



University of  
St Andrews

# Making the most of interference:

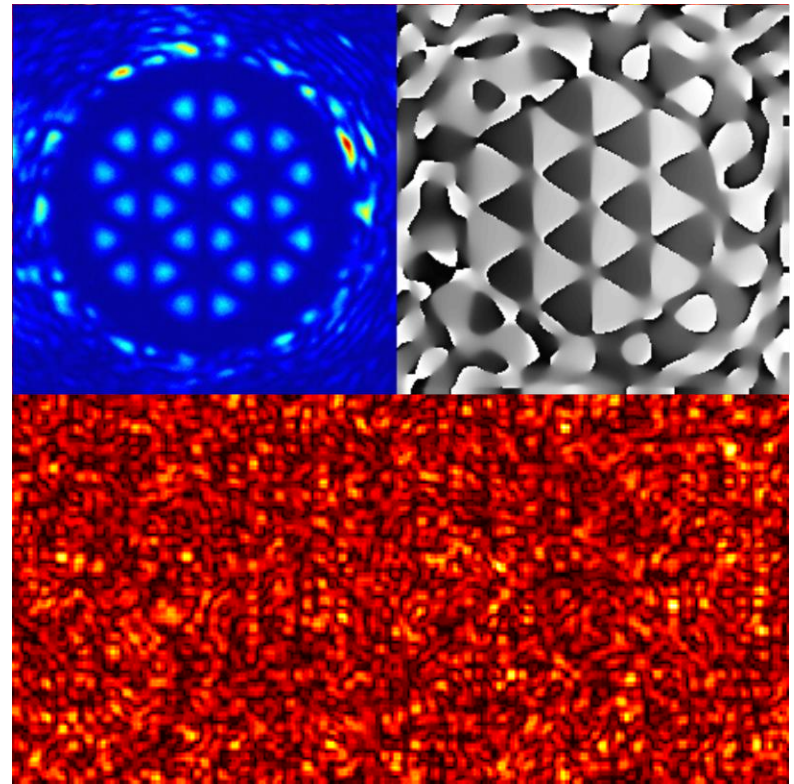
the application of laser speckle and  
computer-generated holography to cold  
atoms, optical trapping and precision  
metrology

**Graham D. Bruce**  
University of St Andrews

 @grahamdbruce

**ColOpt Winter School on collective effects,  
structured light and quantum matter**

**7<sup>th</sup> March 2018**

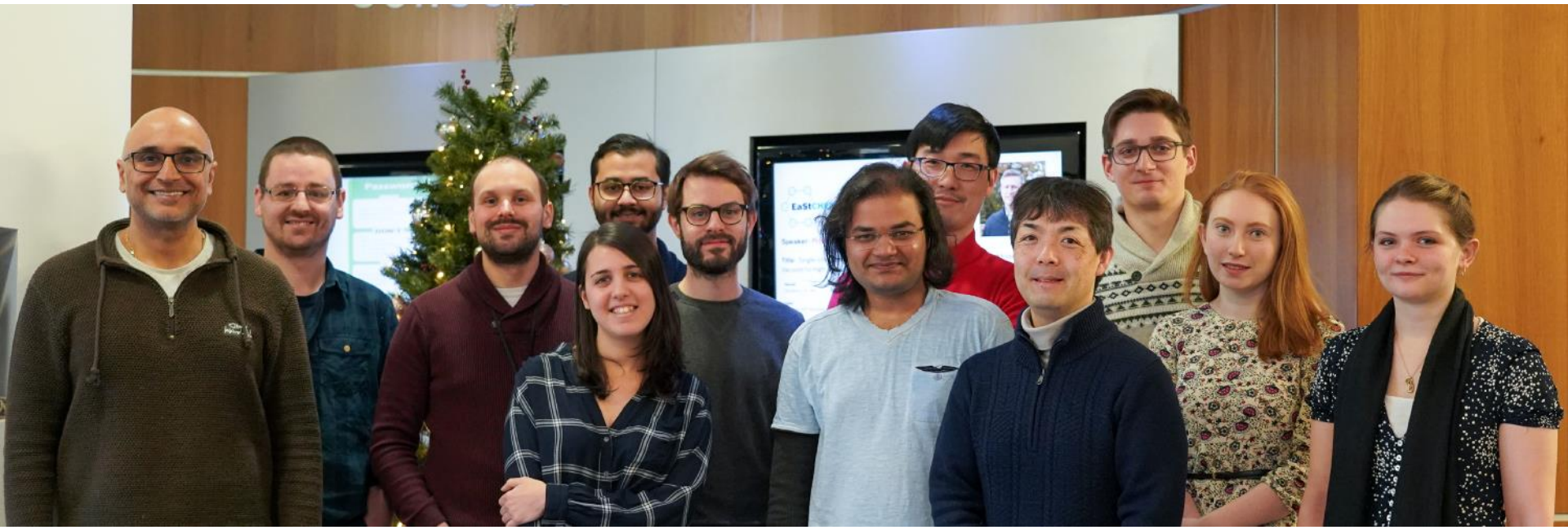
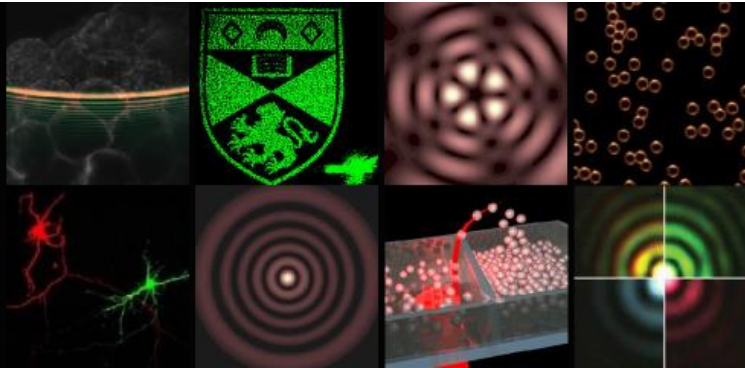
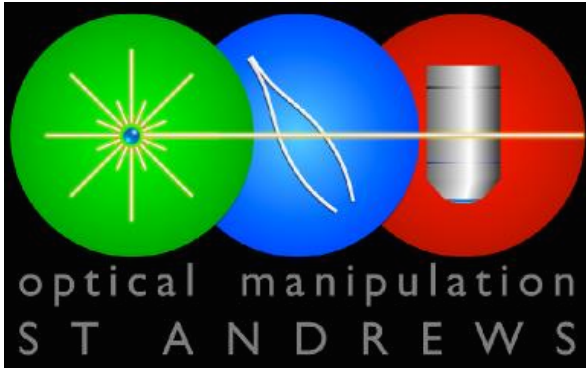




**8000 students at University**  
**47% from outside UK**  
**Physics: 40 academic staff; 60 research staff; 80 PhD students**  
<http://www.st-andrews.ac.uk/physics/index.php>



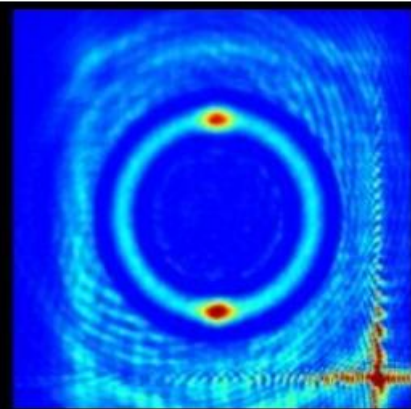
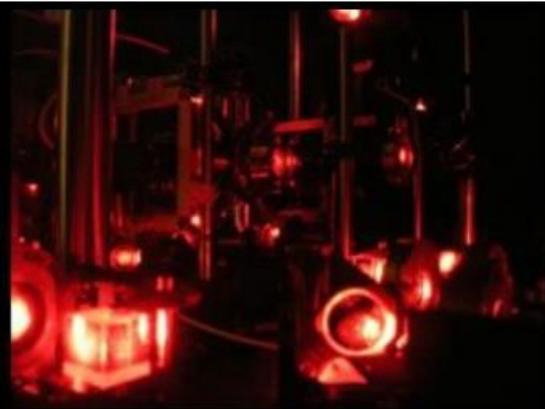
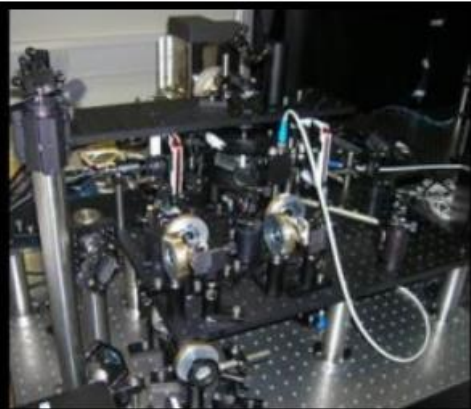






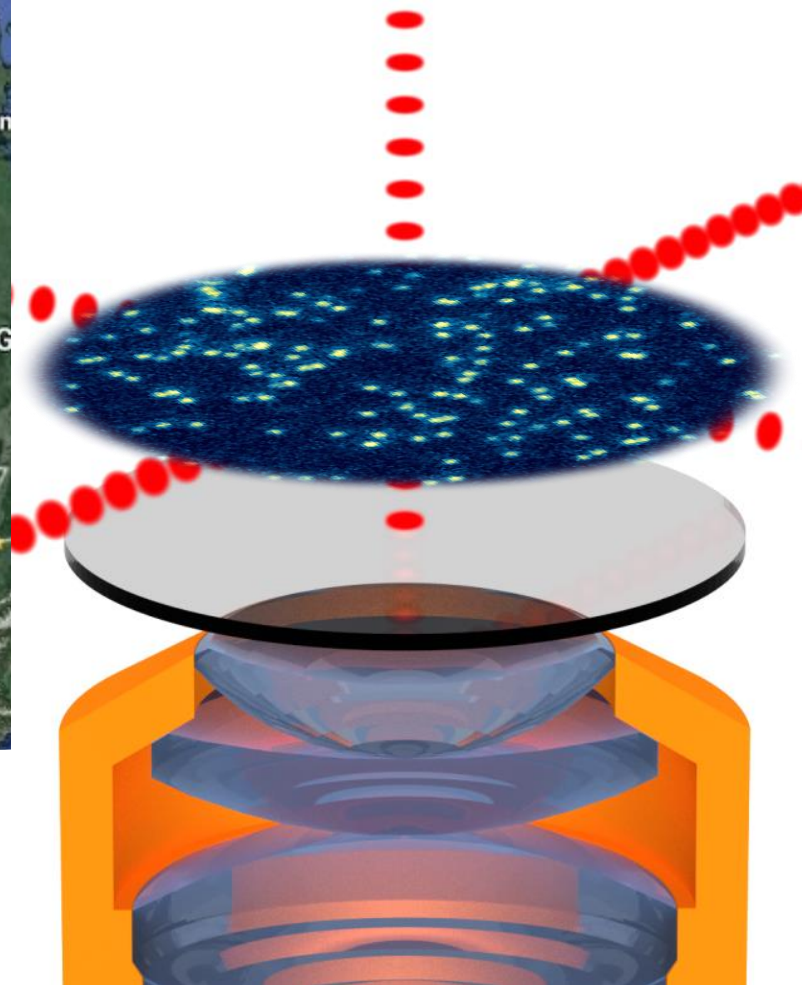
University of St Andrews

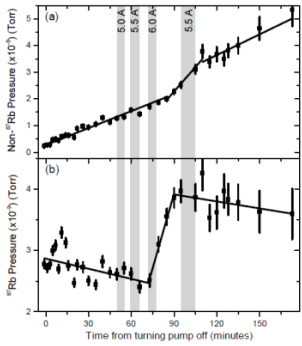
## Cold Atoms Group



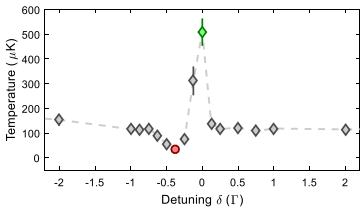
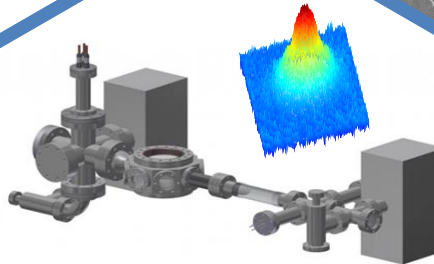
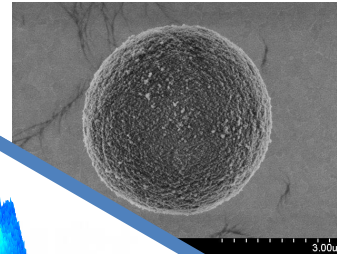
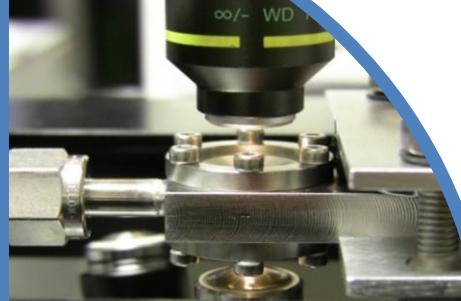
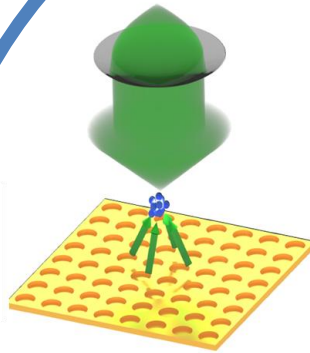


University of  
**Strathclyde**  
Glasgow

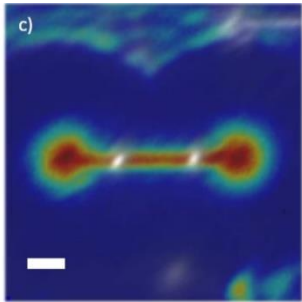




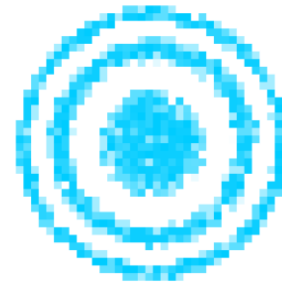
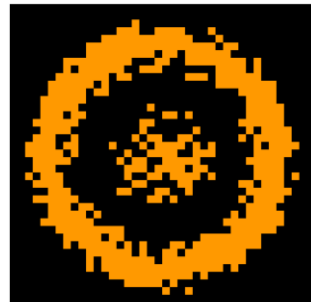
*Sci Rep* **5**, 14729 (2015)  
*Rev Sci Inst* **86**, 093108 (2015)



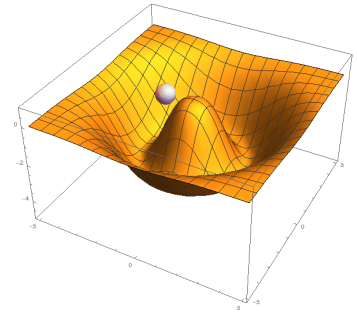
*J Phys B* **50**, 095002 (2017)



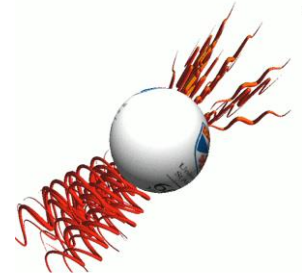
*Opt Express* **25**, 11692 (2017)  
*Opt Express* **23**, 8365 (2015)  
*Opt Express* **22**, 26548 (2014)



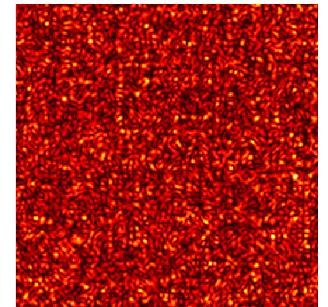
*New J Phys* **18** 075012 (2016)  
*Phys Rev A* **94** 051601 (2016)



*JOSA B* **34**, C14 (2017)  
*Phys Rev A* **94**, 053821 (2016)



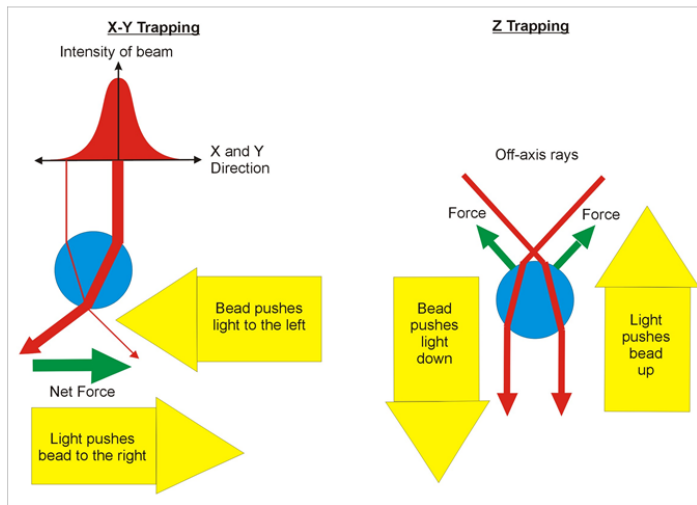
*Nature Comm* **4**, 2374 (2013)



*Opt Lett* **44** 1367 (2019)  
*Nature Comm* **8**, 15610 (2017)  
*Opt Lett* **39** 96 (2014)

