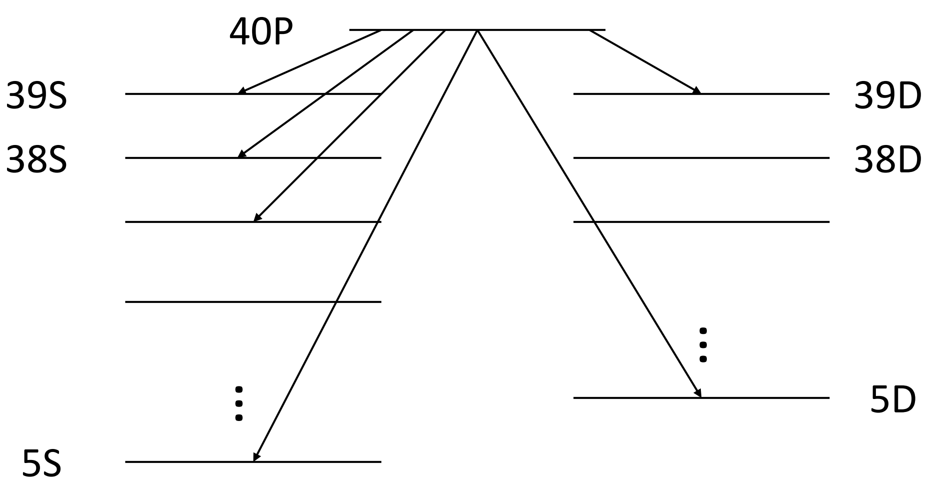
New experimental systems: example

- Ultracold Rydberg atoms

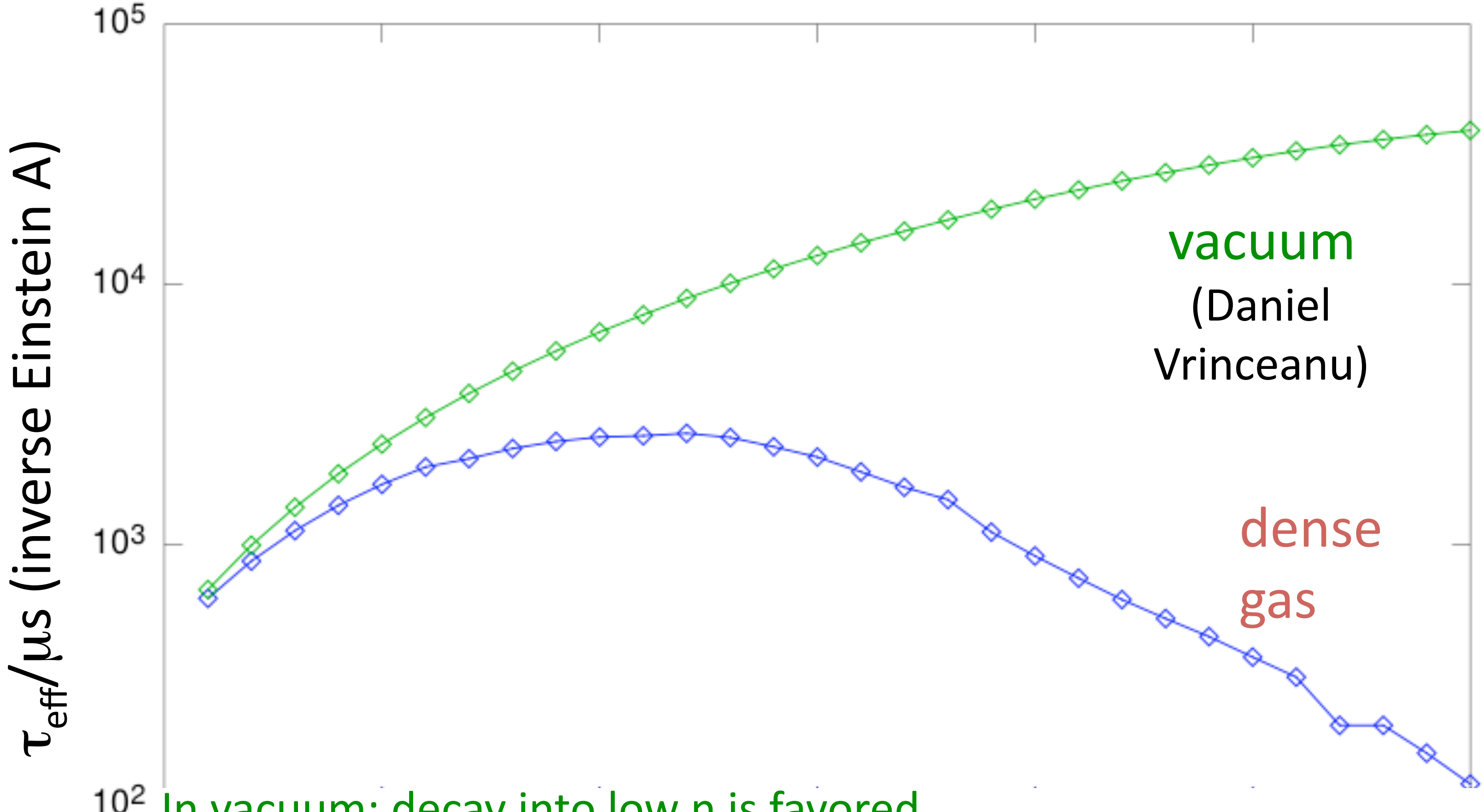


(Phil Gould, Ed Eyler, Uconn)

Rb



Effective decay times from 40P into nS

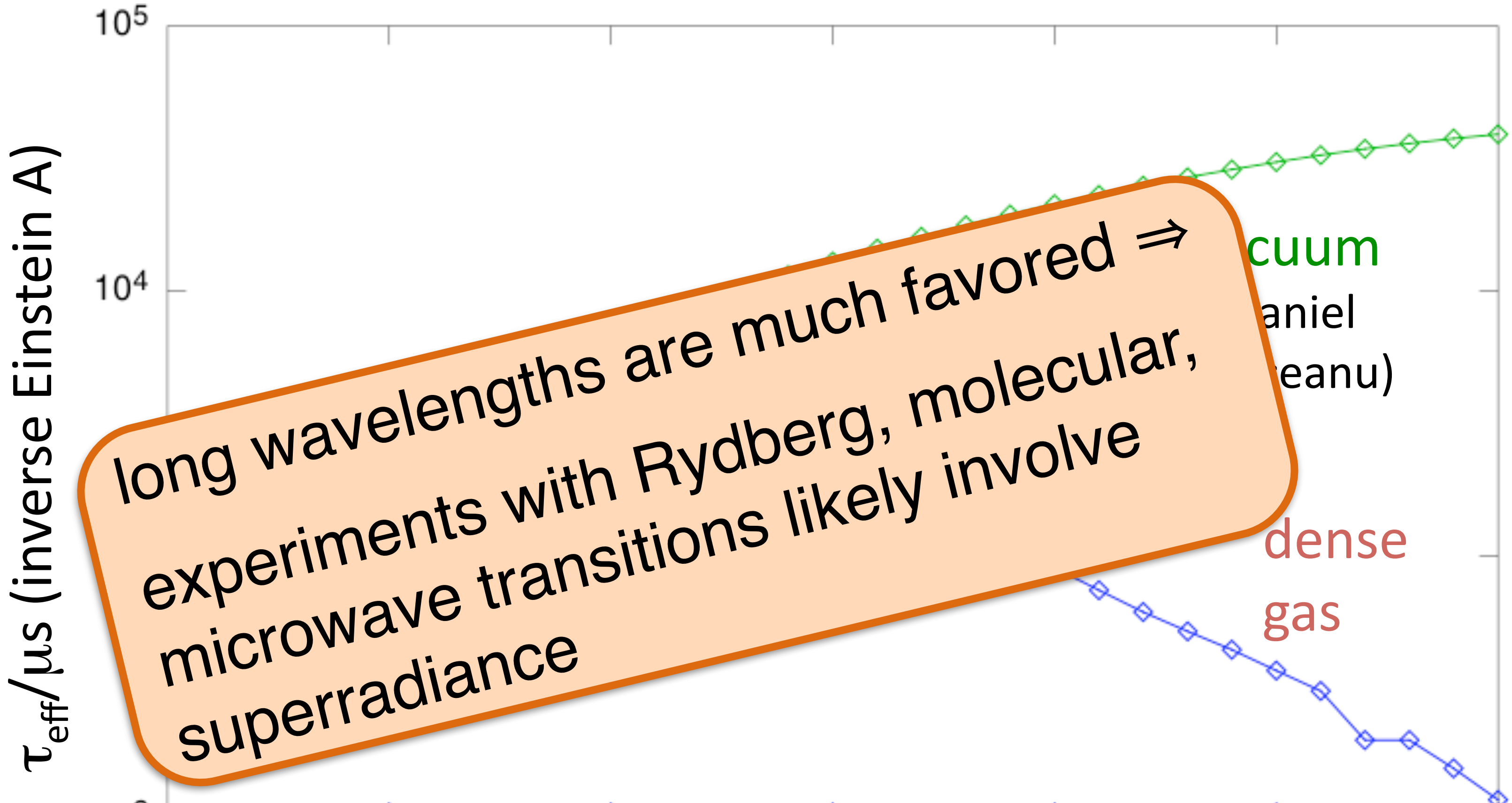


In vacuum: decay into low n is favored

In dense gas: decay into high n is favored ⇨ λ large, n λ^2 r large!

superradiant decay!

Effective decay times from 40P into nS



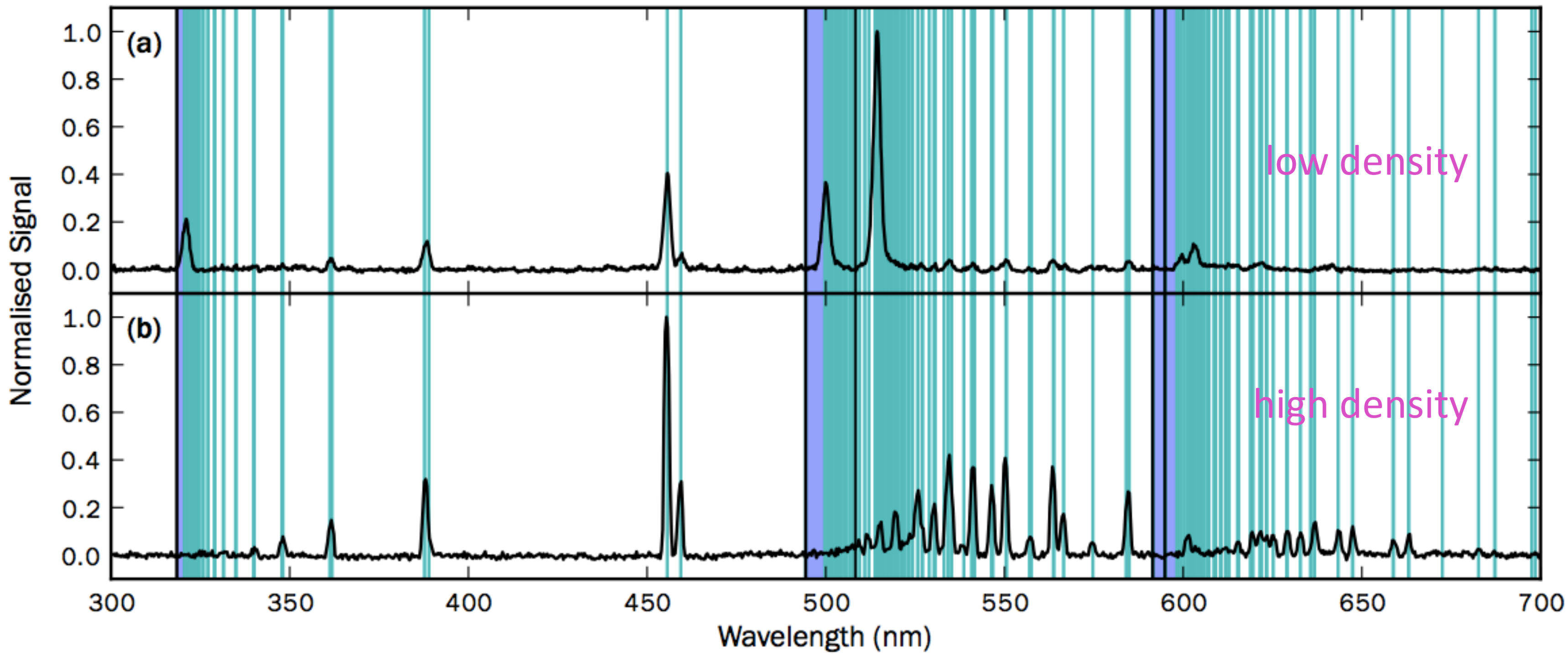
In vacuum: decay into low n is favored

In dense gas: decay into high n is favored ⇨ λ large, n λ^2 r large!

superradiant decay!

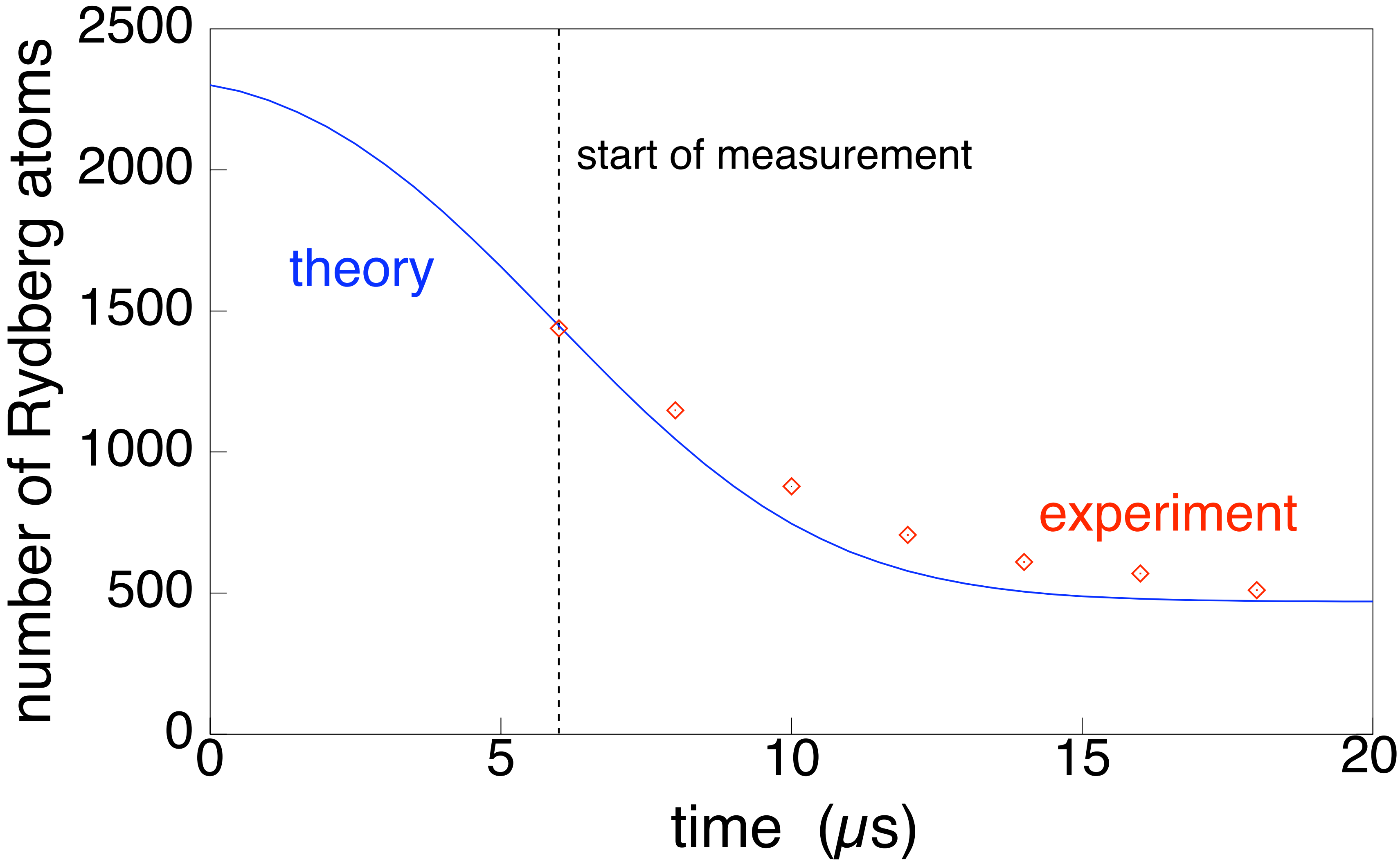
Wang, et al.,PRA 75, 033802 (2007); Lin & Yelin Mol. Phys. 111, 1917 (2013).

Experimental Proof!



Carr, Ritter, Wade, Adams, Weatherill, PRL **111**, 113901 (2013)