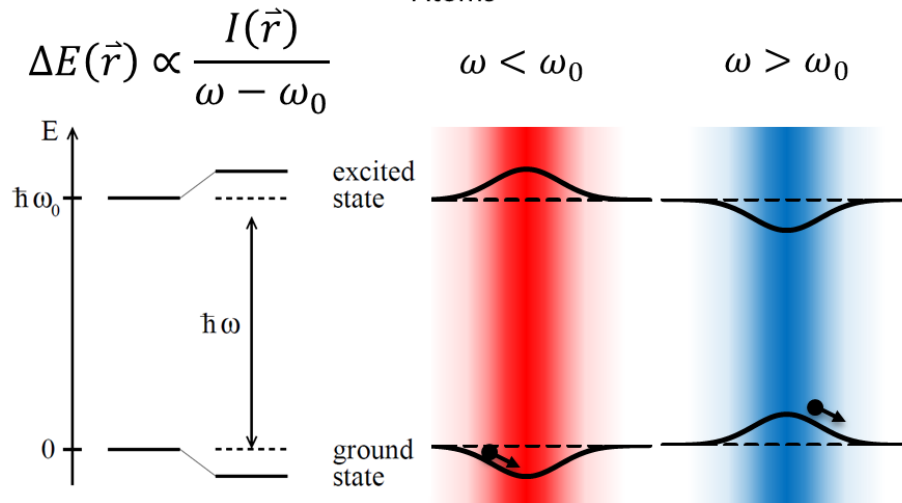
## Optical Gradient Force

### Microparticles



## Optical Dipole Force

### Atoms



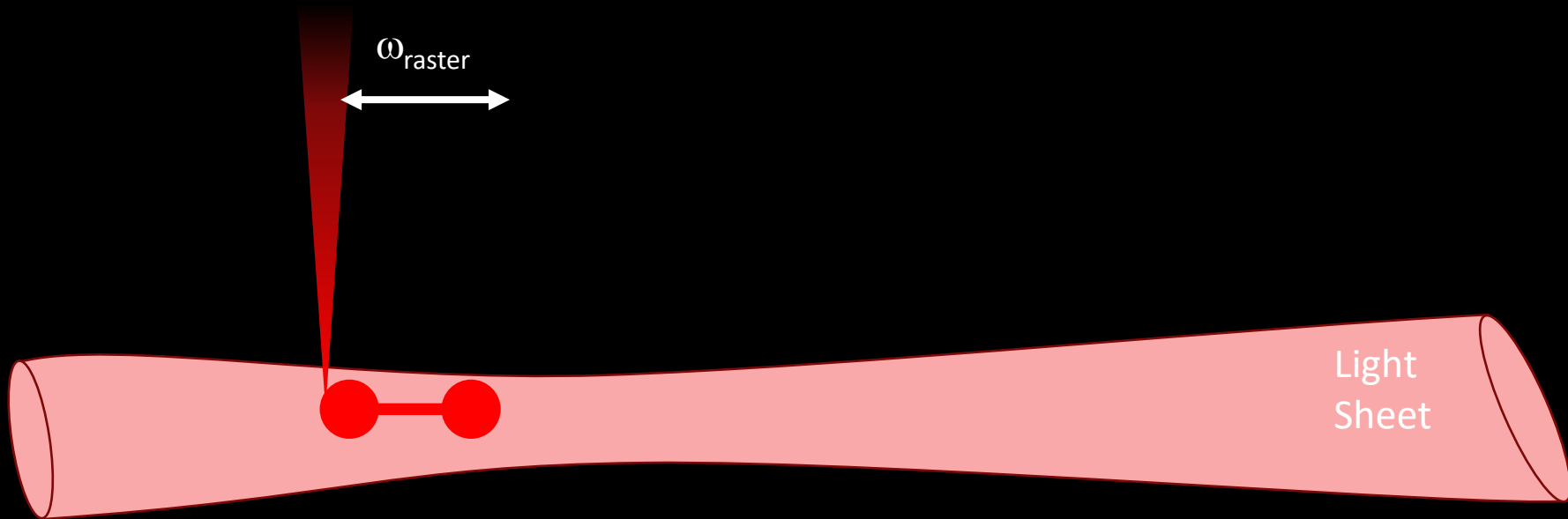
*R Grimm et al, Adv. At. Mol. Opt. Phys. 42, 95-170 (2000)*

How much control do we have over optical trapping geometry?



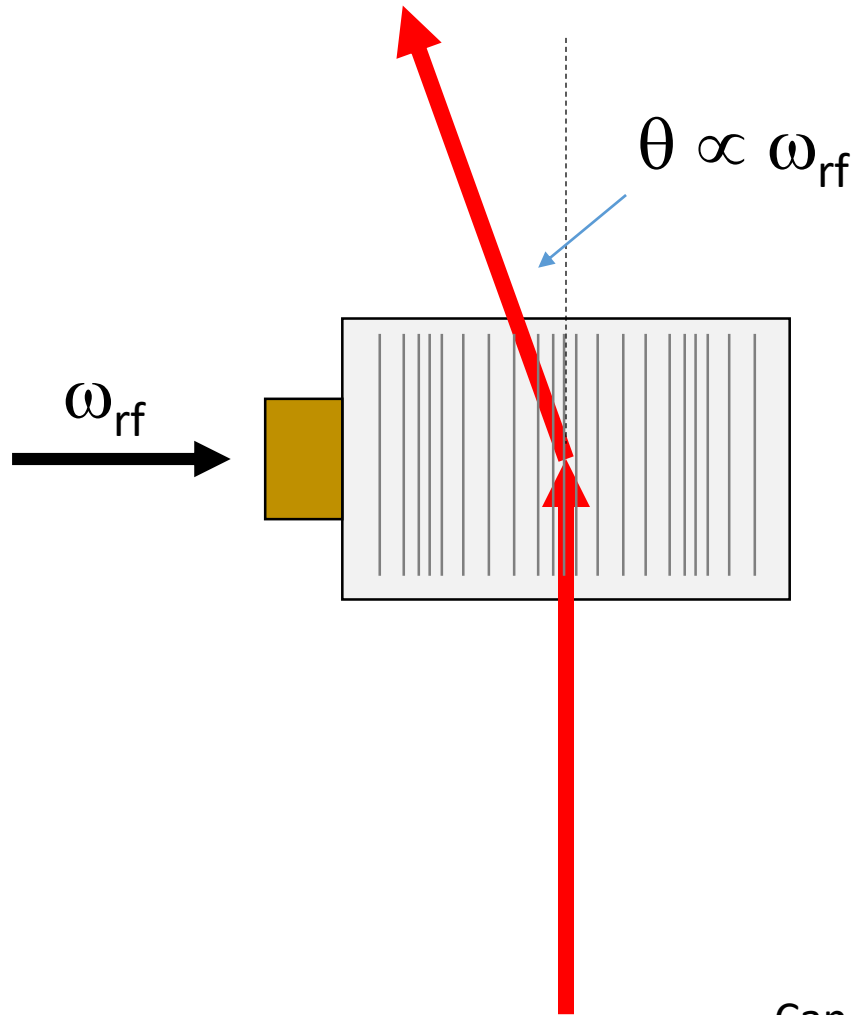
Time  
Averaged

$\omega_{\text{raster}}$



Light  
Sheet

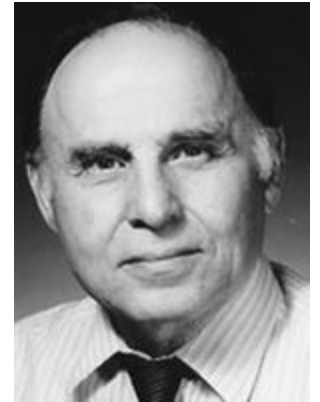
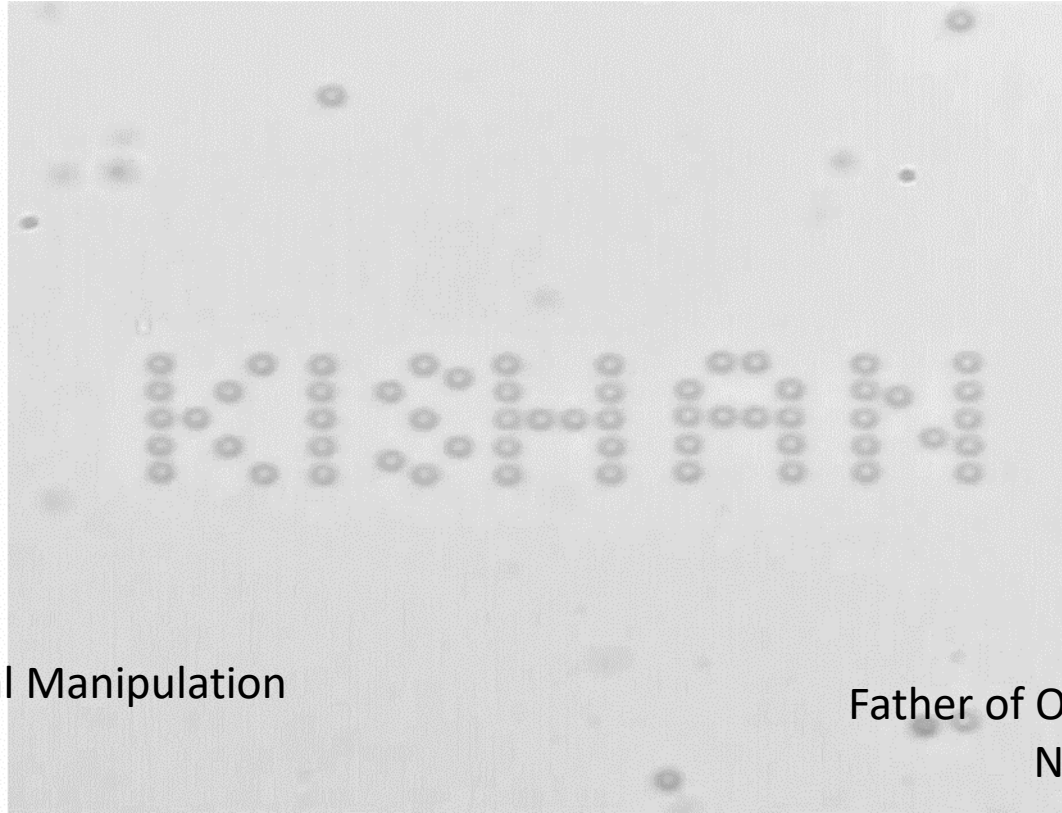
(not necessary for microparticles)



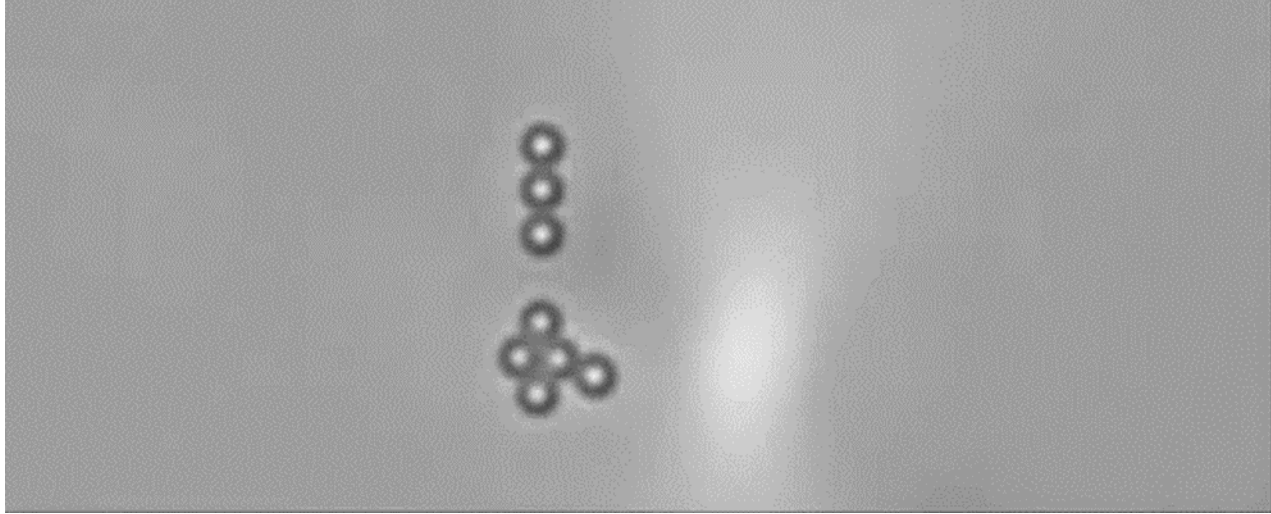
Can be modulated at MHz rates

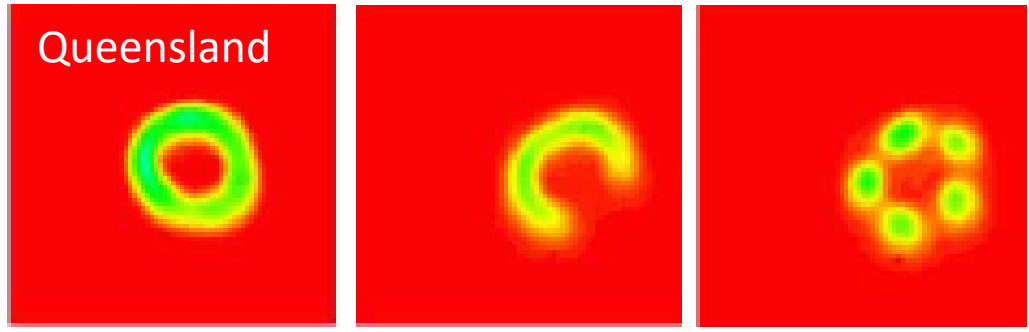
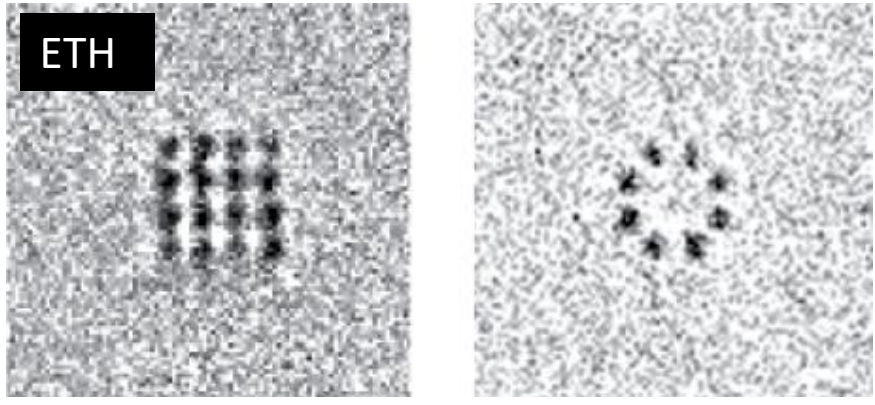
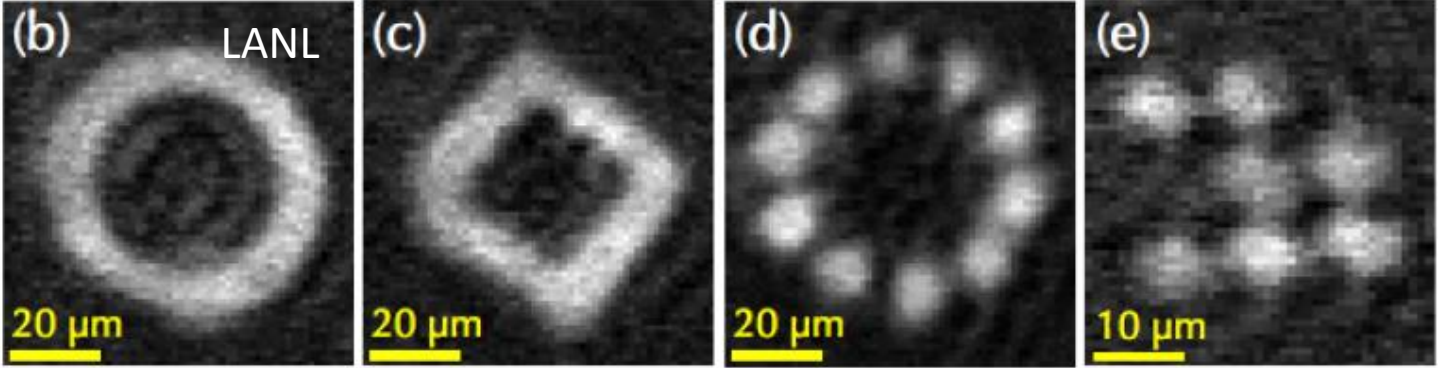