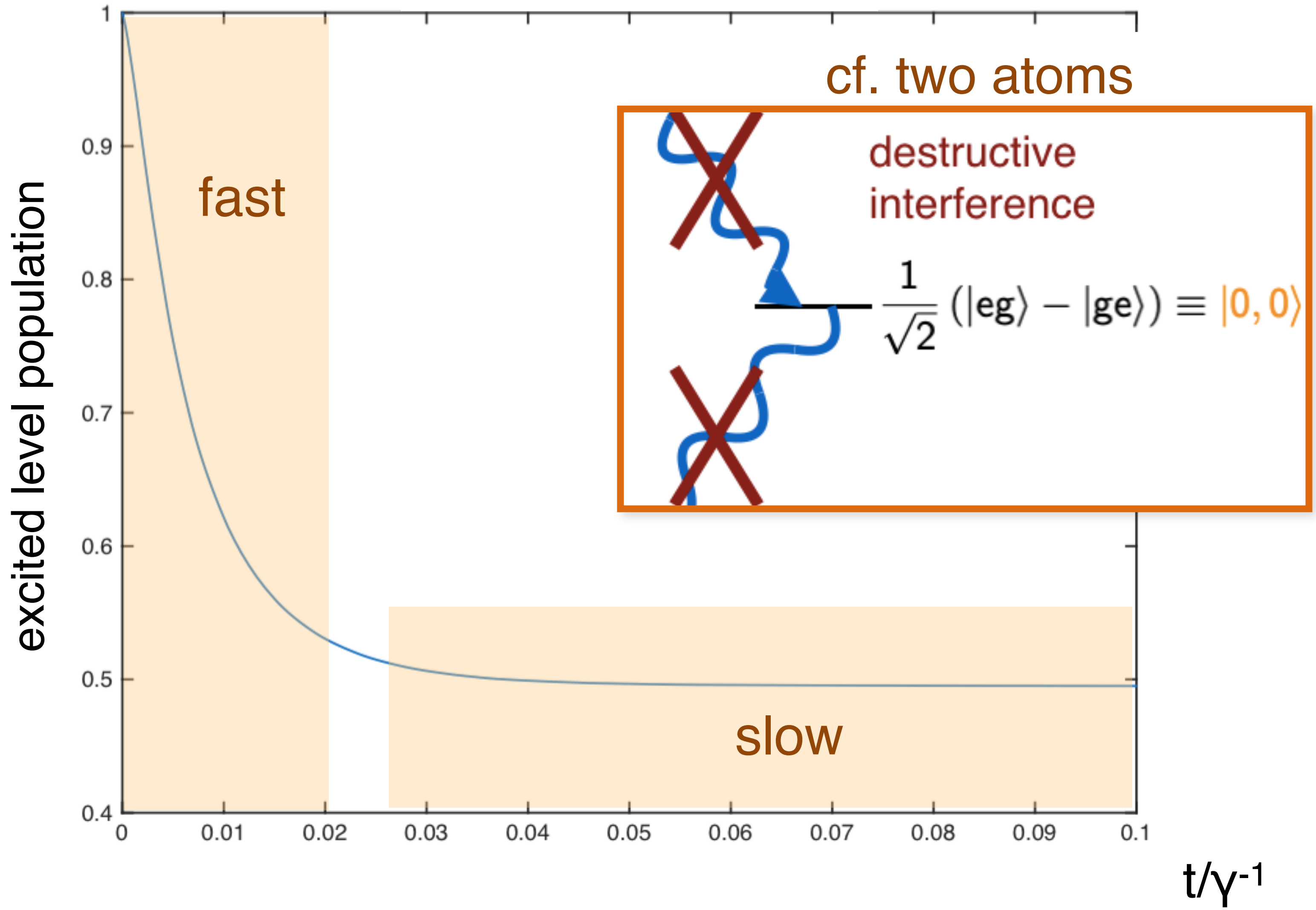
Superradiance in Rydberg systems



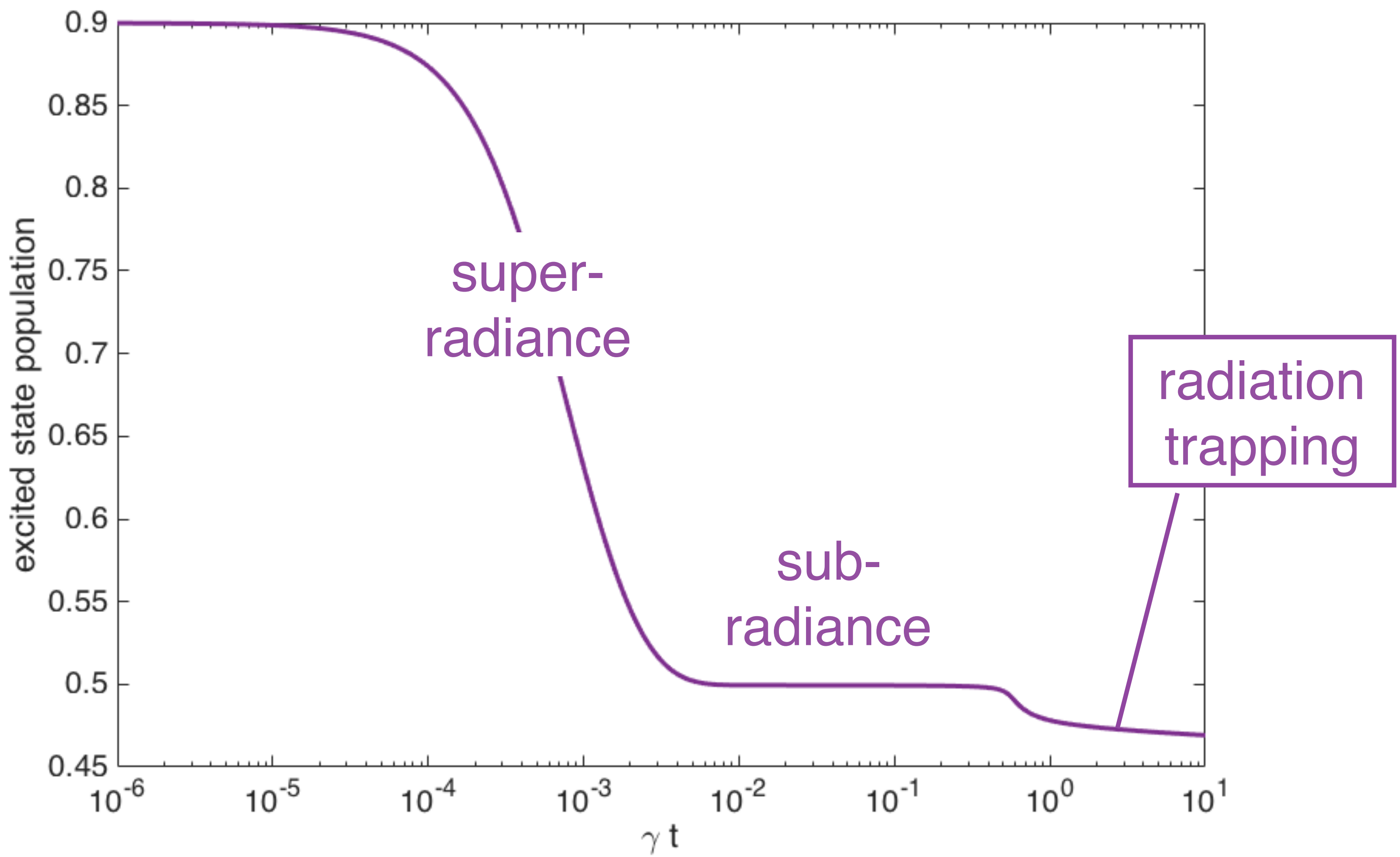
These lectures

- Cooperative effects in complex systems
  - Collective (Lamb) level shifts
  - Subradiance
  - Entanglement
- New application: atomically thin mirrors

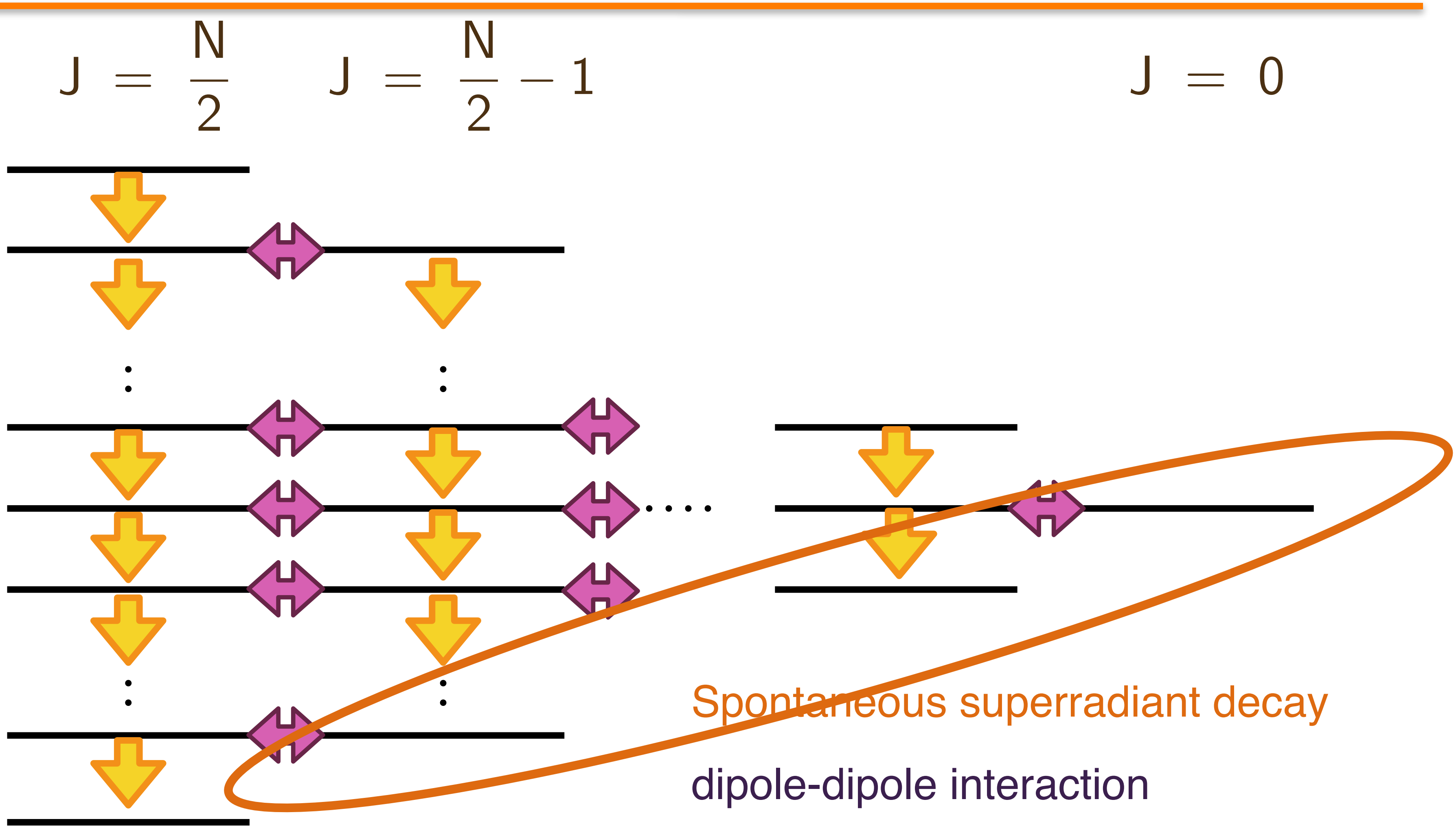
Decay dynamics



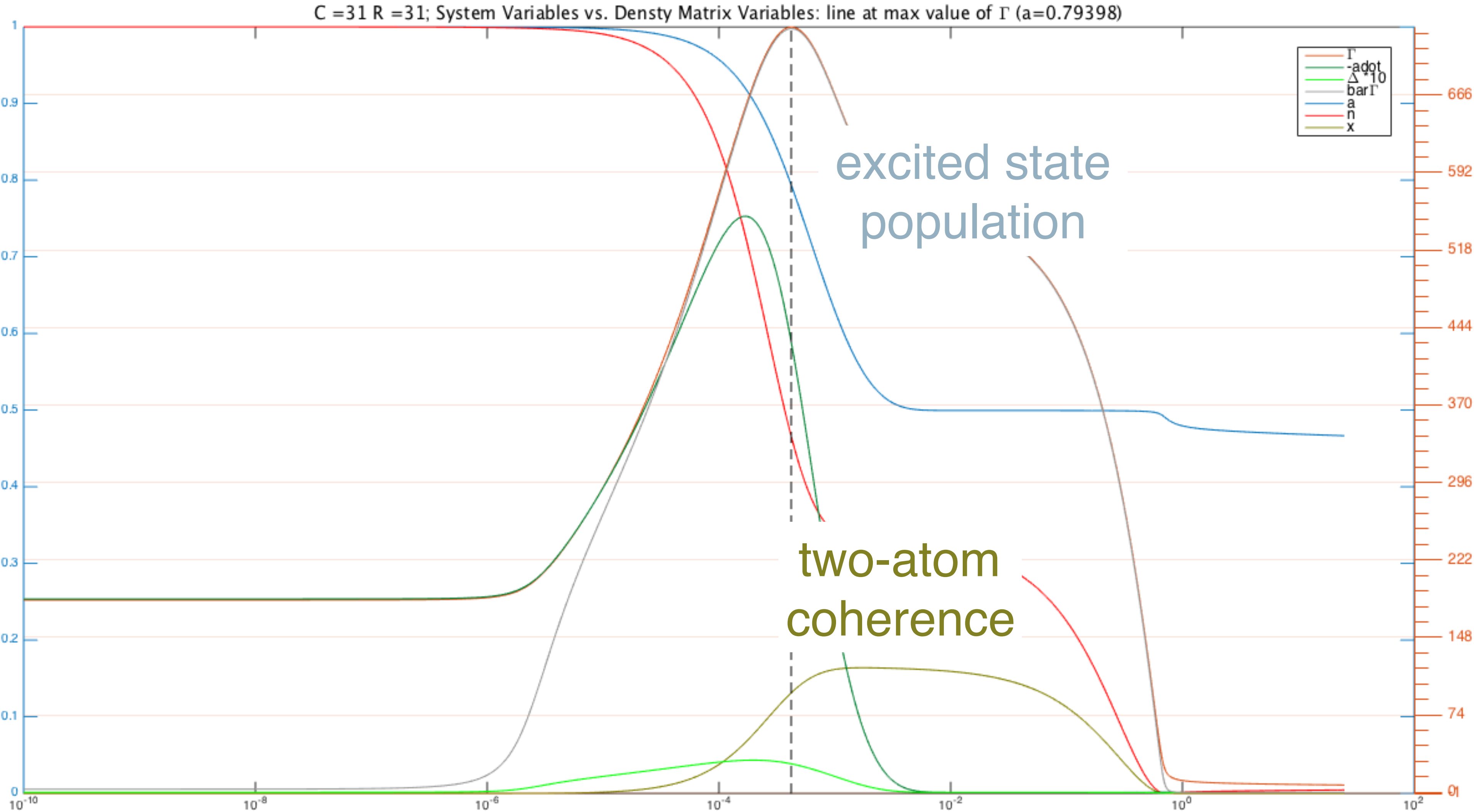
Subradiance?



Subradiant states



Transitions



Subradiance: Outlook

- Dynamics of subradiance = transition to many-body localized state?
  - ➡ Create, manipulate localization
- Engineered subradiance to create stable states without spontaneous decay
  - ➡ Create, stabilize many-body entangled state (Dissipative non-equilibrium physics)

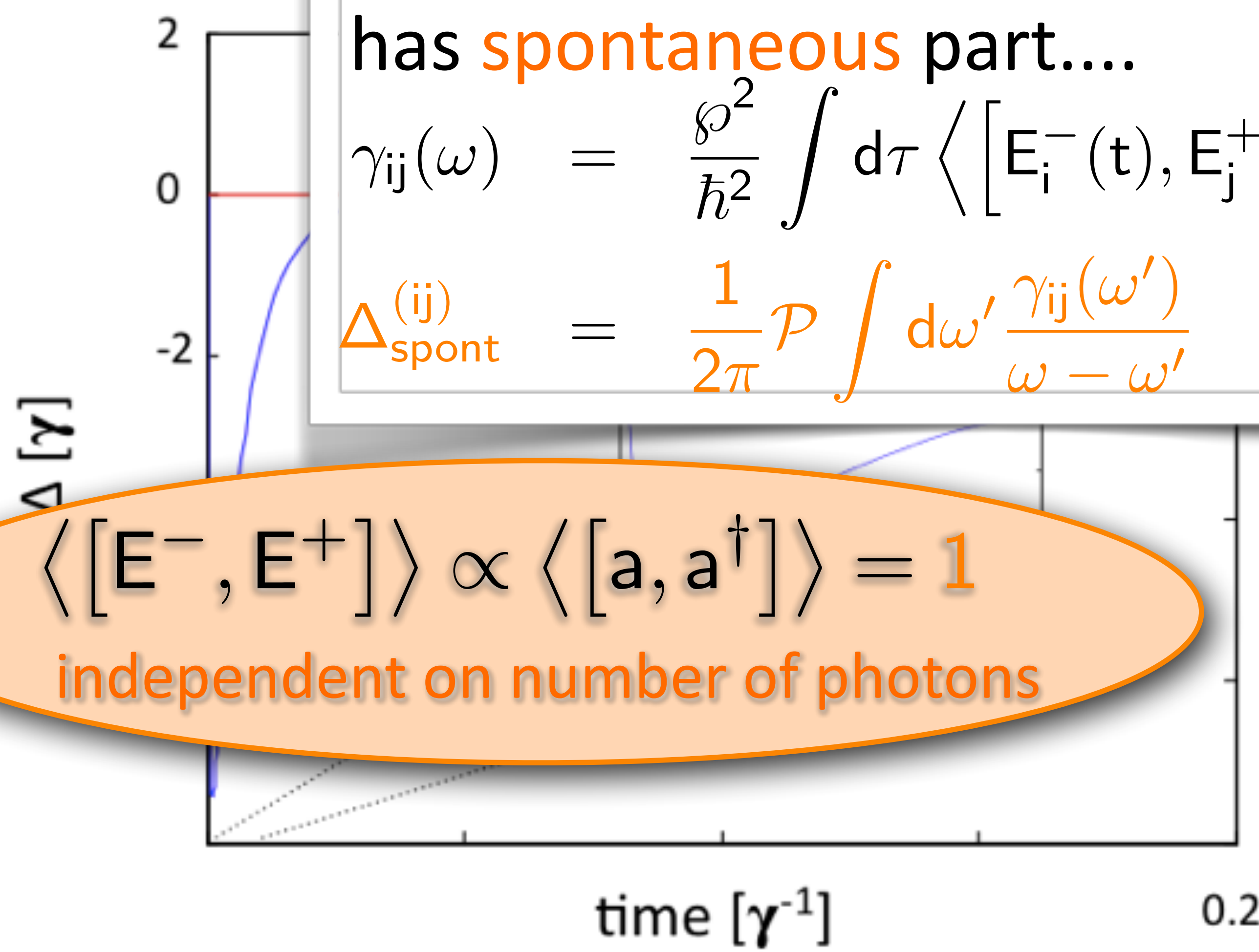
Collective Lamb shift

- "Lamb shift" is the result of interaction with the vacuum fluctuations
- In the case of altered density of states of the "vacuum" (i.e., the surrounding space), the value of the shift changes
- With a high (superradiant) density of radiators, the density of states inside the medium can be considerably altered

"Collective Lamb shift"

Putnam, Lin, Yelin, arXiv:1612.04477

Collective Shift



has spontaneous part....

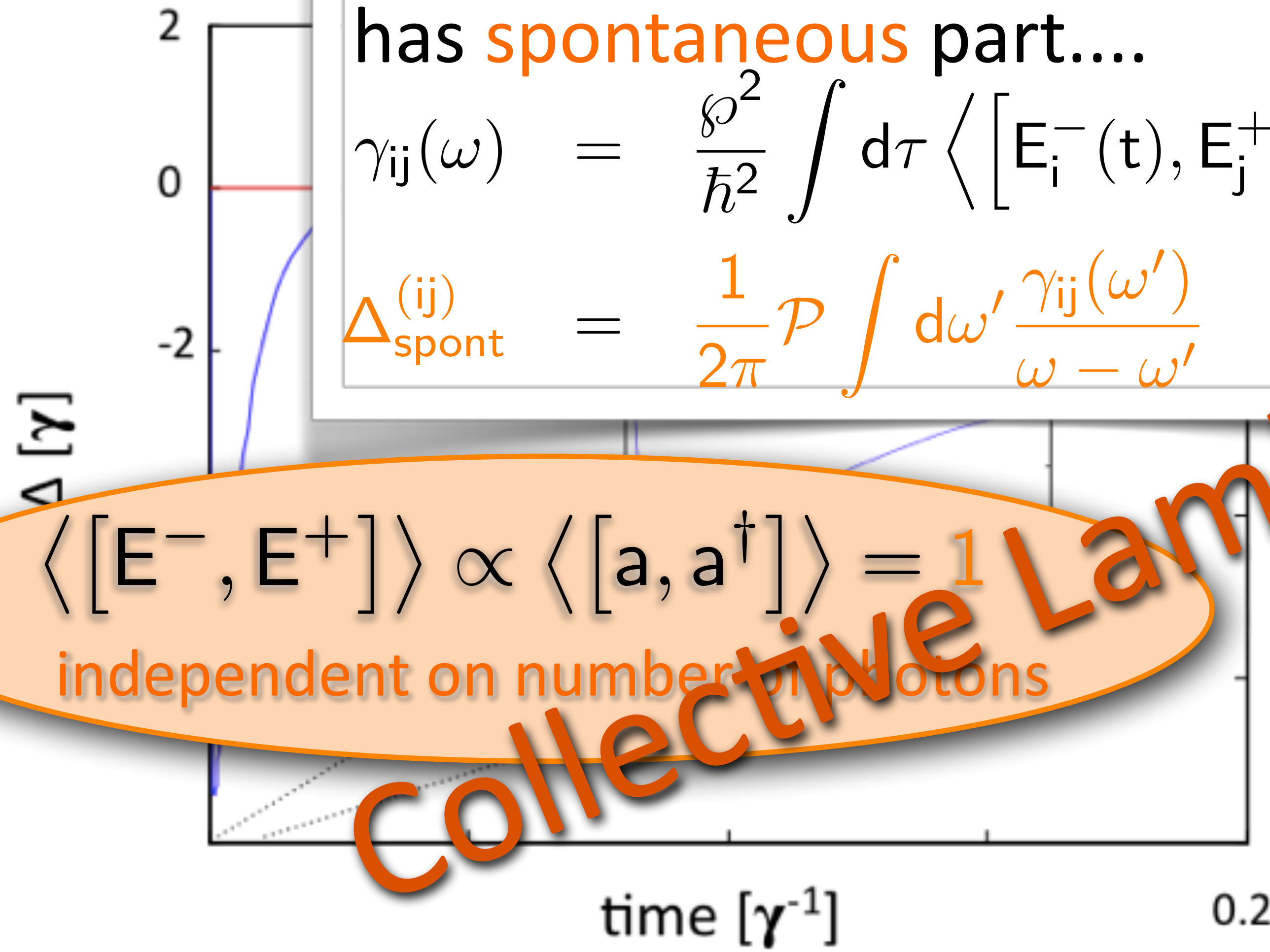
$$\gamma_{ij}(\omega) = \frac{\wp^2}{\hbar^2}\int d\tau \left\langle \left[E_i^-(t), E_j^+(t+\tau) \right] \right\rangle e^{i\omega\tau}$$

$$\Delta_{spont}^{(ij)} = \frac{1}{2\pi}\mathcal{P}\int d\omega' \frac{\gamma_{ij}(\omega')}{\omega - \omega'}$$

$$\left\langle \left[E^-, E^+ \right] \right\rangle \propto \left\langle \left[a, a^\dagger \right] \right\rangle = 1$$

independent on number of photons

Collective Shift



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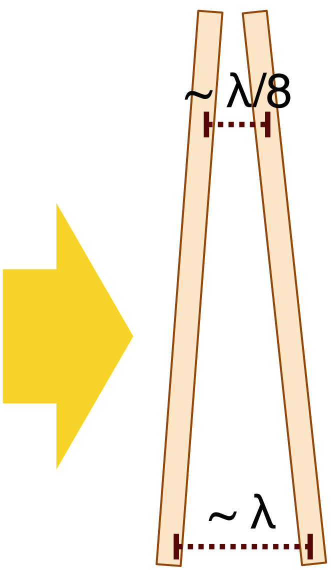
Collective Lamb Shift

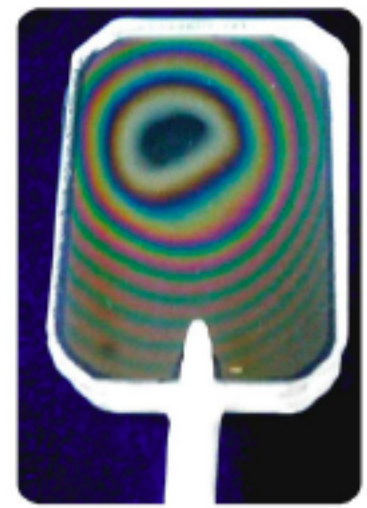
Collective Lamb Shift



cavity
with variable
thickness

Keaveney et al.,PRL **108**, 173601 (2012), theory: Lin, Li, Yelin, in prep.