Kishan Dholakia  
Professor of Optical Manipulation  
St Andrews



Arthur Ashkin  
Father of Optical Tweezers  
Nobel Prize 2018





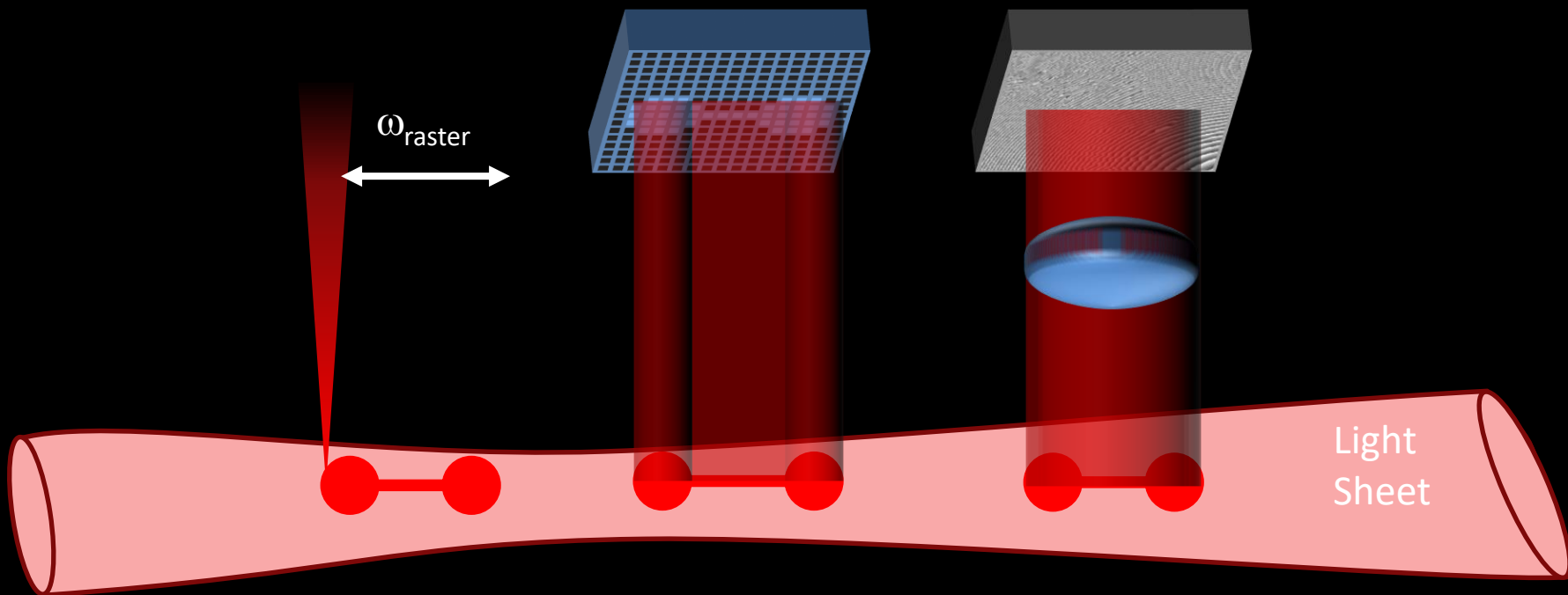
Time  
Averaged

Amplitude  
Mask

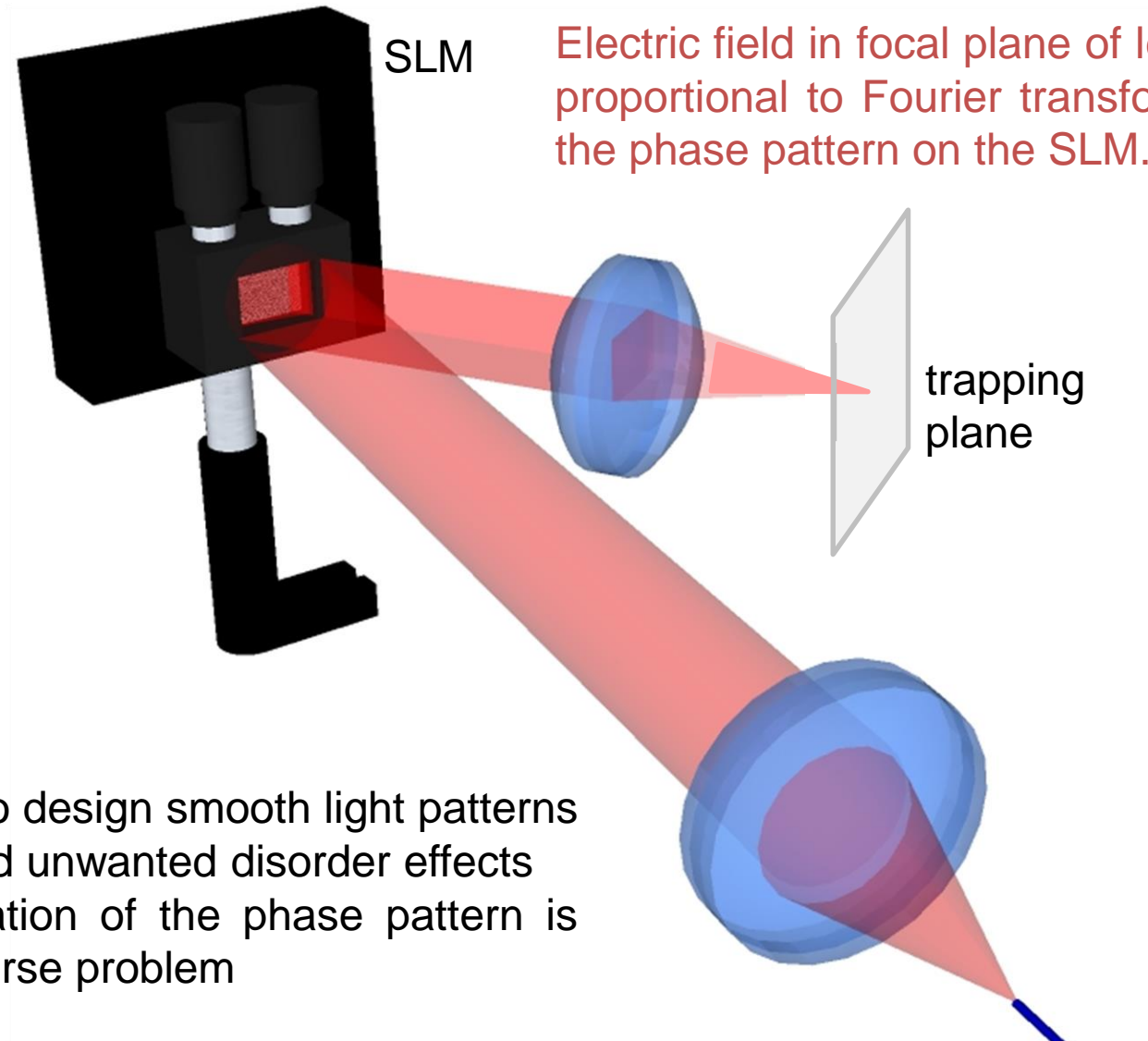
Hologram

$\omega_{\text{raster}}$

Light  
Sheet

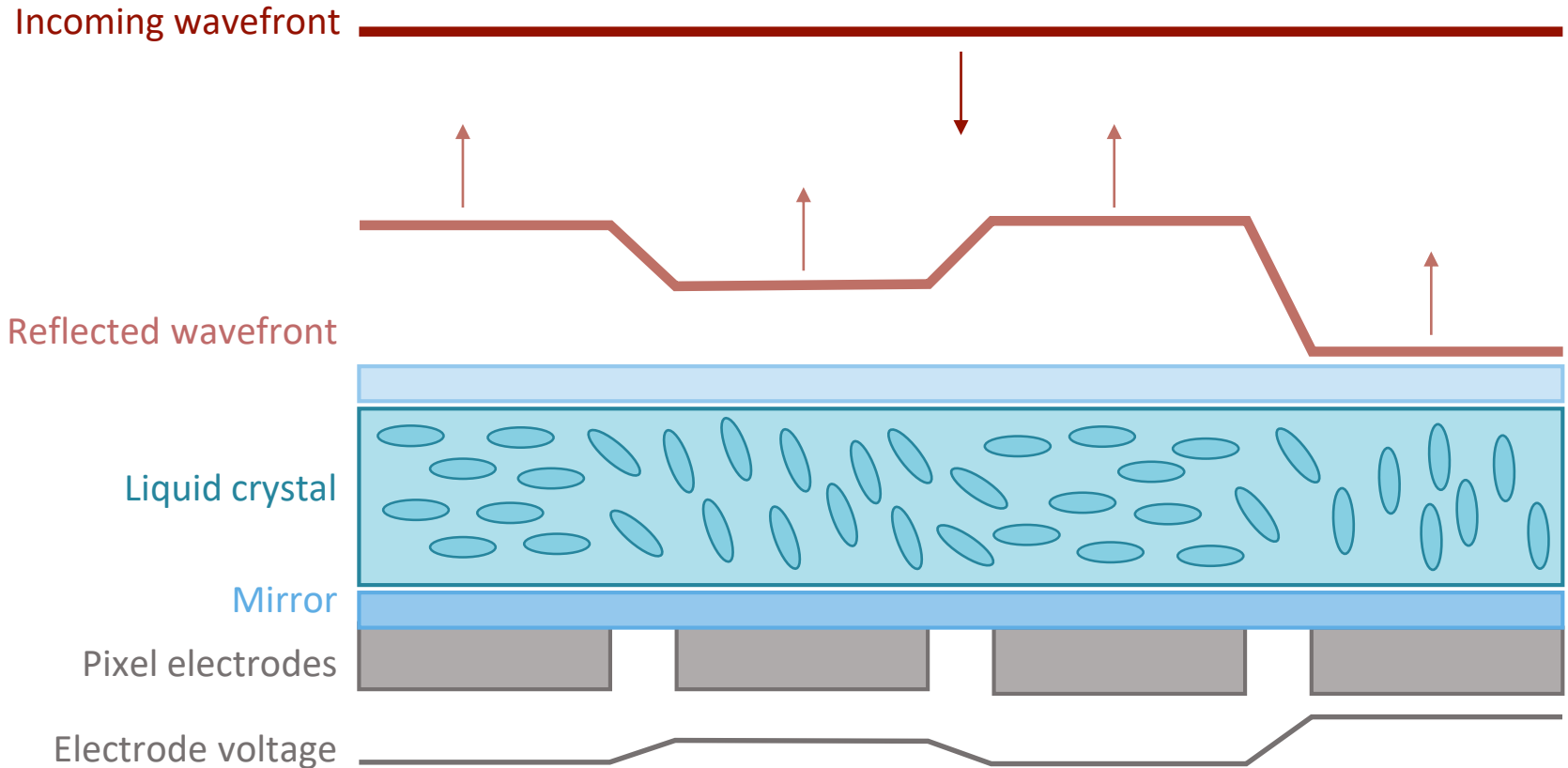


# Computer Generated Holography



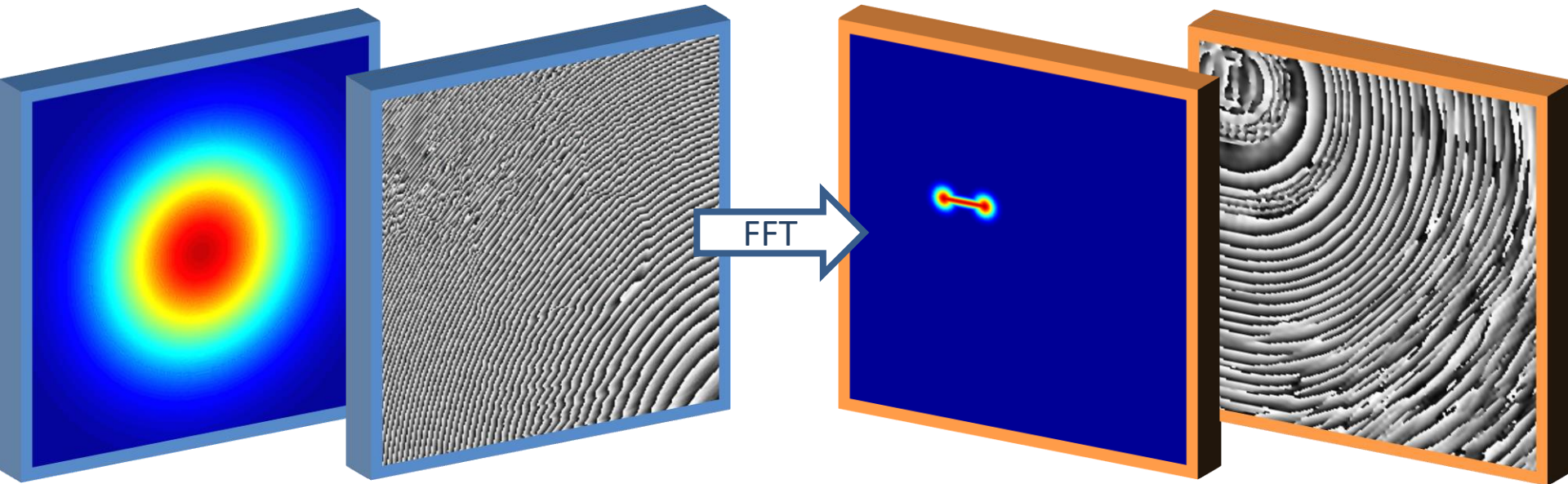
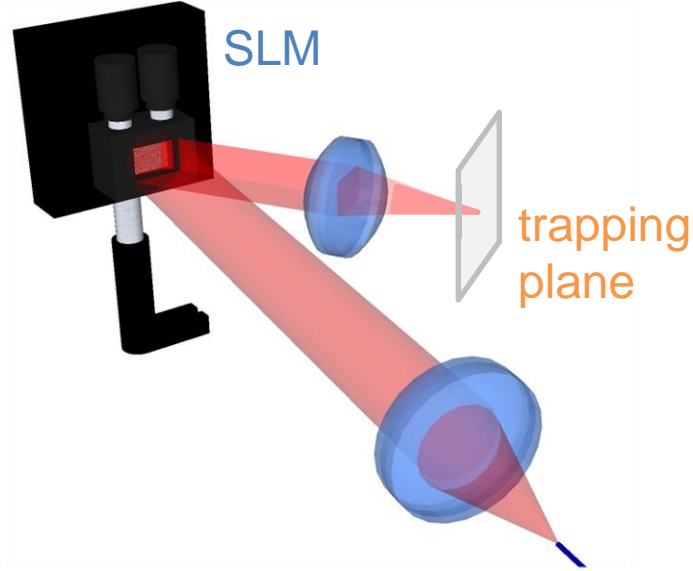
- Want to design smooth light patterns to avoid unwanted disorder effects
- Calculation of the phase pattern is an inverse problem