Collective Lamb Shift



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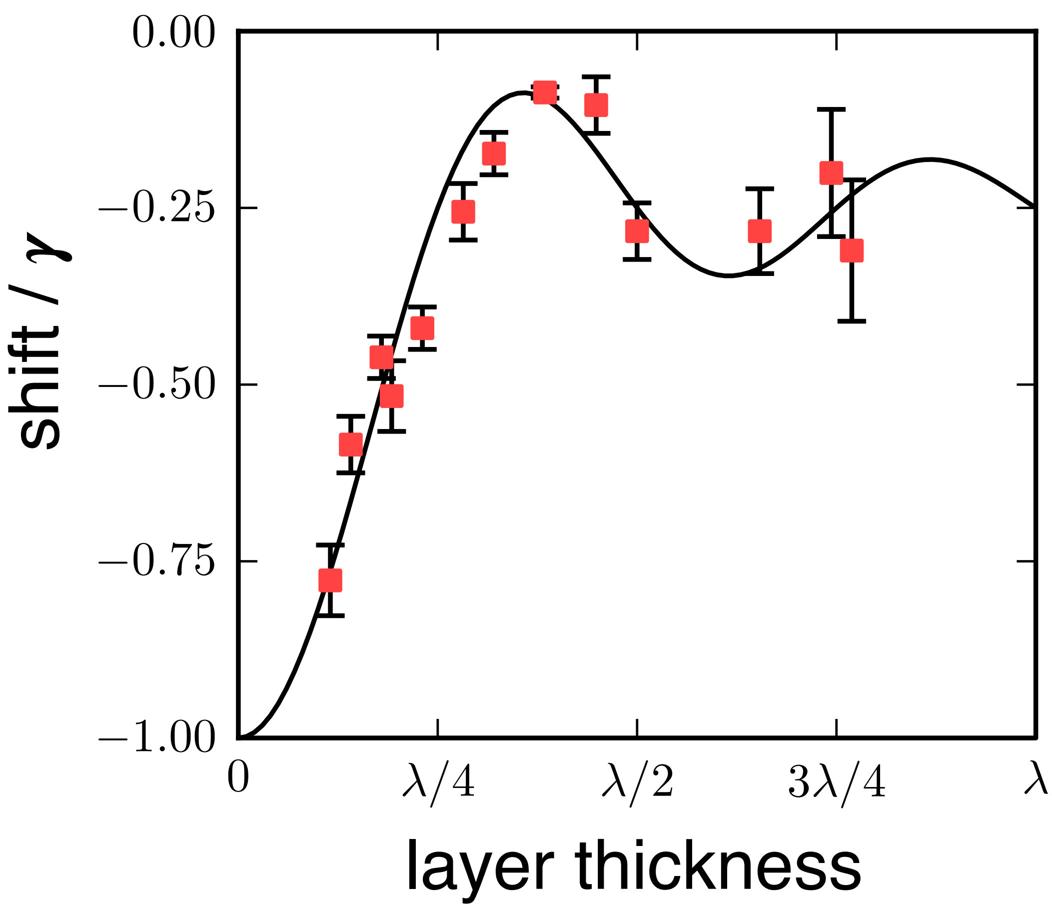

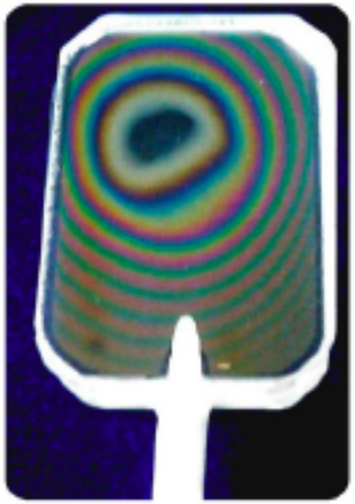

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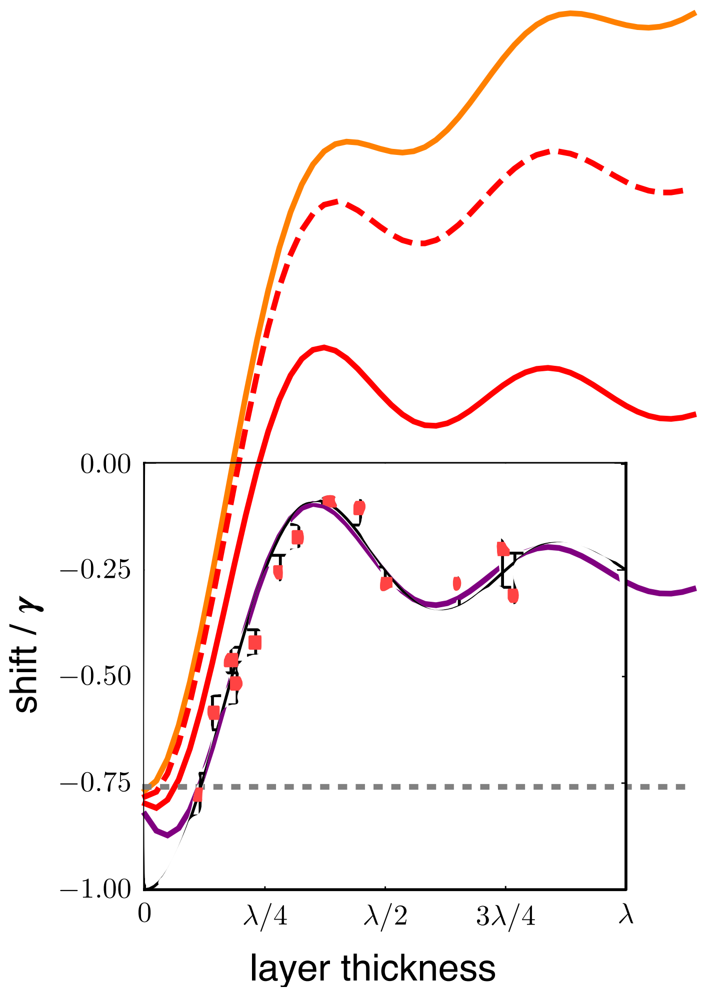
Collective Lamb Shift

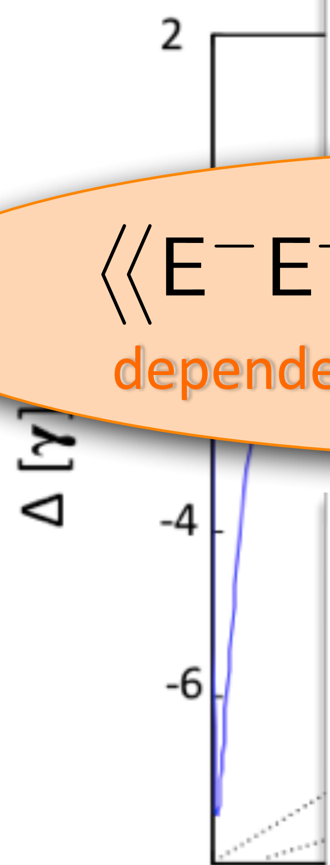


Optical depth = highest

Optical depth = lowest

Keaveney et al.,PRL **108**, 173601 (2012), theory: Lin, Li, Yelin, in prep.

Collective Shift



has spontaneous part....

$$\frac{\wp^2}{\ldots} \int d\tau \left\langle \left[E_i^-(t), E_j^+(t+\tau) \right] \right\rangle e^{i\omega\tau}$$

$$\langle\langle E^- E^+ \rangle\rangle \overset{\propto}{\sim} \langle\langle a^\dagger a \rangle\rangle = n$$

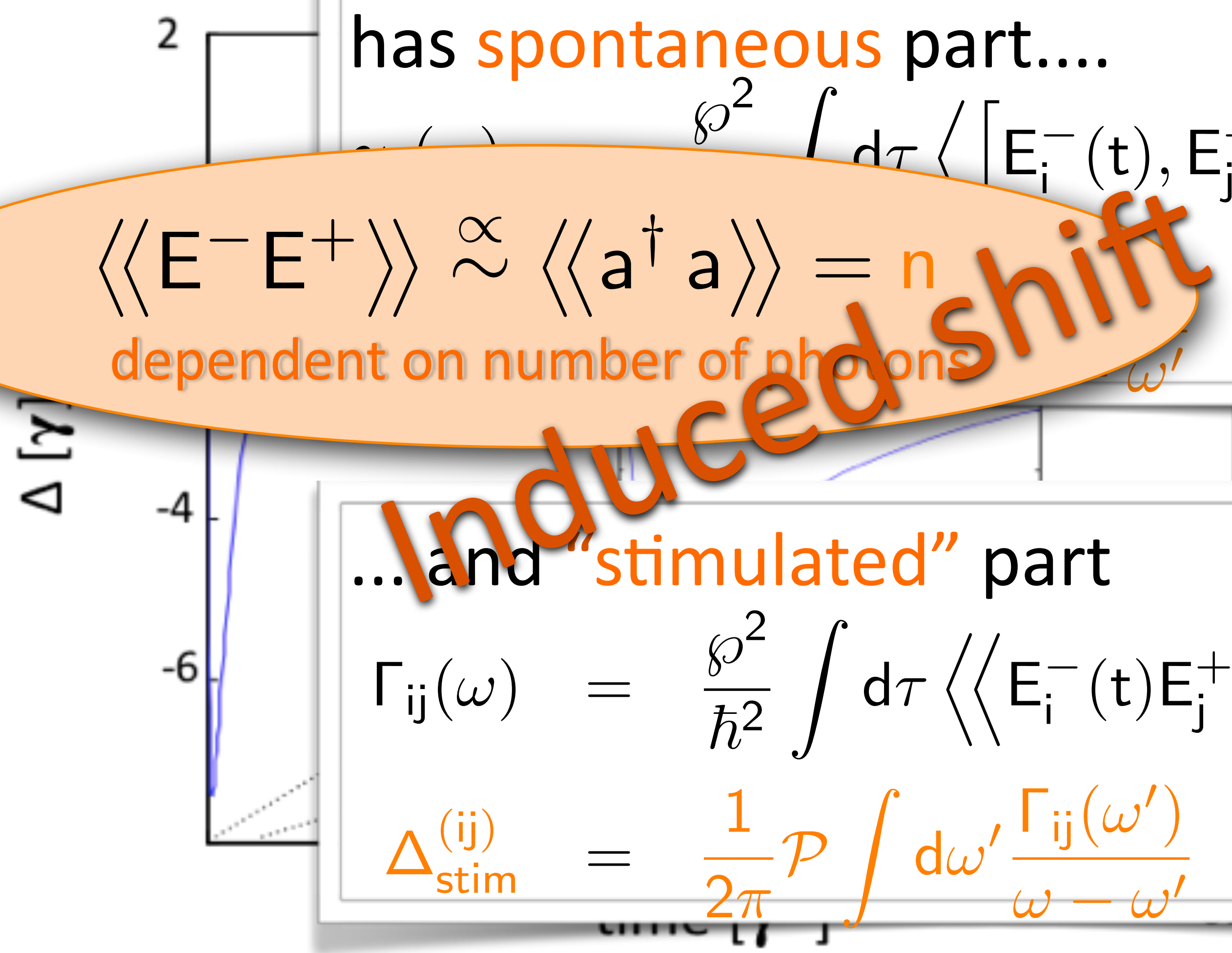
dependent on number of photons

... and "stimulated" part

$$\Gamma_{ij}(\omega) = \frac{\wp^2}{\hbar^2} \int d\tau \left\langle\left\langle E_i^-(t) E_j^+(t+\tau) \right\rangle\right\rangle e^{i\omega\tau}$$

$$\Delta_{stim}^{(ij)} = \frac{1}{2\pi} \mathcal{P} \int d\omega' \frac{\Gamma_{ij}(\omega')}{\omega - \omega'}$$

Collective Shift



has spontaneous part....

$$\ldots \frac{\wp^2}{\ldots} \int d\tau \left\langle \left[E_i^-(t), E_j^+(t+\tau) \right] \right\rangle e^{i\omega\tau}$$

$$\langle\langle E^- E^+ \rangle\rangle \overset{\propto}{\sim} \langle\langle a^\dagger a \rangle\rangle = n$$

dependent on number of photons

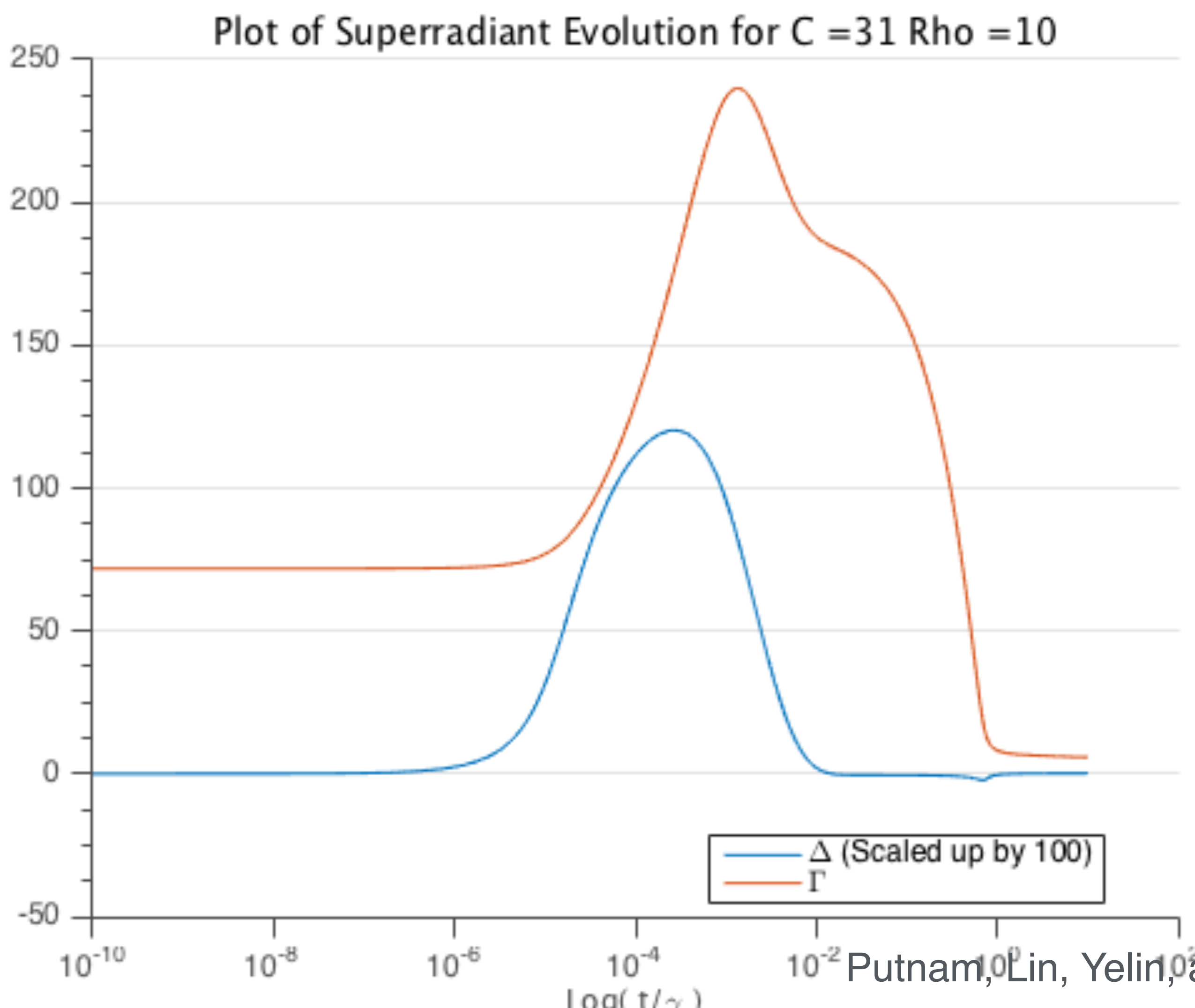
Induced shift

... and "stimulated" part

$$\Gamma_{ij}(\omega) = \frac{\wp^2}{\hbar^2} \int d\tau \left\langle\left\langle E_i^-(t) E_j^+(t+\tau) \right\rangle\right\rangle e^{i\omega\tau}$$

$$\Delta_{\text{stim}}^{(ij)} = \frac{1}{2\pi} \mathcal{P} \int d\omega' \frac{\Gamma_{ij}(\omega')}{\omega - \omega'}$$

Collective Shift: decay of inverted TLS



Putnam, Lin, Yelin, arXiv:1612.04477

These lectures

- Cooperative effects in complex systems
  - Collective (Lamb) level shifts
  - Subradiance
  - Entanglement
- New application: atomically thin mirrors

Superradiance and Entanglement

Does (Dicke) superradiance need/create entanglement?

NO

Wolfe, Yelin, PRL 112, 140402 ('14)

Example: 2-atom Dicke

- PPT (Peres-Horodecki) criterion:

  Eigenvalues of partial positive transpose ≥ 0

$$\rho = \sum_{\mathsf{ijkl}} \mathsf{p}^{\mathsf{ij}}_{\mathsf{kl}} |\mathsf{i}\rangle\langle \mathsf{j}| \otimes |\mathsf{k}\rangle\langle \mathsf{l}|$$

$$\rho^{\mathsf{PPT}} = \sum_{\mathsf{ijkl}} \mathsf{p}^{\mathsf{ij}}_{\mathsf{kl}} |\mathsf{i}\rangle\langle \mathsf{j}| \otimes |\mathsf{l}\rangle\langle \mathsf{k}|$$

Superradiance and Entanglement

How to define/calculate many-particle entanglement?

O Gühne, G Tóth

Multipartite entanglement in four-qubi… …lass states

YK Bai, ZD Wang

Exact and asymptotic measur… …ite pure-state entanglement

CH Bennett, S Popesc… …, JA Smolin

Geometric measur… …ment and applications to bipartite and m… …ntum states

TC Wei, PM…

Scalable m… entanglement of trapped ions

H Häffner, W… …sel, CF Roos, J Benhelm, M Chwalla

~ 10,000 for "definition of multipartite entanglement"

Superradiance and Entanglement

Does (Dicke) superradiance need/create entanglement? (Initial state: no entanglement)

Dicke superradiant time evolution = separable states

constructive proof

Wolfe, Yelin, PRL 112, 140402 ('14)

Superradiance and Entanglement

Does (Dicke) superradiance need/create entanglement? (Initial state: no entanglement)

Dicke superradiant states	=	separable states

Wolfe, Yelin, Wolfe, Yelin, PRL 112, 140402 ('14)

Superradiance and Entanglement

Does (Dicke) superradiance need/create entanglement? (Initial state: no entanglement)

our system: mixed state of N-atom Dicke states with N+1 known independent coefficients p_i

=

compare to mixture of symmetric product states of N (two-level) atoms (needs N+1 coefficients y_i)

(N+1) - dim. equation system

Wolfe, Yelin, Wolfe, Yelin, PRL 112, 140402 ('14)

Form of equations

- General Dicke states:

$$\rho_{\mathrm{GDS}} = \sum_{\mathbf{n}} \chi_{\mathbf{n}} \left| D_{\mathbf{n}} \right\rangle \left\langle D_{\mathbf{n}} \right|$$

- Separable diagonally symmetric:

$$\rho_{\mathrm{SDS}} = N! \sum_{\mathbf{n}} \sum_{j=1}^{j_{\max}} \frac{x_j {y_j}^{n_0} (1 - y_j)^{n_1}}{n_0! n_1!} \left| D_{\mathbf{n}} \right\rangle \left\langle D_{\mathbf{n}} \right|$$

Superradiance and Entanglement

Does (Dicke) superradiance need/create entanglement? (Initial state: no entanglement)

our system: mixe… of
N-atom Dicke st…
N+1 known inde…
coefficie…

…npare to mixture of
…netric product states of
…two-level) atoms (needs
N+1 coefficients y_i)

condition:
all coefficients
$0 \leq p_i \leq 1$

…uation
system

Wolfe, Yelin, PRL 112, 140402 ('14)

Superradiance and Entanglement

Does (Dicke) superradiance need/create entanglement? (Initial state: no entanglement)

our system: mixed state of N-atom Dicke states with N+1 known indepen... coefficients p_i

=

com... mixture of symm... states of N (t... ms (needs N... ents y_i)

(works!)

condition: all coefficients $0 \le p_i \le 1$

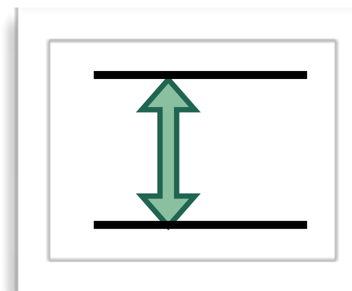
...+1) - dim. equation system

Wolfe, Yelin, PRL 112, 140402 ('14)

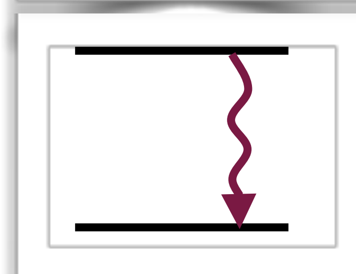
Superradiance and Entanglement

Driven superradiant system:

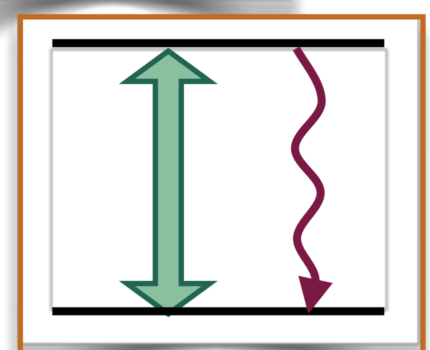
- Driving alone does not entangle atoms



- Superradiance alone does not entangle atoms



- Driving and superradiance together entangle atoms!



Fuzzy Bunny?

Spin Squeezing

- Correlated ("squeezed") spins could improve resolution in one direction ("quadrature").