# Computer Generated Holography

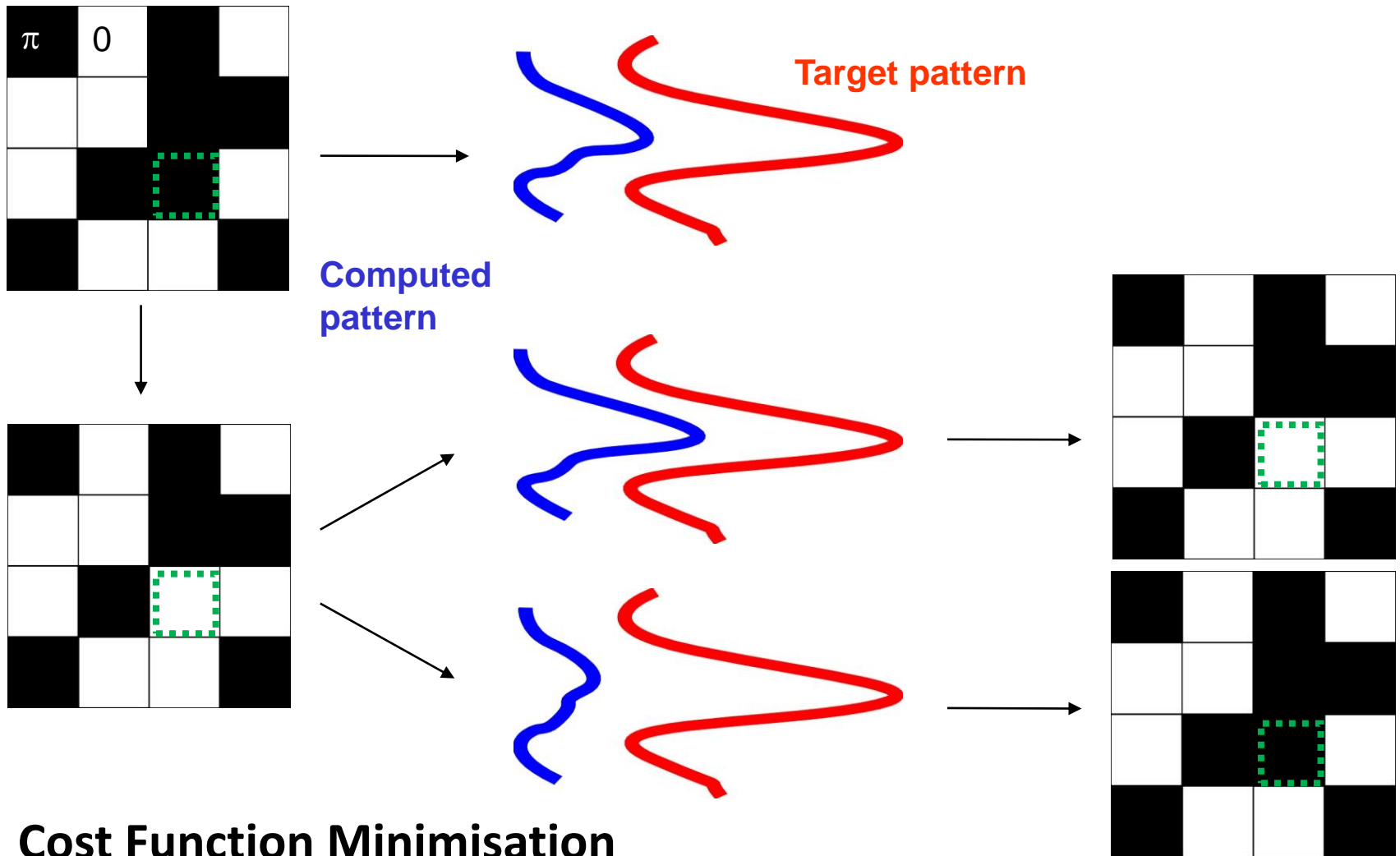




# Calculating Holograms



# Direct Binary Search Algorithm

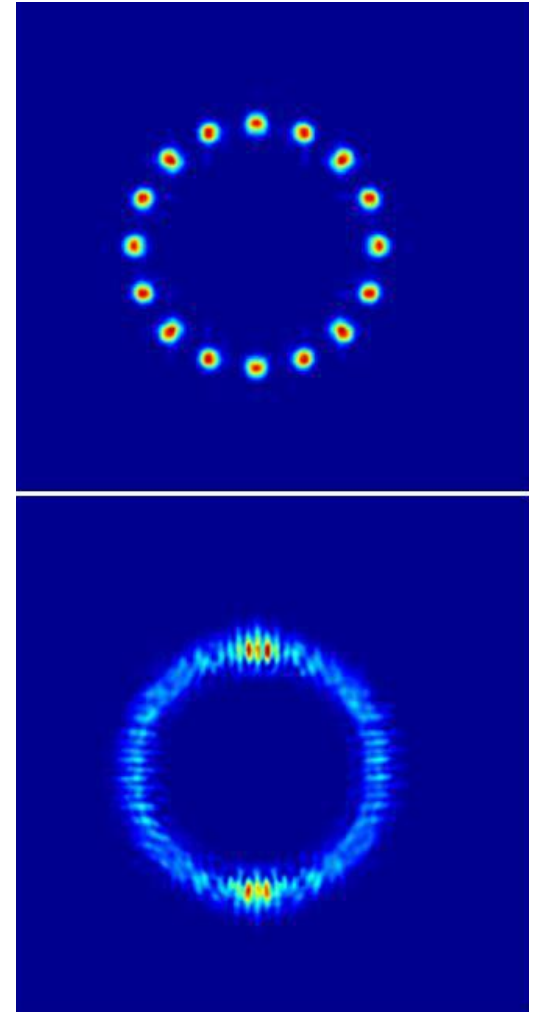
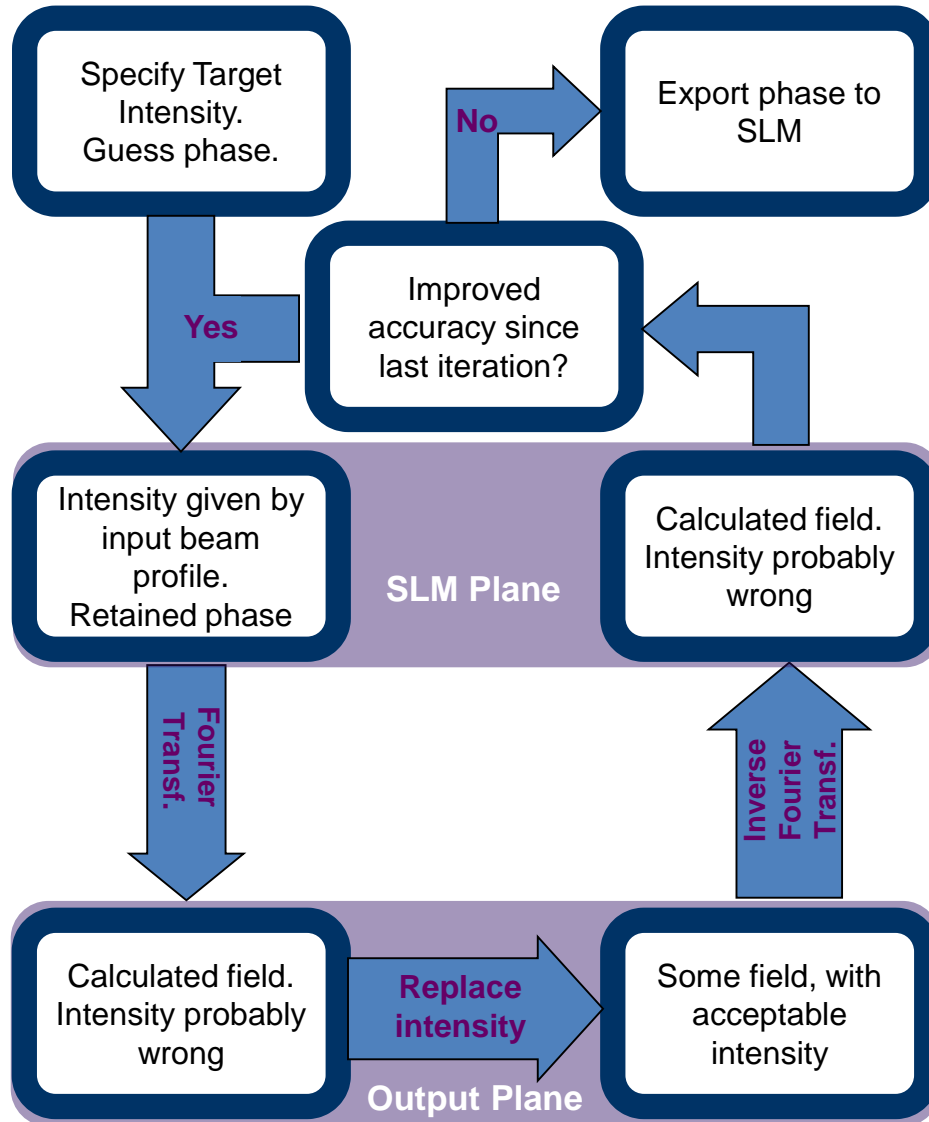


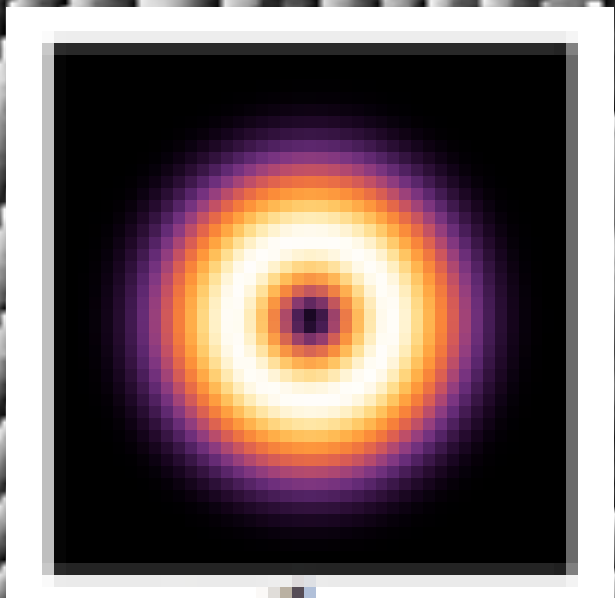
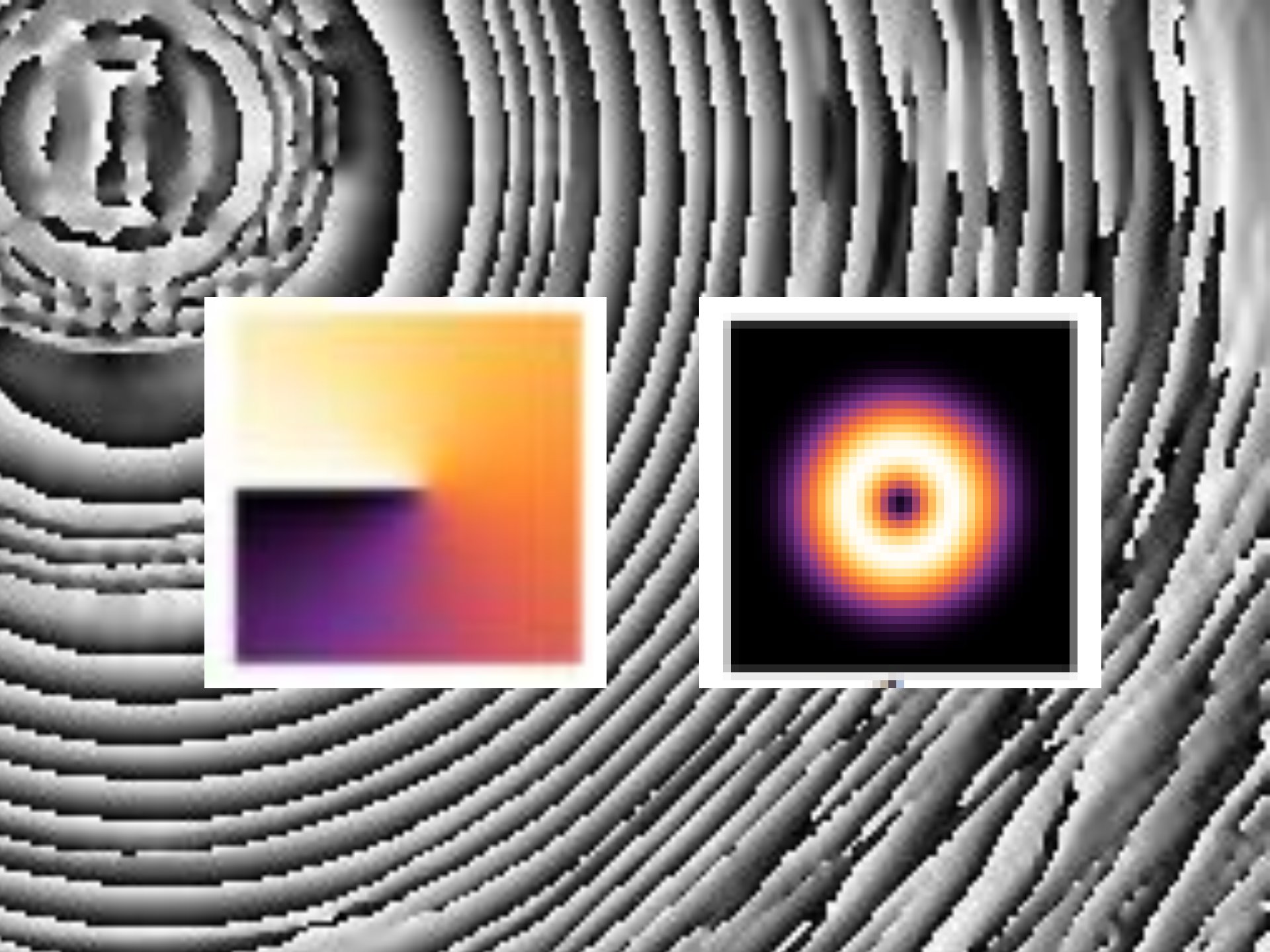
## Cost Function Minimisation

Example cost function:  $C = T - |E_{\text{out}}|^2$

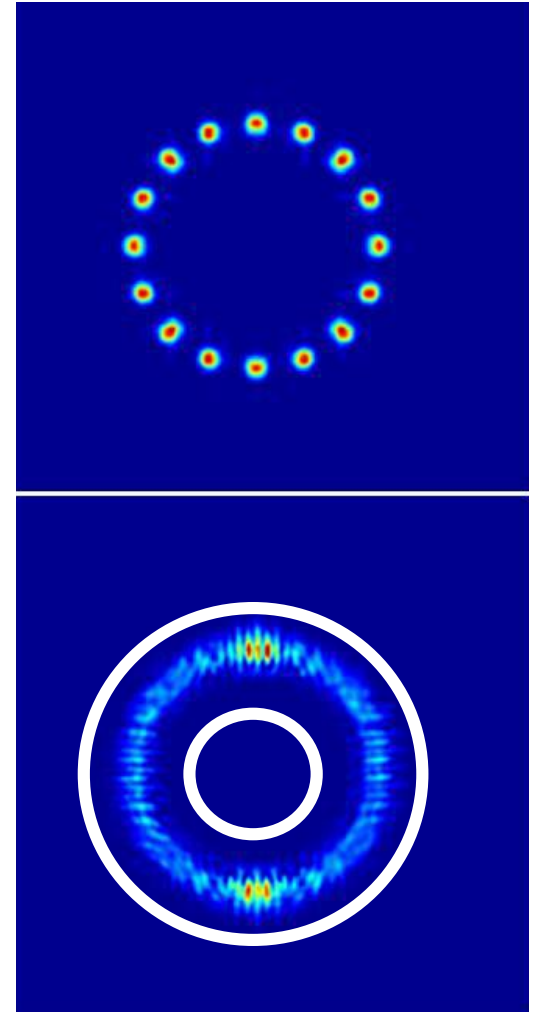
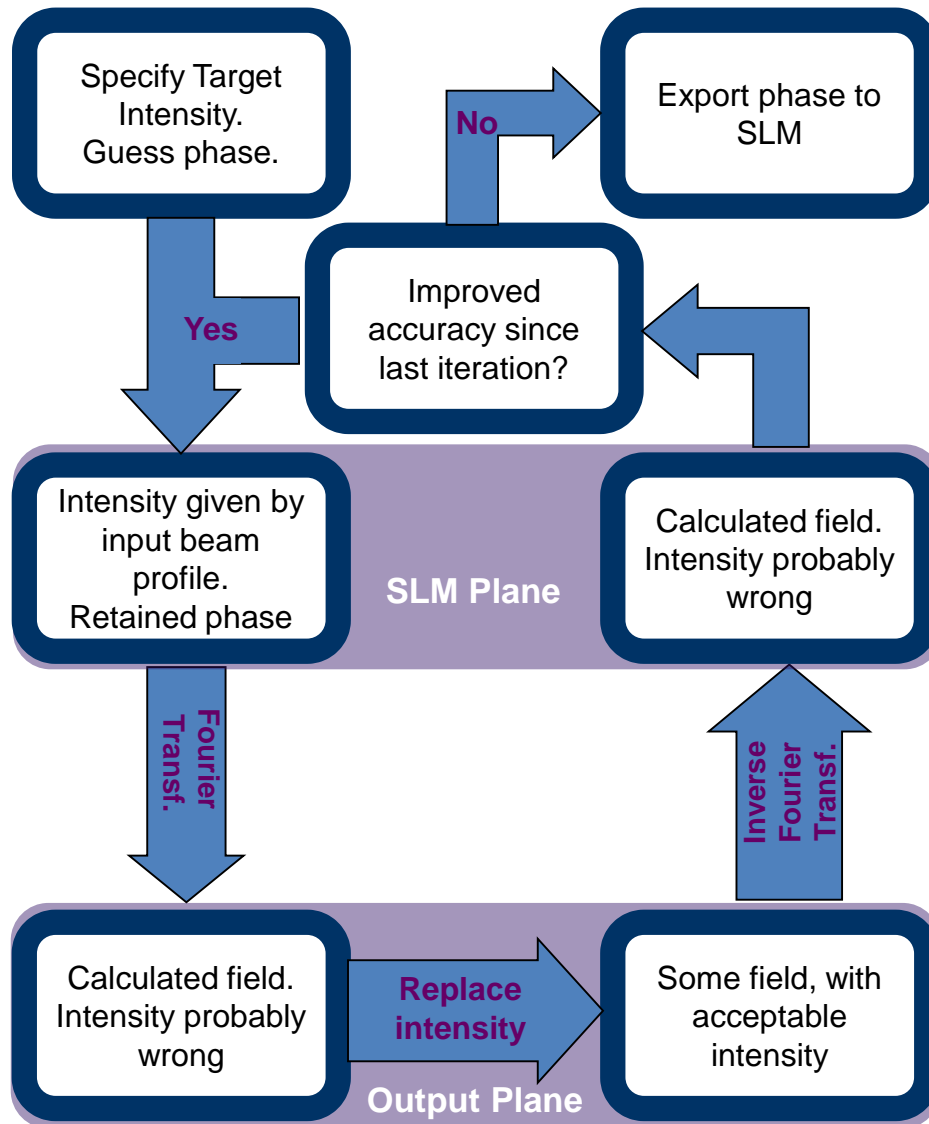
Iterative, unguided optimisation