(Spin) Squeezing

- How to measure squeezing/measurement improvement?

$$\xi^2 \equiv \frac{\text{optimal variance}}{\text{unsqueezed optimal variance}}$$

Kitagawa, Ueda, PRA 47, 5138 (93)

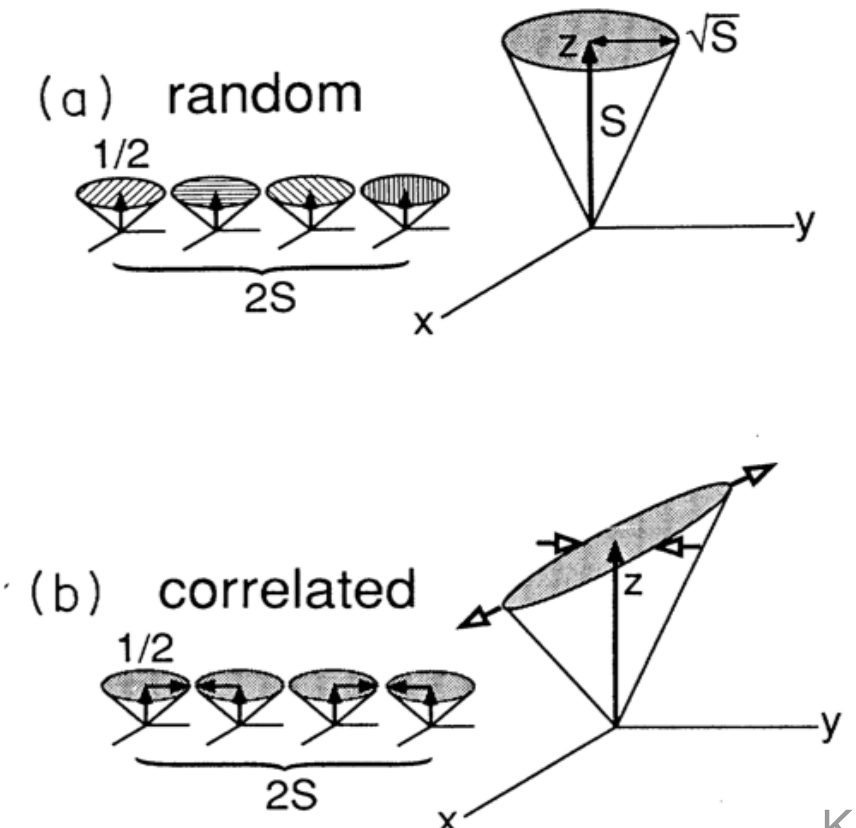
Spin squeezing

Old problem: How to improve metrology by spin squeezing ensembles

➡ Groups of Bigelow, Kuzmich, Lewenstein, Mølmer, Polzik, Sanders, Sørensen, Vuletic, Wineland,…

Spin Squeezing

- Correlated ("squeezed") spins could improve resolution in one direction ("quadrature").



Kitagawa, Ueda, PRA 47, 5138 (93)

(Spin) Squeezing

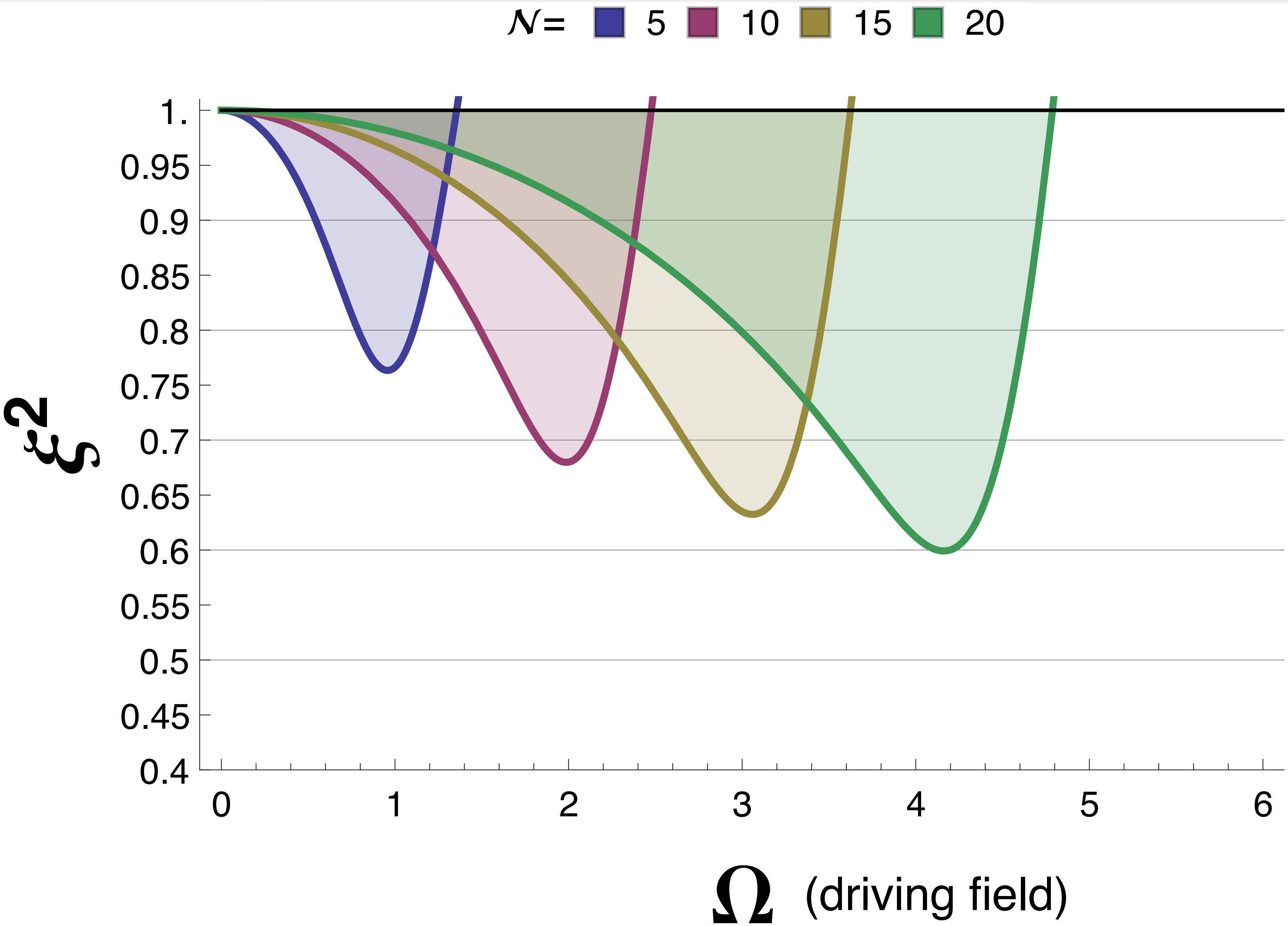
- How to measure squeezing/measurement improvement?

$$\xi^2 \equiv \frac{\text{optimal variance}}{\text{unsqueezed optimal variance}}$$

$$\xi^2 = \frac{N}{2}\Big[\langle J_1^2 + J_2^2\rangle - \sqrt{\langle J_1^2 - J_2^2\rangle^2 + \langle J_1 J_2 + J_2 J_1\rangle^2}$$

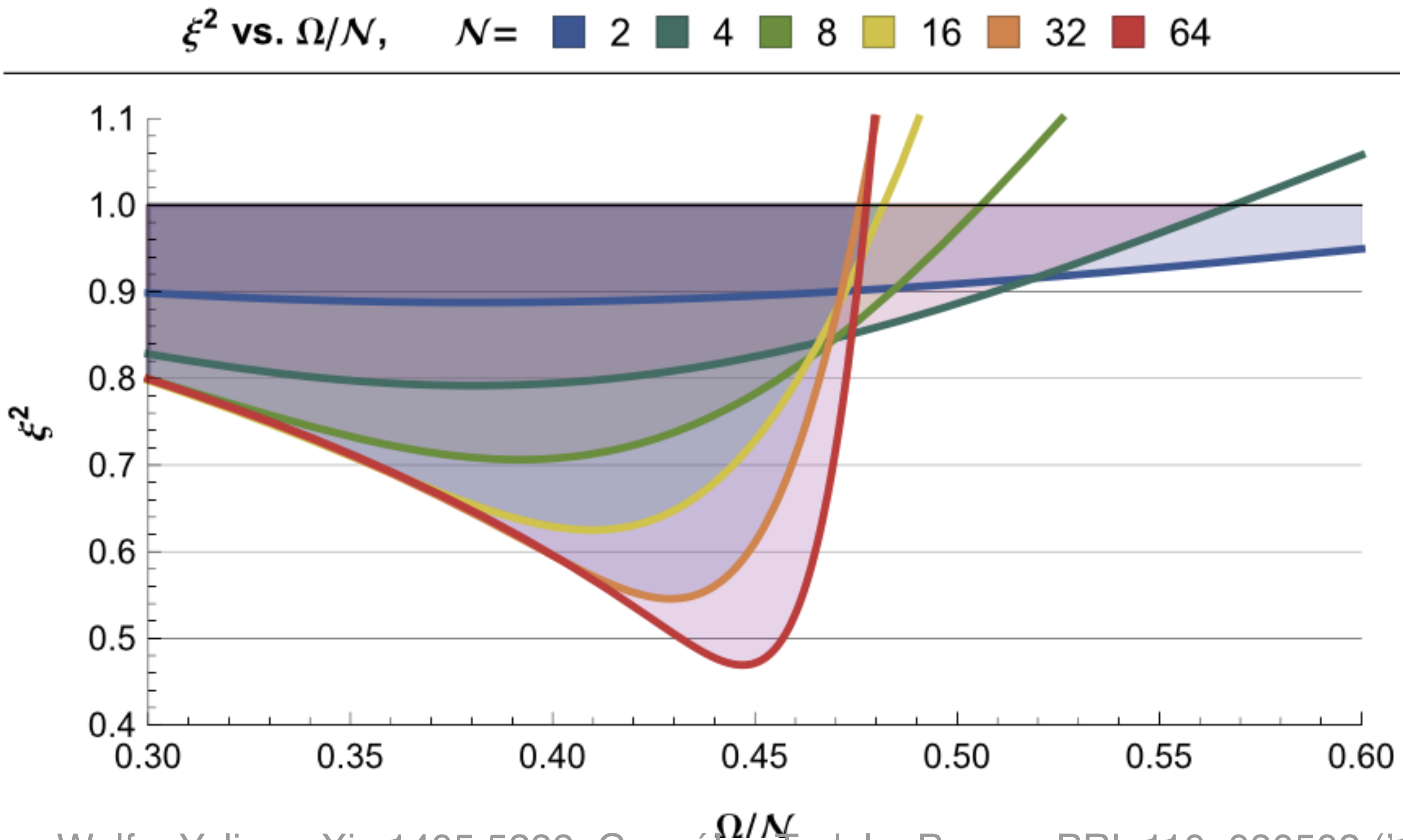
(J_1, J_2, are uncertainties in the two directions orthogonal to the total spin **J**)