# Iterative Fourier Transform Algorithm

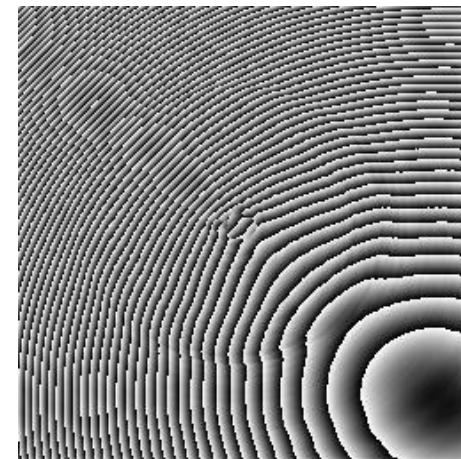
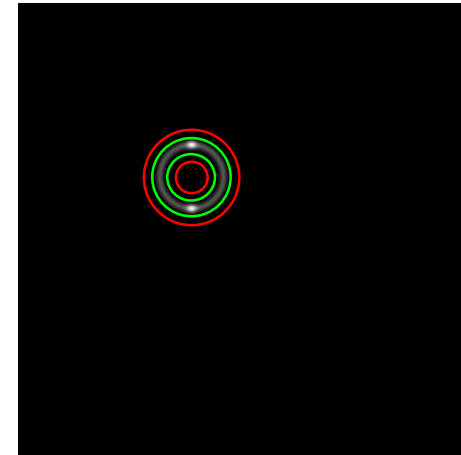
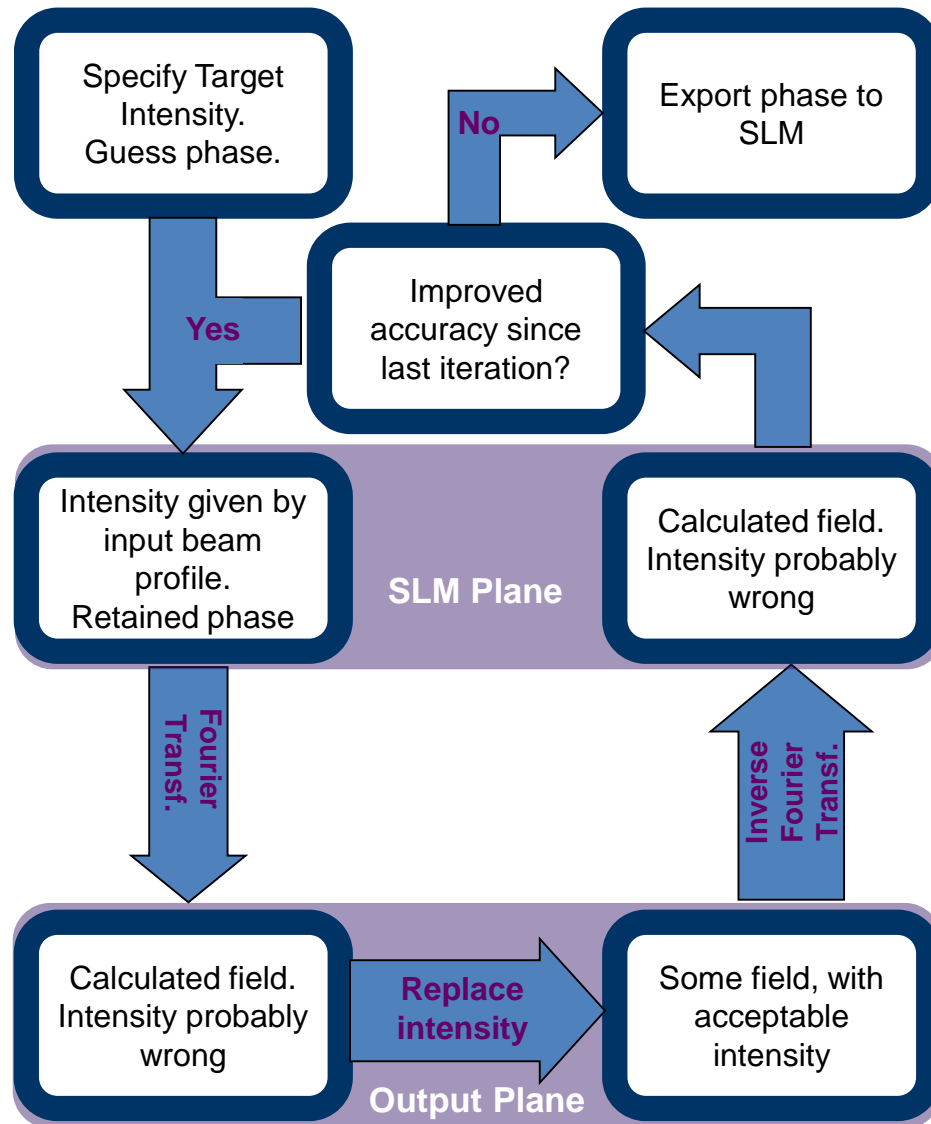




# Iterative Fourier Transform Algorithm

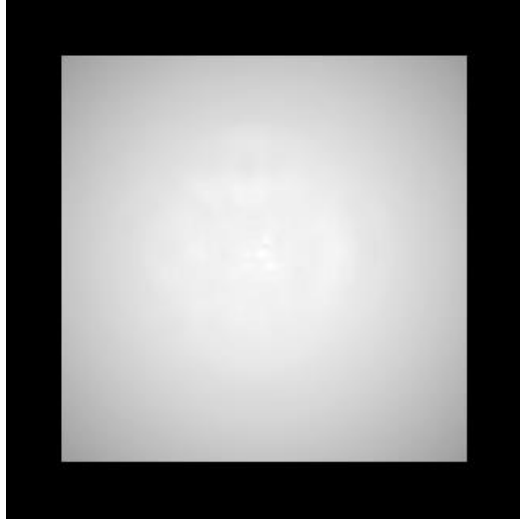


# Iterative Fourier Transform Algorithm

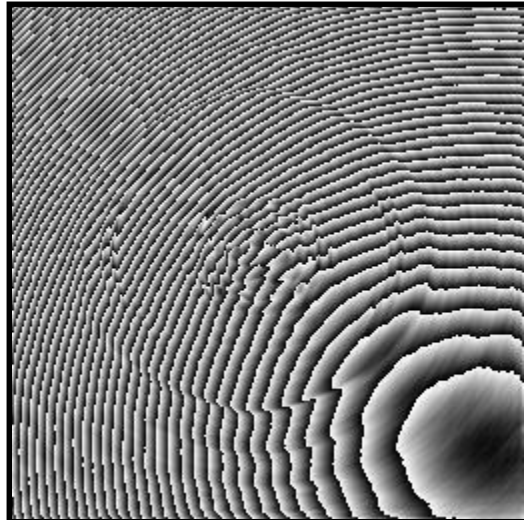
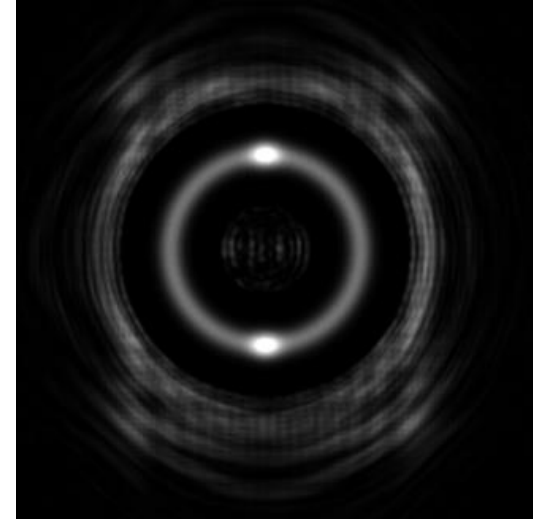


# MRAF Algorithm

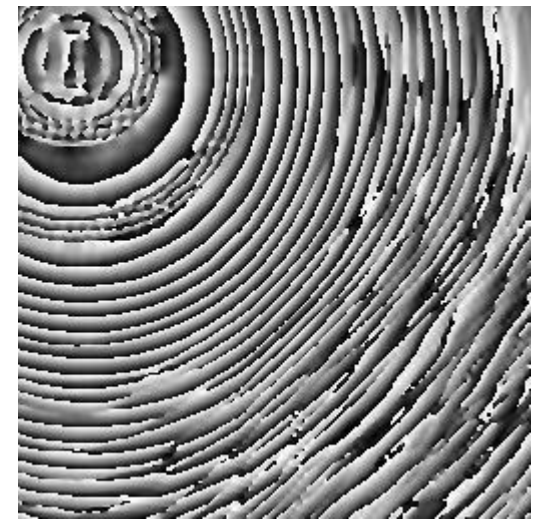
SLM plane



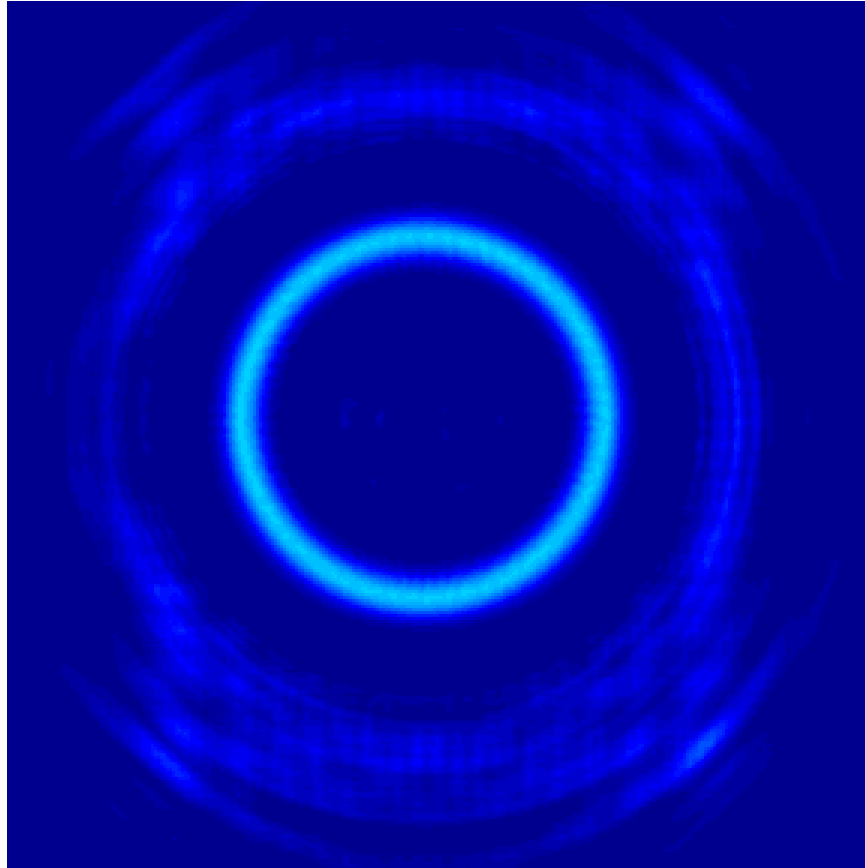
Fourier plane



$i = 116$

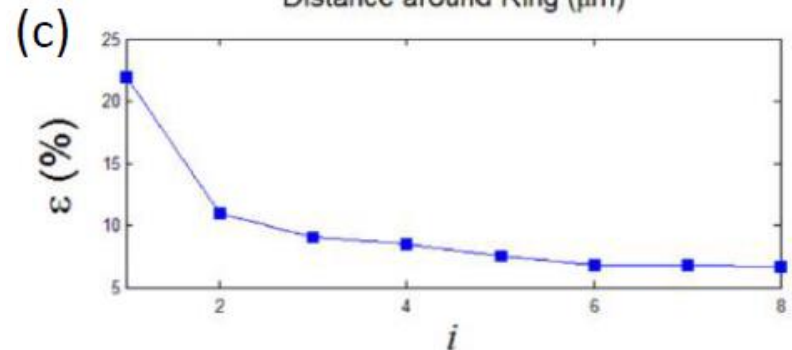
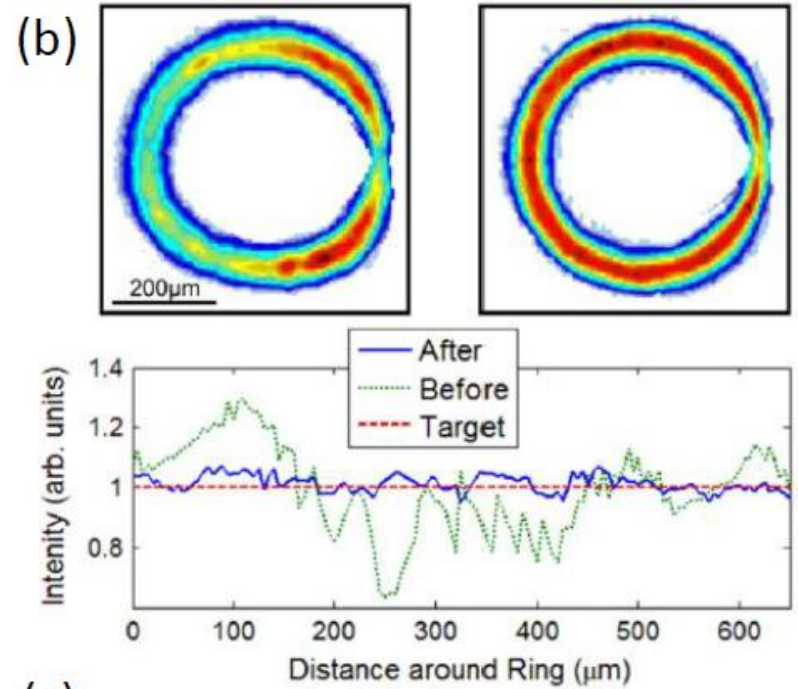
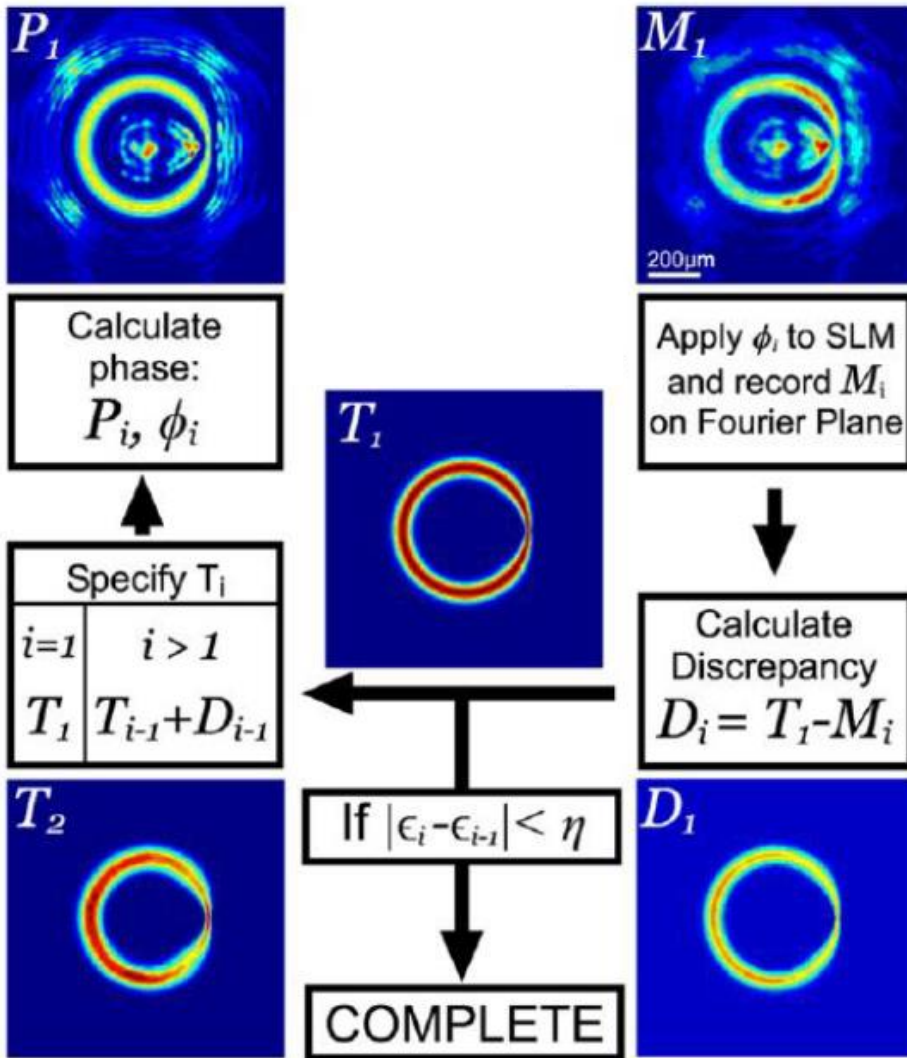


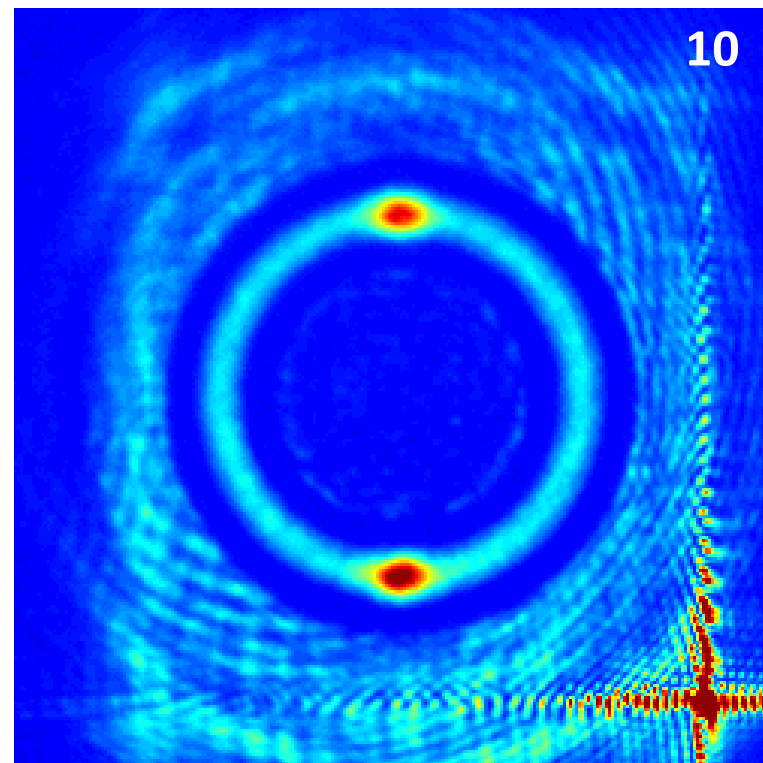
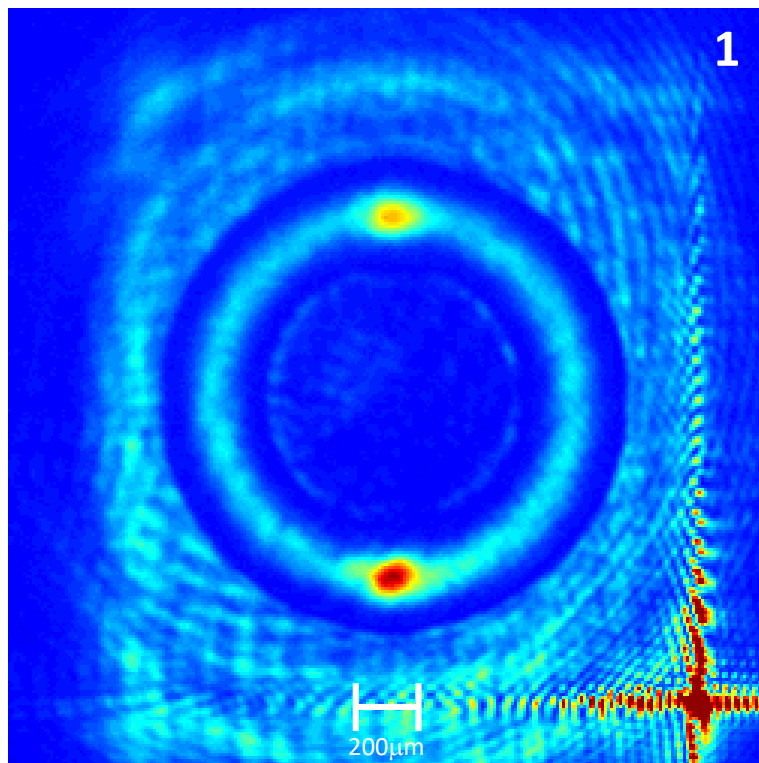
# Inducing Superflow





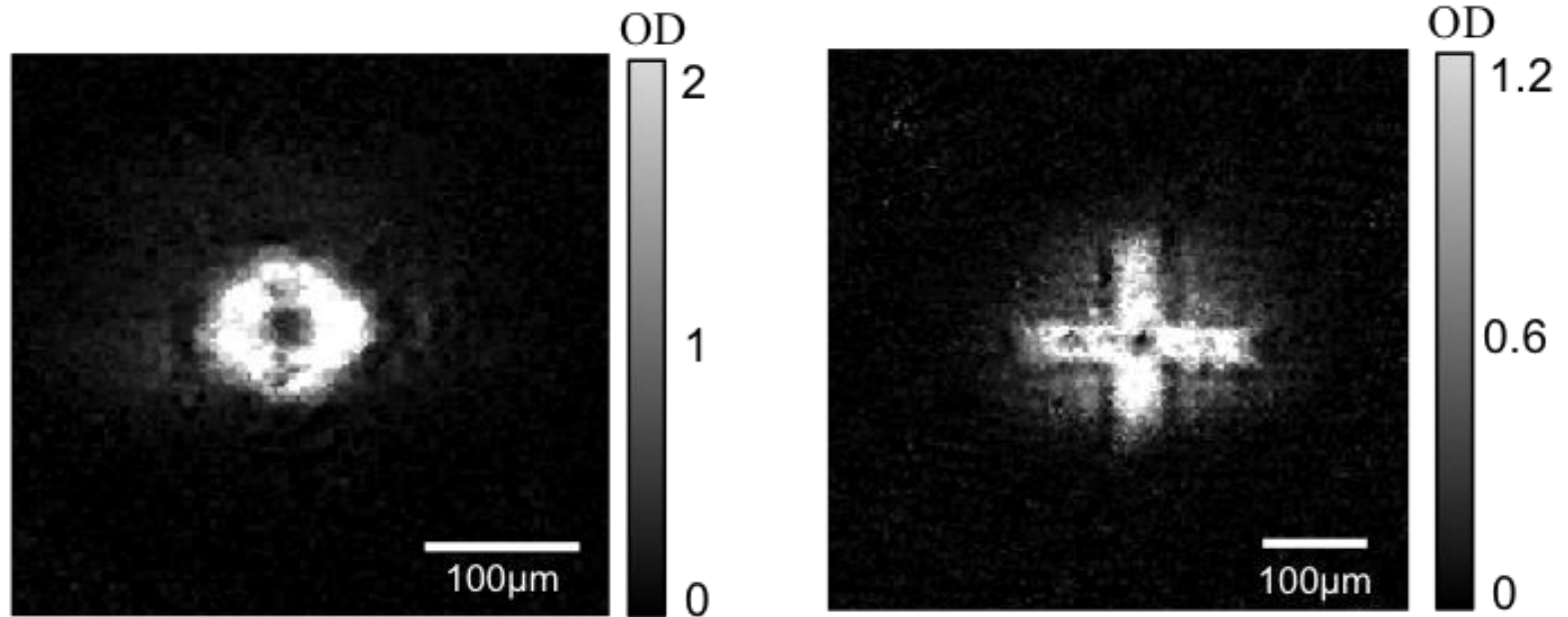
# Experimental implementation





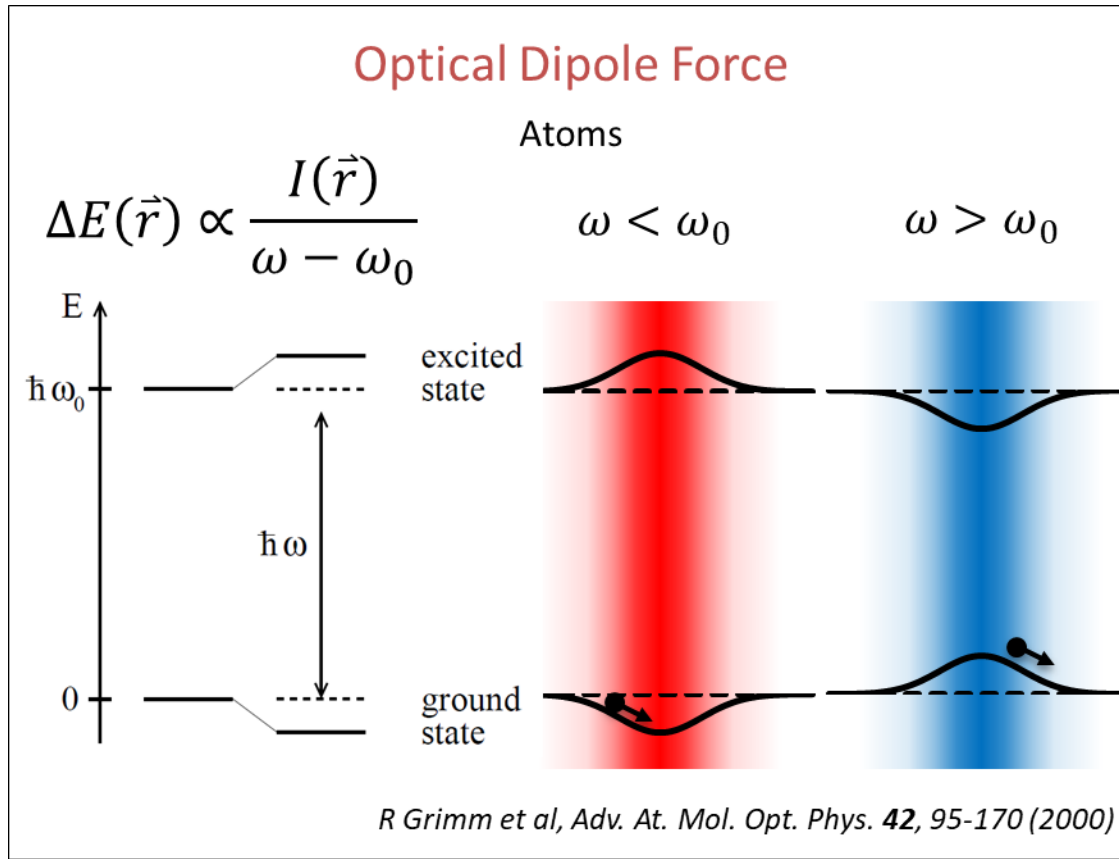
**Quality progression within 10 iterations.**

# Can use atoms as a probe!

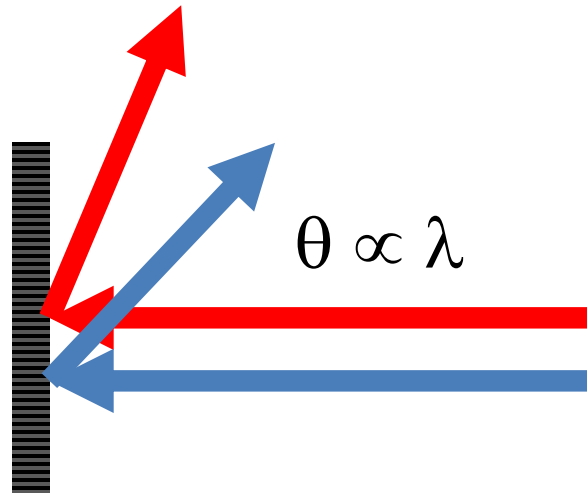


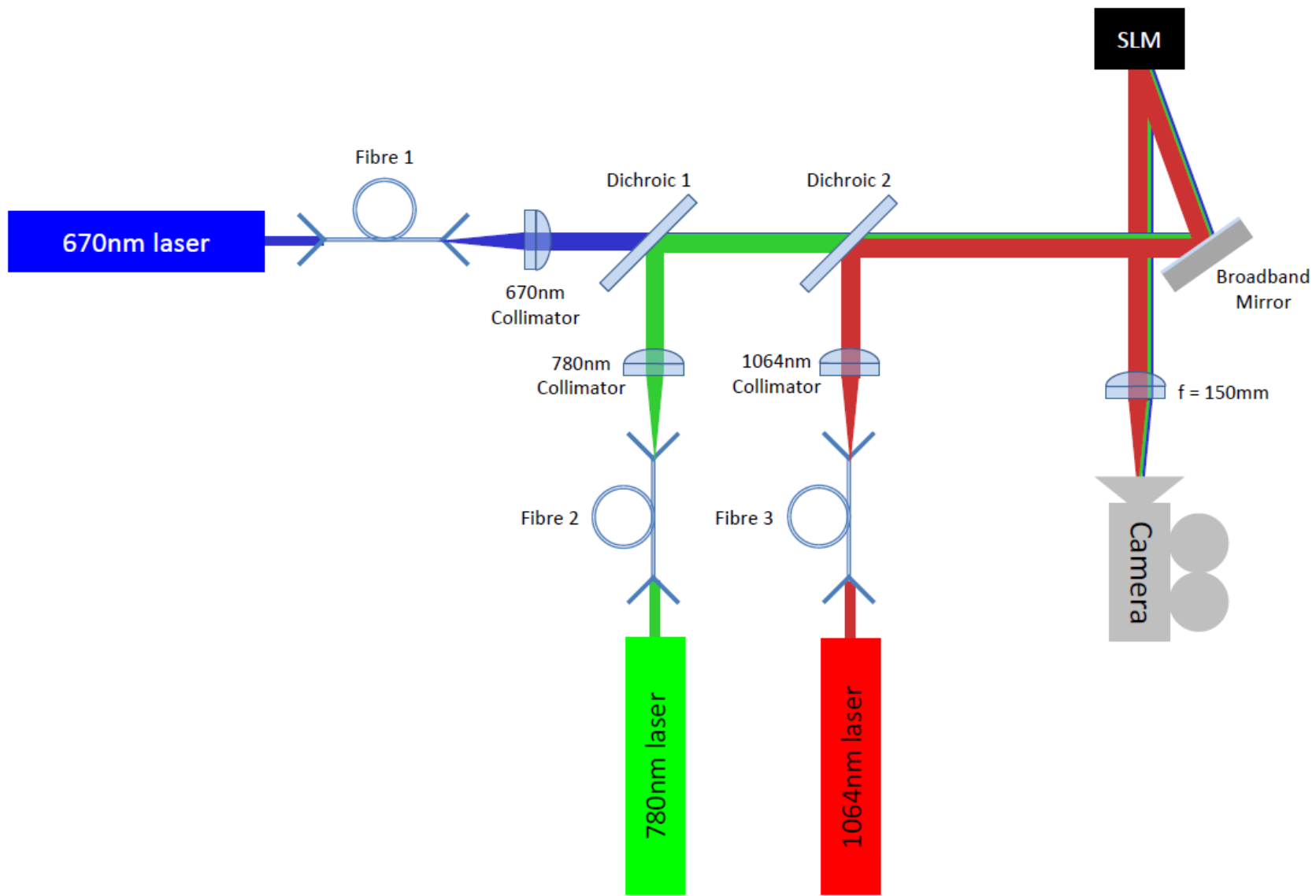
$^{87}\text{Rb}$  BEC,  $10^5$  atoms,  $T/T_c \sim 0.1$

# Multi-wavelength holography

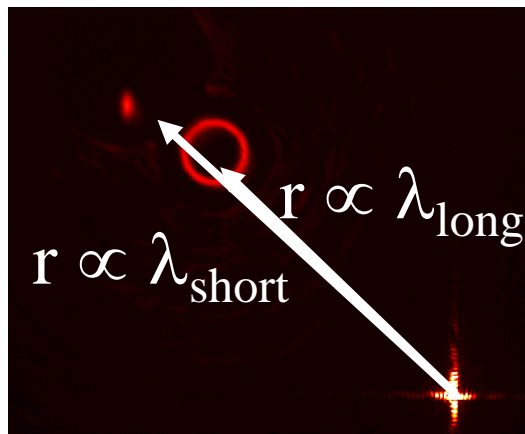


Remember, we use the SLM like a diffraction grating, and separate our pattern of interest from the unmodulated light

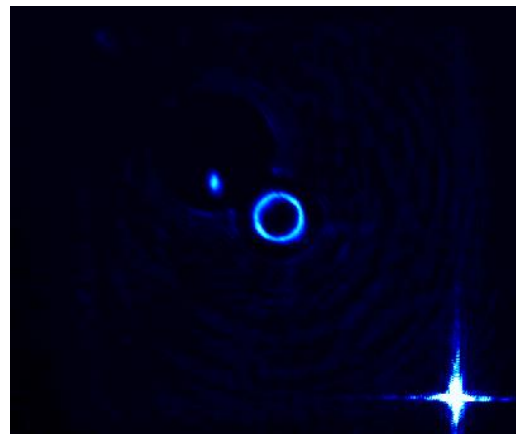




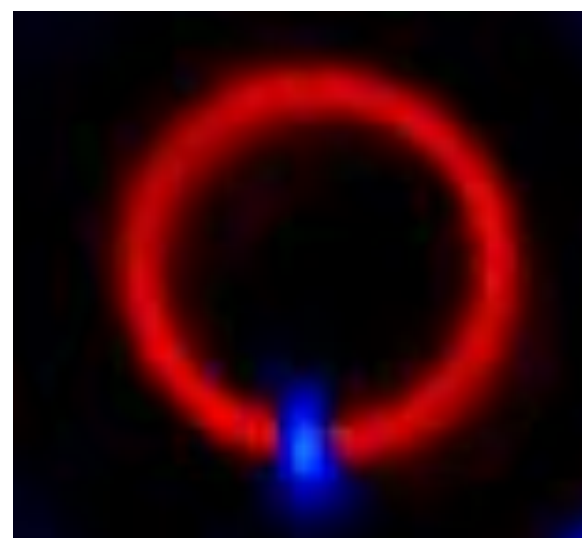
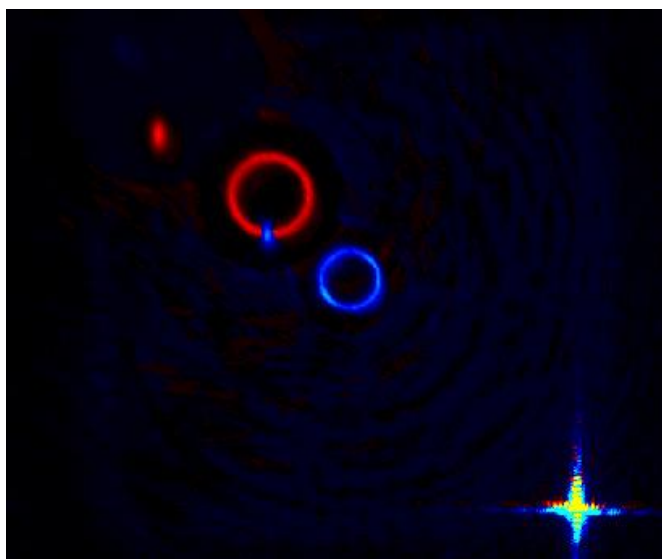
# Multi-wavelength holography