Superradiant Spin Squeezing



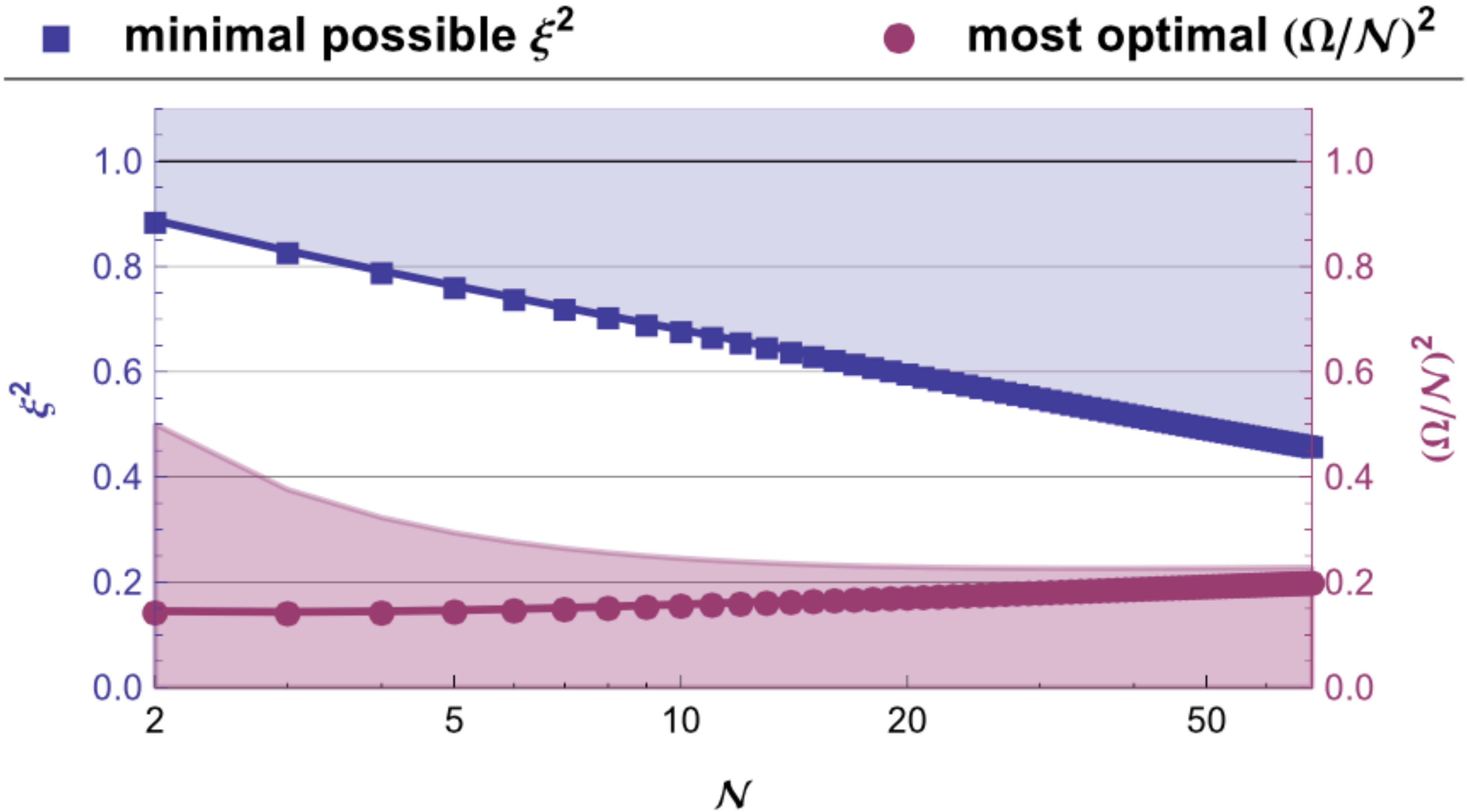
Wolfe, Yelin, arXiv:1405.5288, González Tudela, Porras, PRL 110, 080502 ('13)

Superradiant Spin Squeezing



Wolfe, Yelin, arXiv:1405.5288, González Tudela, Porras, PRL 110, 080502 ('13)

Best case for Dicke ensemble



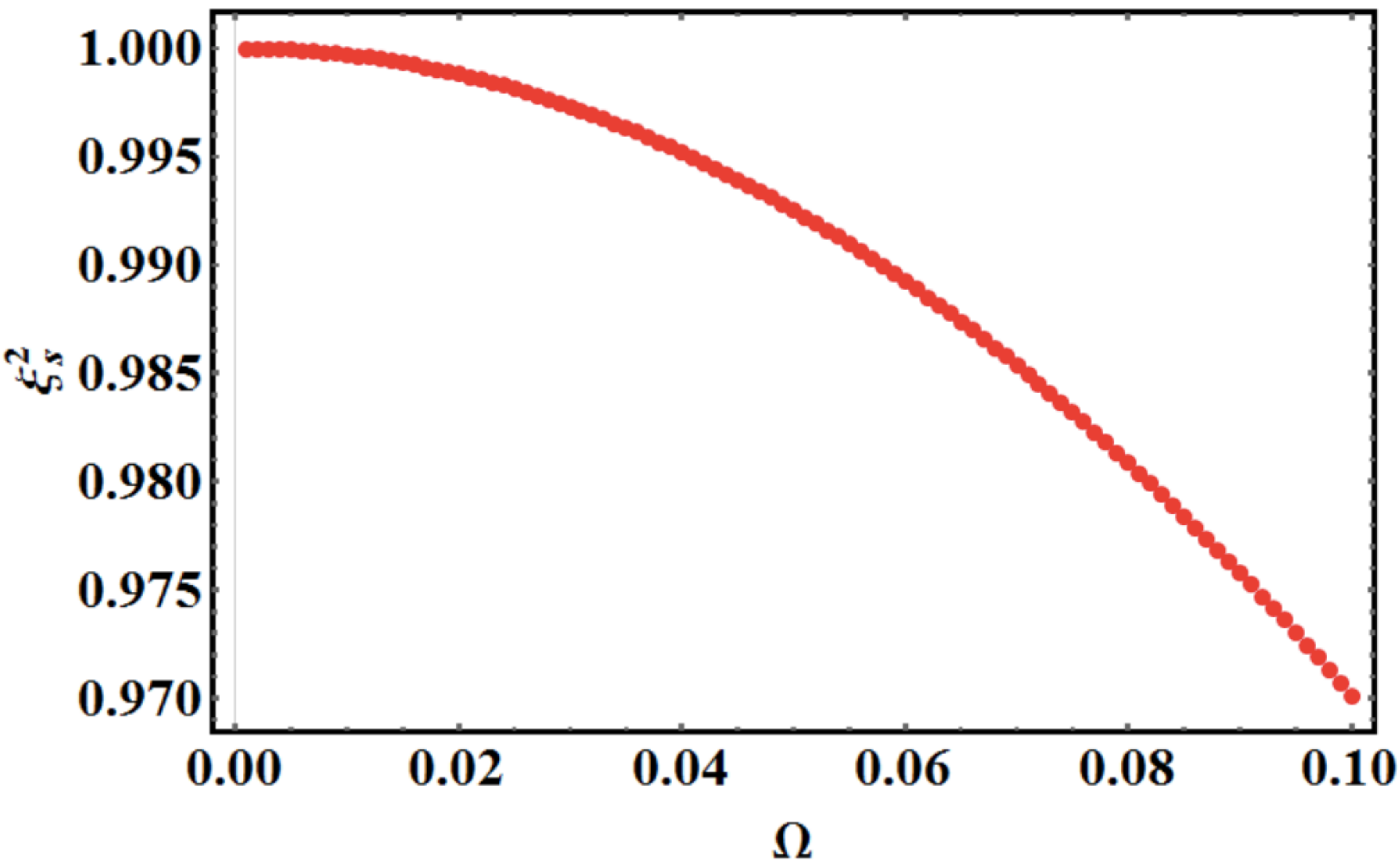
Wolfe, Yelin, arXiv:1405.5288, González Tudela, Porras, PRL 110, 080502 ('13)

What about realistic systems?

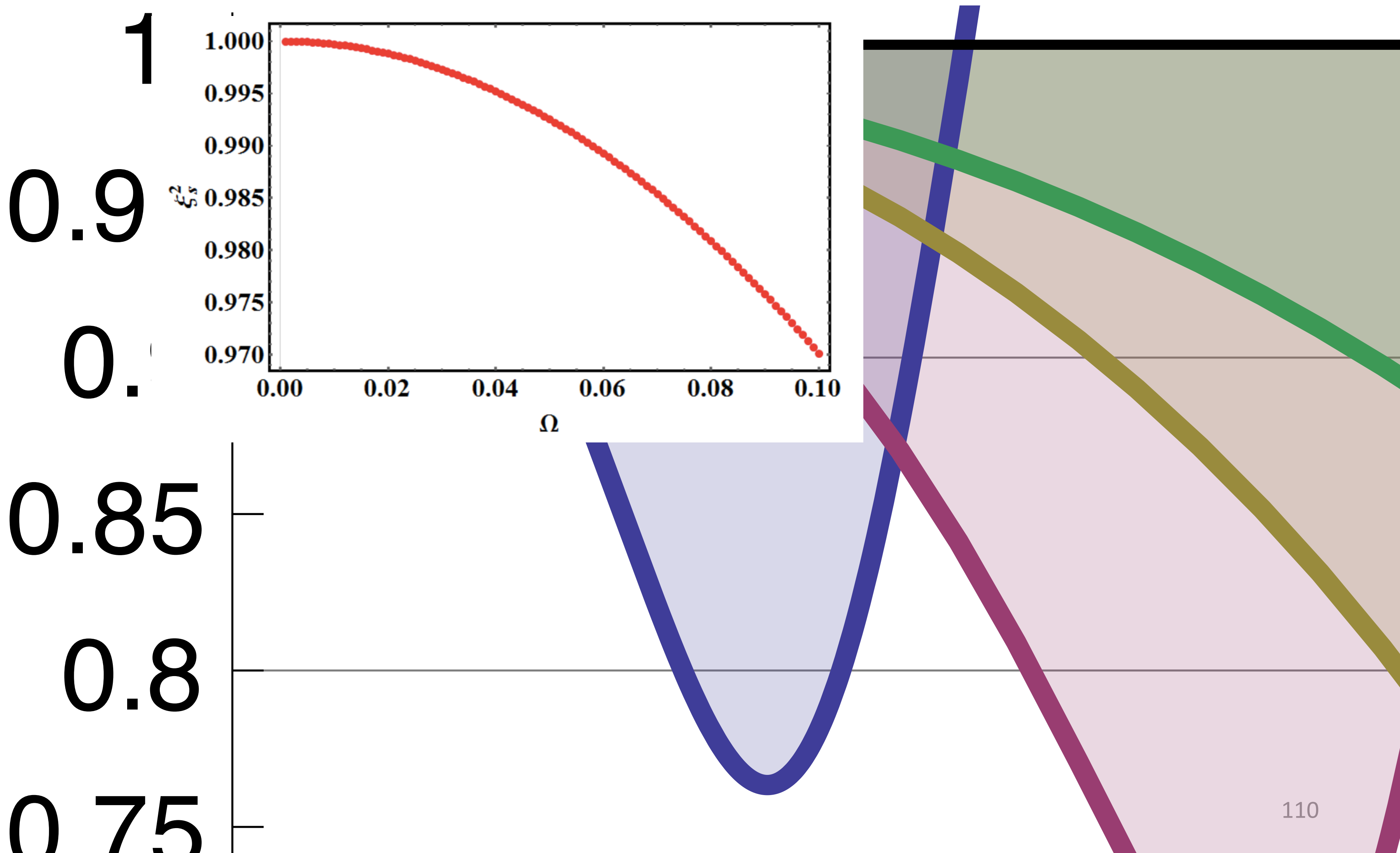
- Dicke: 3 parameters (N, Γ, Ω)
- Realistic systems: (OD, rel. density, Γ, Ω, γ_{ij}, Δ, δ_{ij})
- Is it possible to find parallels?
- minimize "incoherent" aspects?

➡ key: spontaneous decay, shift instead of induced!

Spin squeezing in realistic systems?



Spin squeezing in realistic systems?



Full dynamics (all degrees of freedom of atoms, fields)

$$H = H_{atoms} + H_{field} - \sum \mathbf{p}_i \mathbf{E}_i$$

two probe atoms

atoms

Thank you!