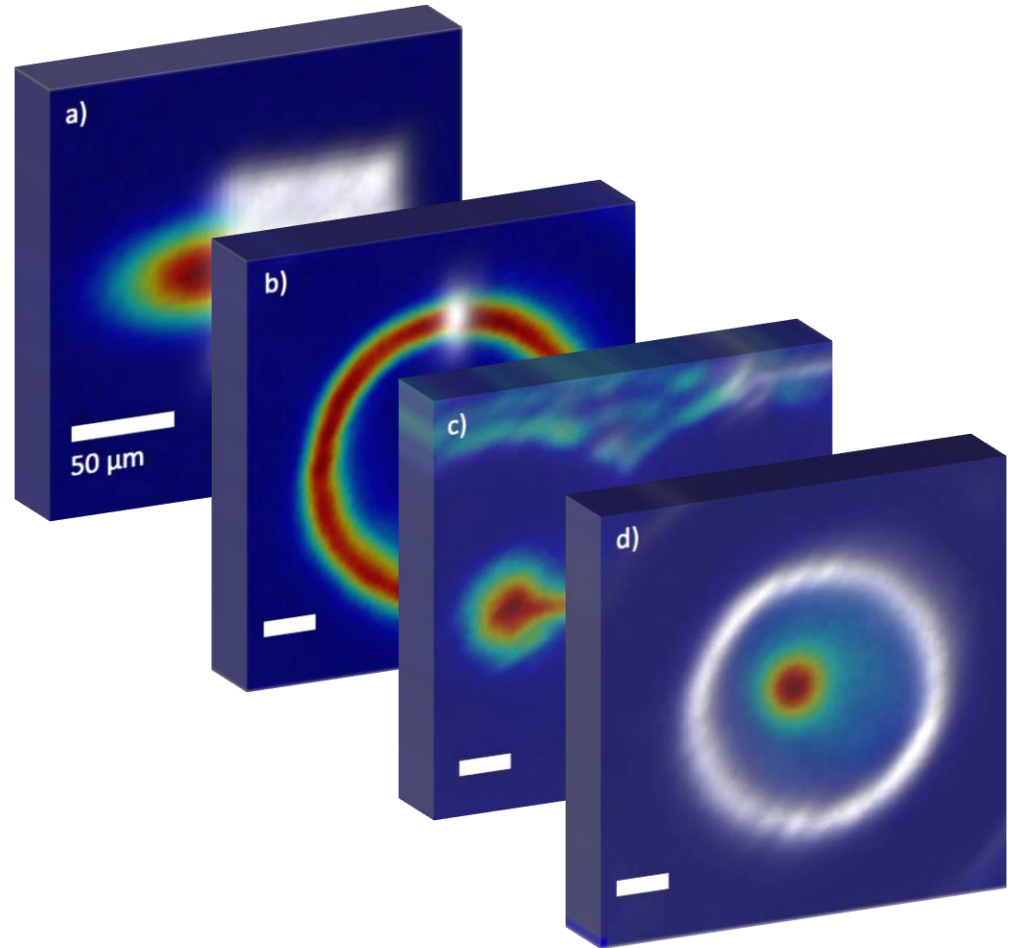
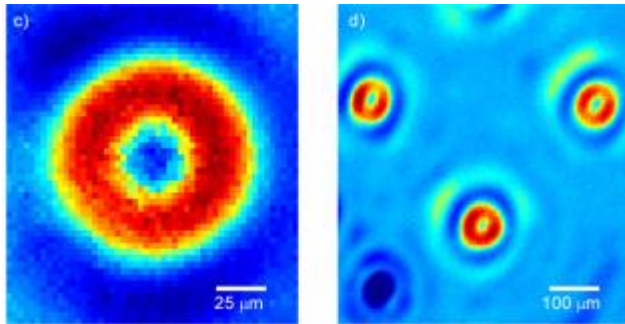
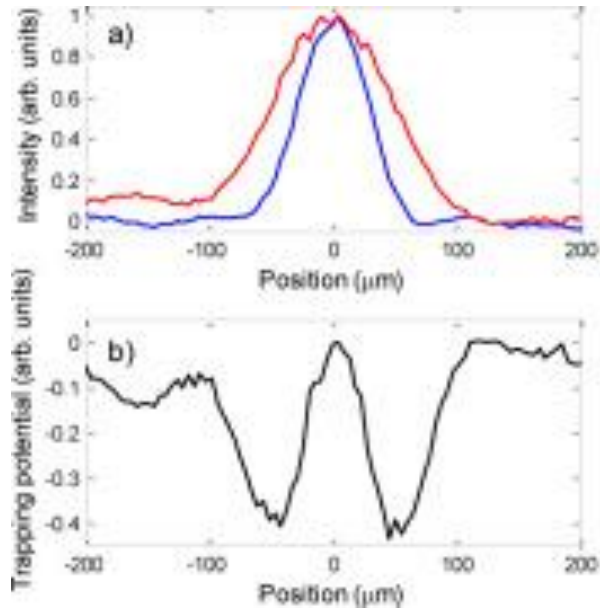
$\lambda = 1064\text{nm}$



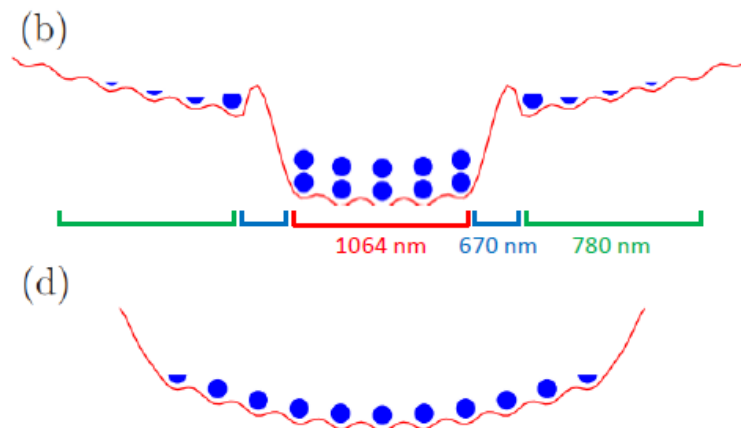
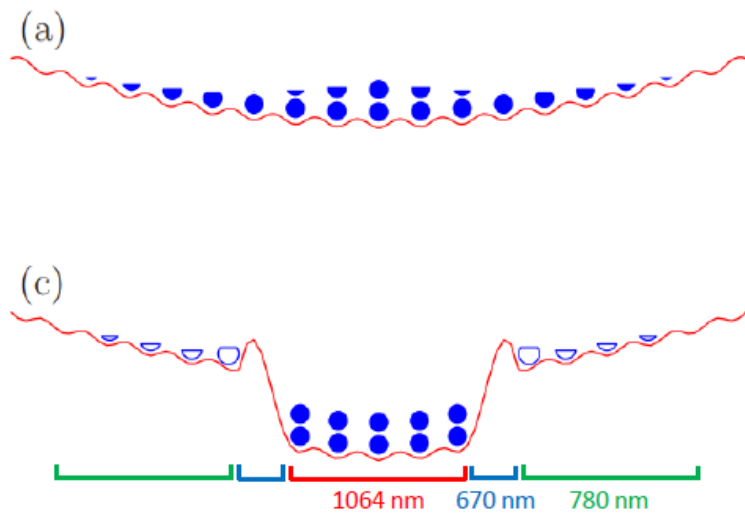
$\lambda = 780\text{nm}$



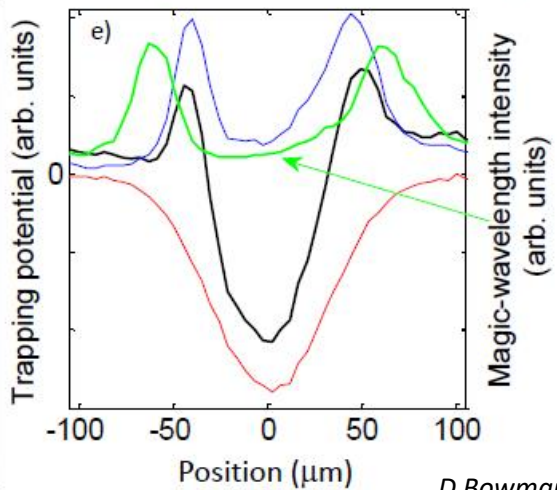
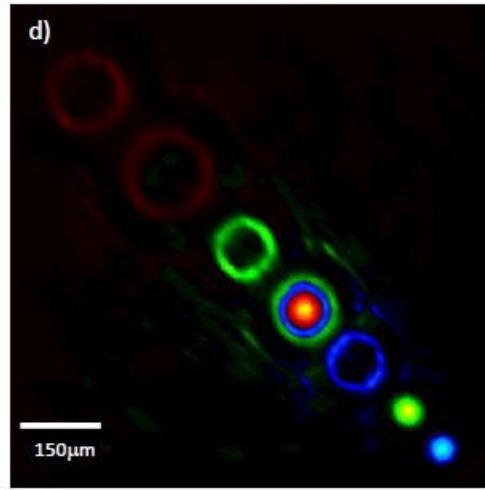
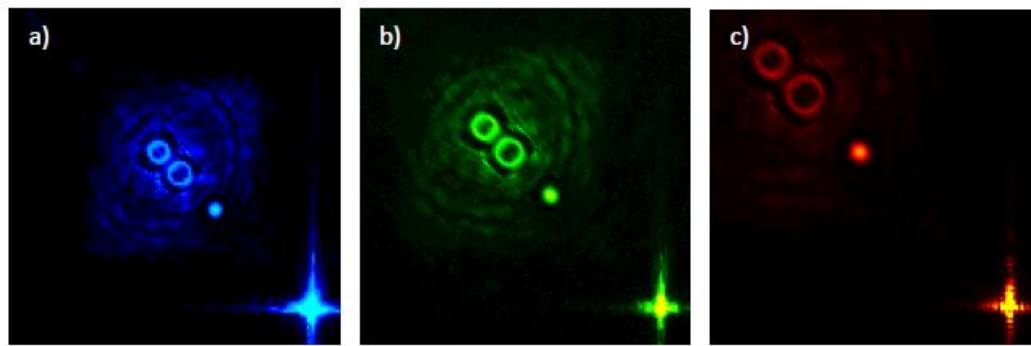
# Multiwavelength Holography







Bernier et al, PRA 79, 061601 (2009).



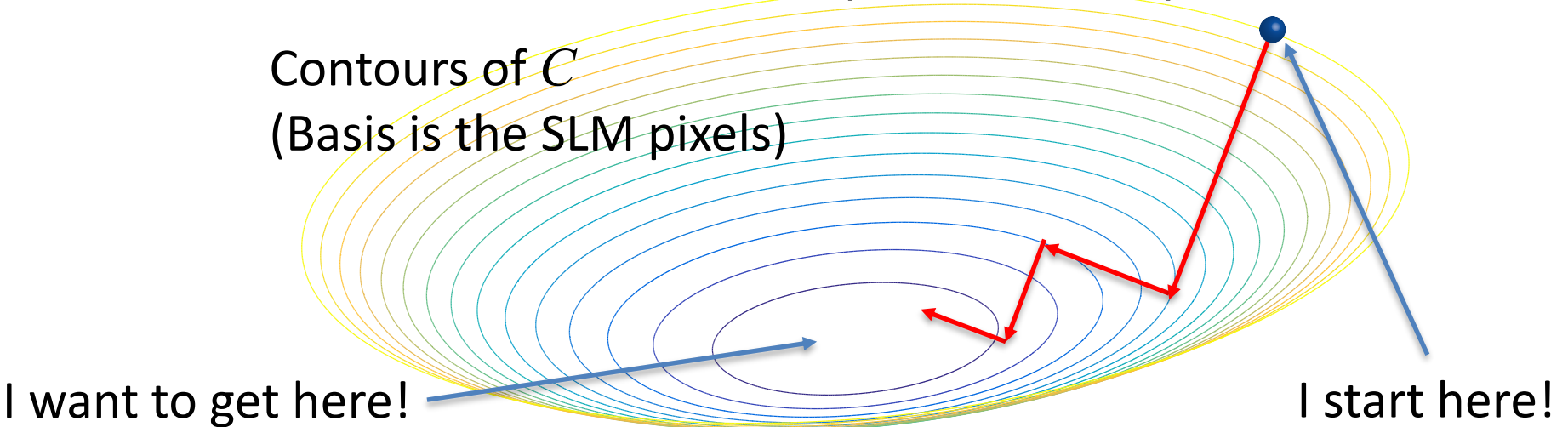
D Bowman et al Opt Express 23, 8365 (2015)

# Steepest Descent

## Cost Function Minimisation

Example cost function:  $C = \sum_{n,m} |T_{n,m} - E_{n,m}^{(out)}|^2$

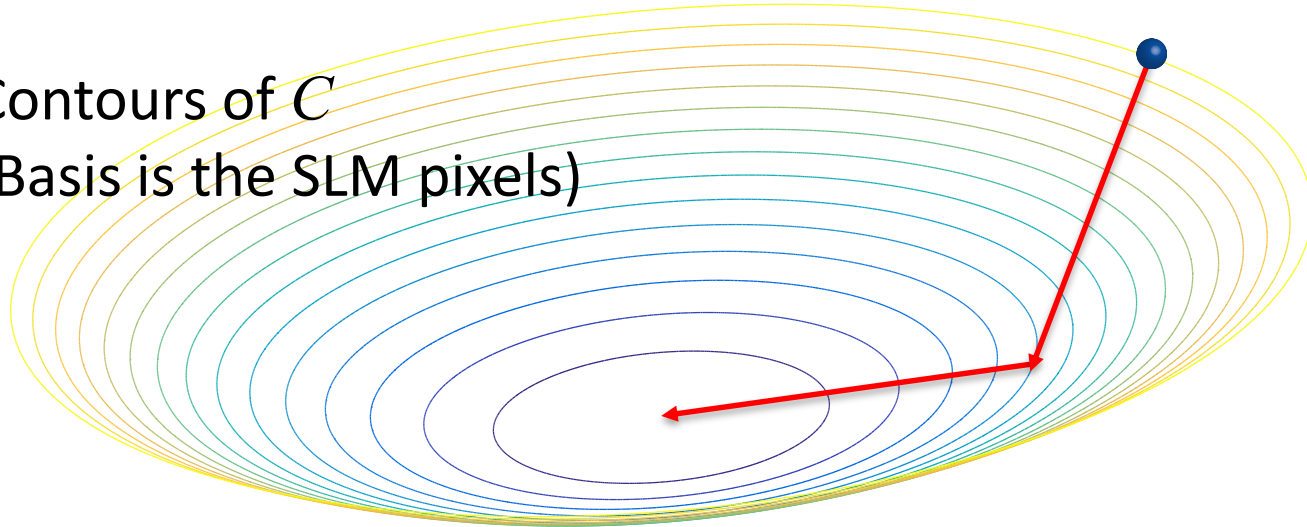
Contours of  $C$   
(Basis is the SLM pixels)



1. Move along a line, initially perpendicular to the contour (steepest descent)
2. Stop when I am parallel to nearest contour
3. Change direction to move perpendicular to contour and repeat until I reach the minimum

# Conjugate gradient

Contours of  $C$   
(Basis is the SLM pixels)

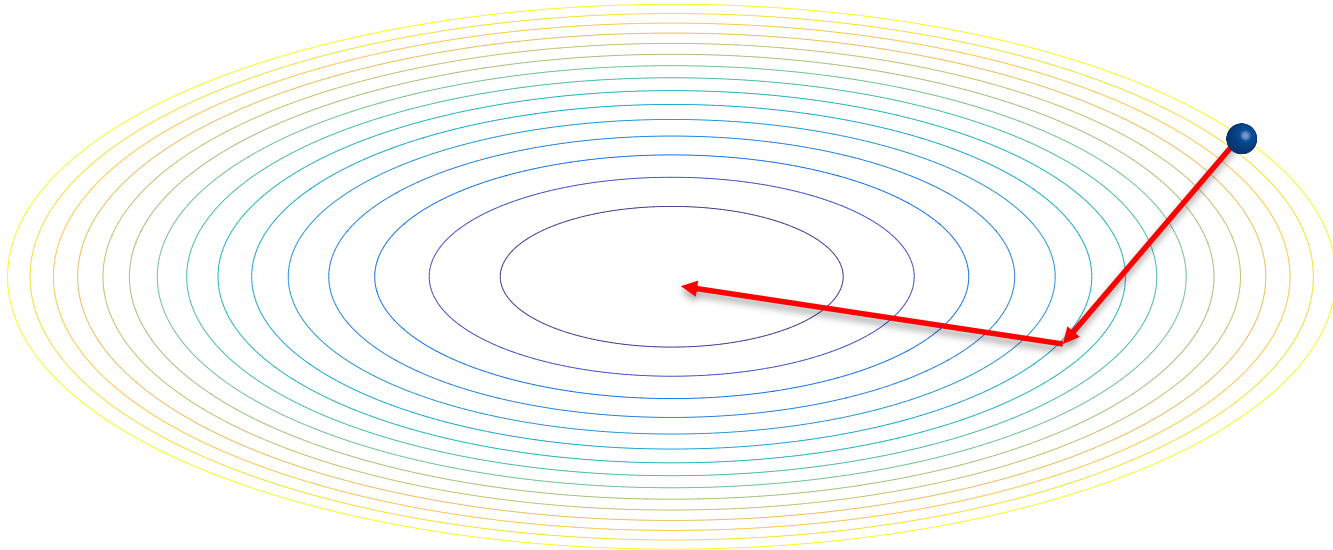


Wouldn't it be better if we could avoid going back over the same directions?

We could take the conjugate direction!

# Conjugate gradient

Contours of  $C$   
(Basis is the SLM pixels)



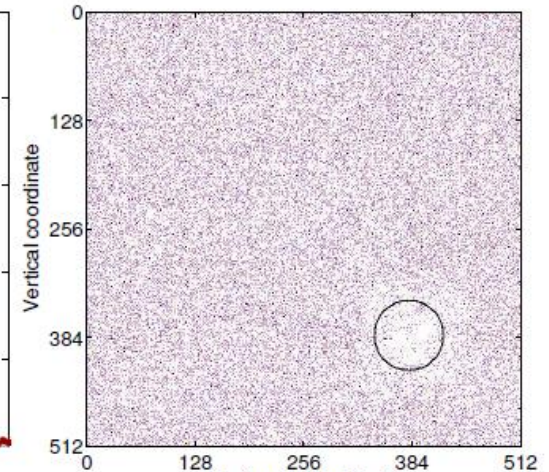
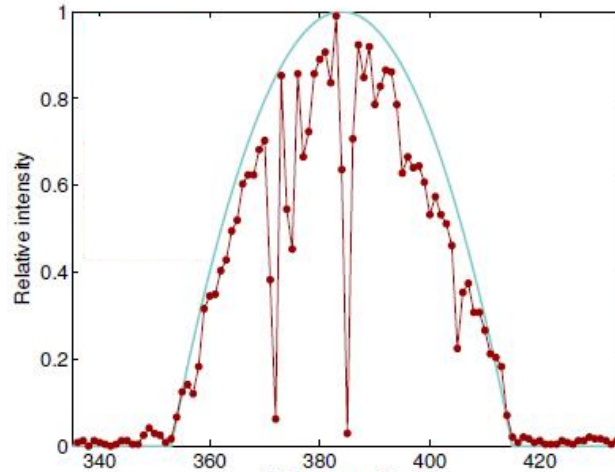
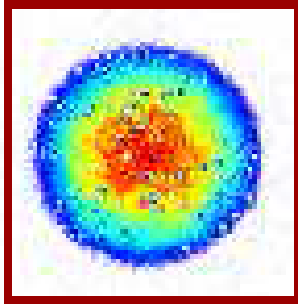
(also known as the C-orthogonal direction)

- Guarantees convergence in fewer than  $N$  steps, where  $N$  is the size of my basis vector (number of SLM pixels)
- Robust against initial position

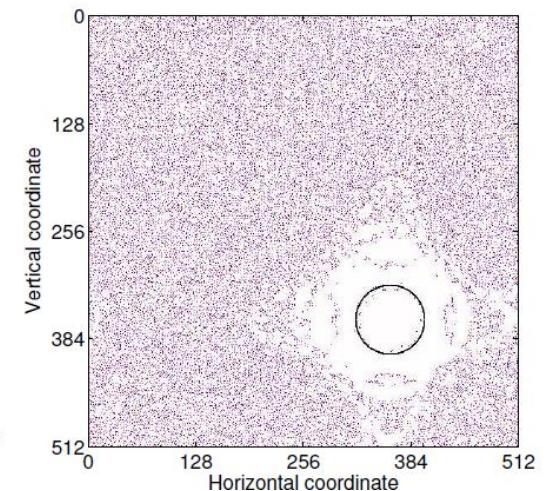
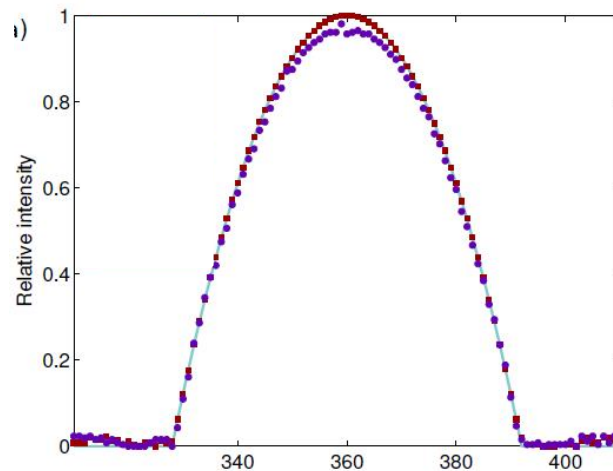
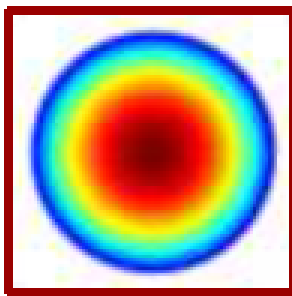
# Cost Function Choice

The power of this approach is flexibility in cost function choice.

$$C = \sum_{nm} \left( T_{nm} - |E_{out,nm}|^2 \right)^2$$



$$C_b = \sum_{nm} \left\{ \left( T_{nm} - |E_{nm}^{(out)}|^2 \right)^2 + \left[ \left( |E_{nm}^{(out)}|^2 - |E_{n(m\pm 1)}^{(out)}|^2 \right)^2 + \left( |E_{nm}^{(out)}|^2 - |E_{(n\pm 1)m}^{(out)}|^2 \right)^2 \right] \right\}$$



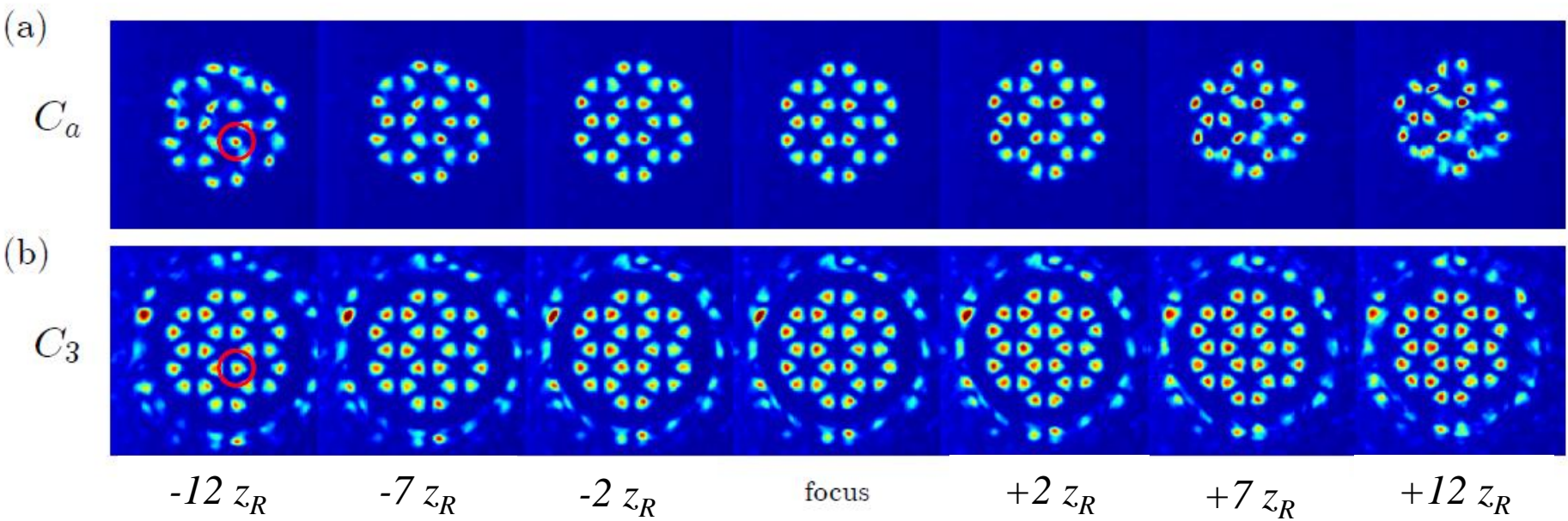
# Controlling Amplitude and Phase

- Clearly, to achieve smooth potentials, control over amplitude and phase is desirable

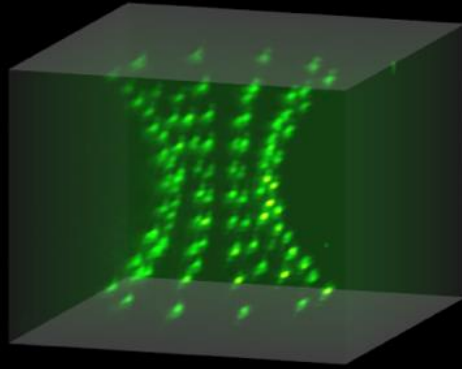
$$C_a = \sum_{nm} \left( T_{nm} - |E_{out,nm}|^2 \right)^2$$

$$C_3 = 10^d \left( 1 - \sum_{n,m} \operatorname{Re} \left\{ \left| \tilde{\tau}_{n,m}^* \tilde{E}_{n,m}^{\text{out}} \right| \right\} \right)^2$$

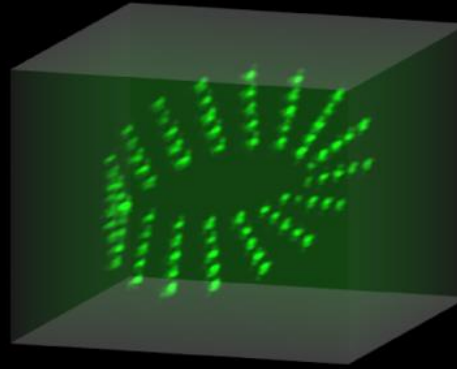
$$= 10^d \left( 1 - \sum_{n,m} \sqrt{\tilde{I}_{n,m} \tilde{T}_{n,m}} \cos(\Phi_{n,m} - \varphi_{n,m}) \right)^2$$



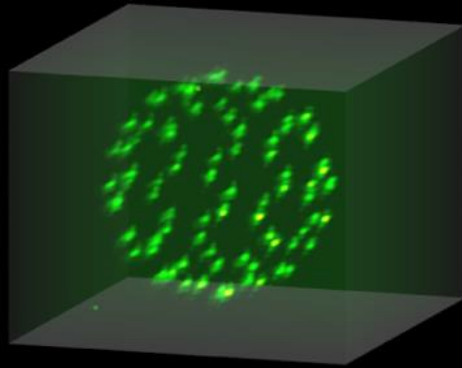
**a** Hyperboloid (90 sites)



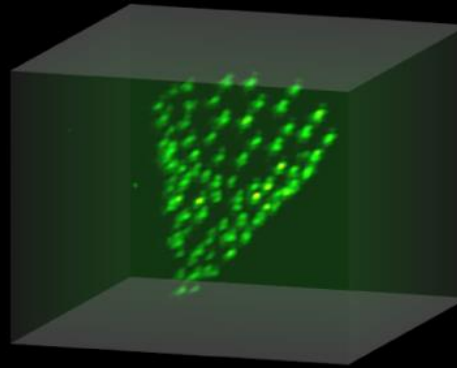
**b** Möbius strip (85 sites)



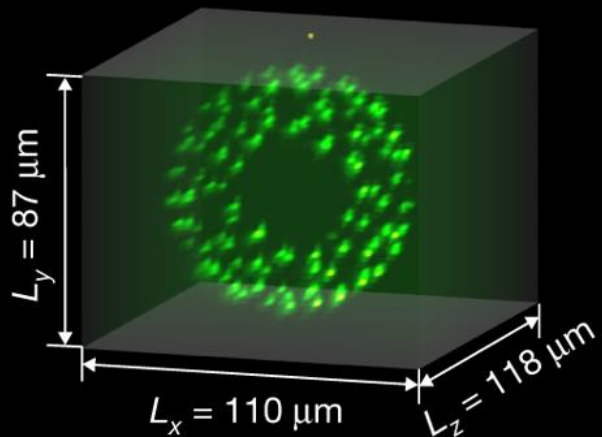
**c**  $C_{84}$  fullerene-like (84 sites)



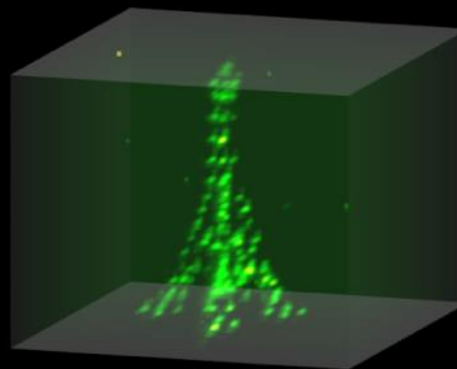
**d** Cone (100 sites)



**e** Torus (120 sites)

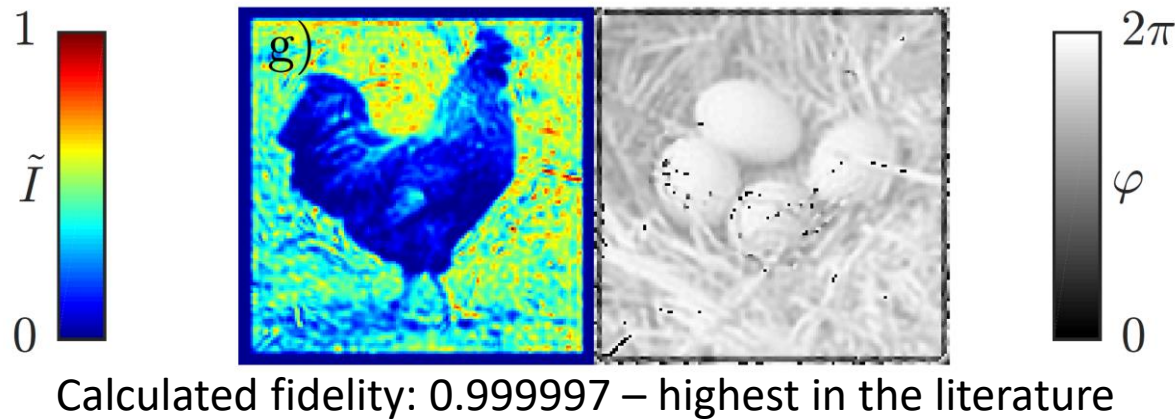


**f** Eiffel tower (126 sites)



# Controlling Amplitude and Phase

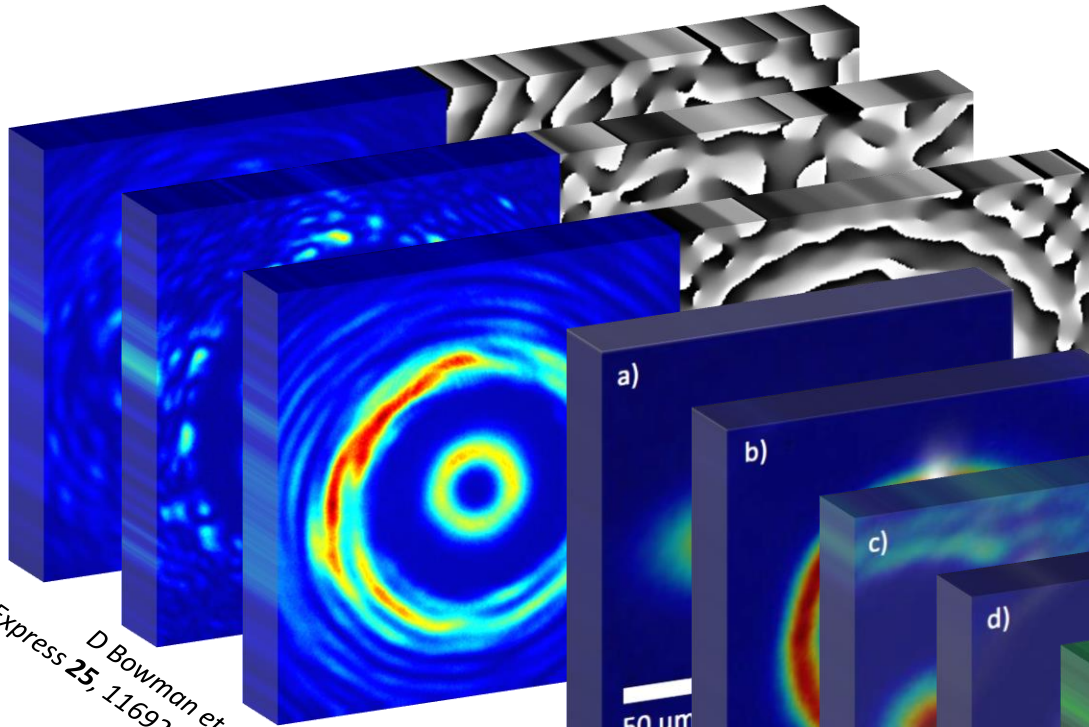
- With this method, we can produce completely uncorrelated images in amplitude and phase



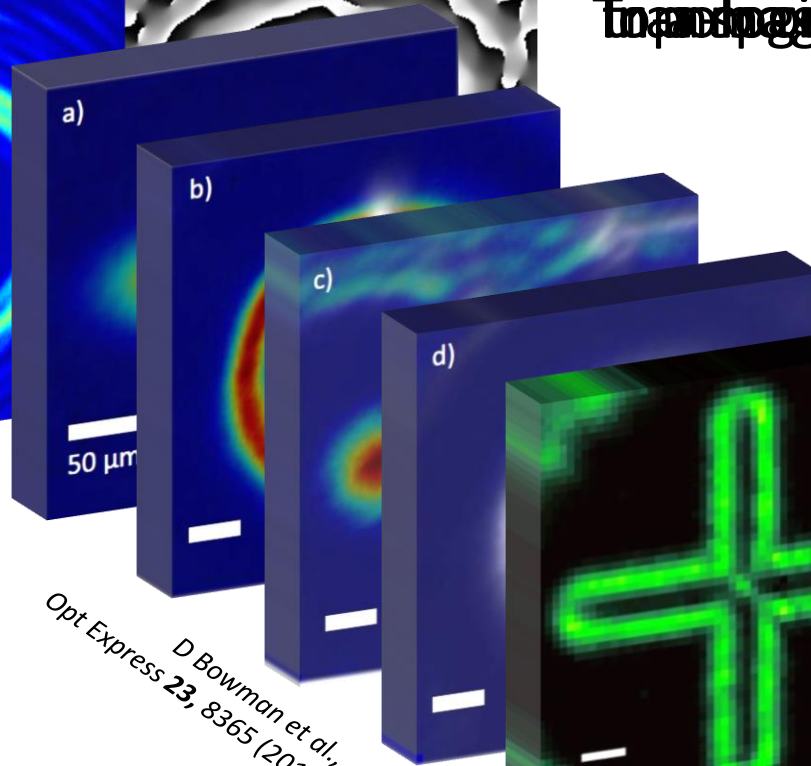
- Furthermore, phase can be used to provide additional forces on trapped objects



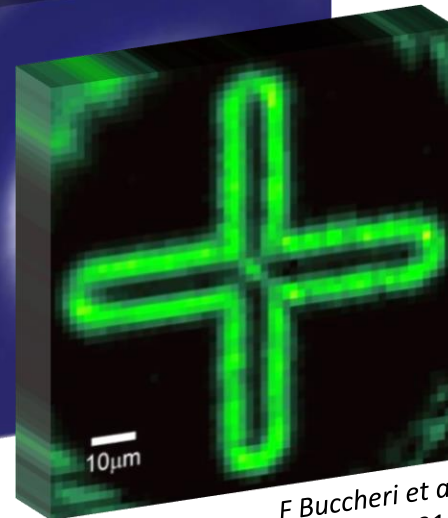
# Holographic Optical Potentials



*Opt Express* **25**, 11692 (2017)  
D Bowman et al.,



*Opt Express* **23**, 8365 (2015)  
D Bowman et al.,



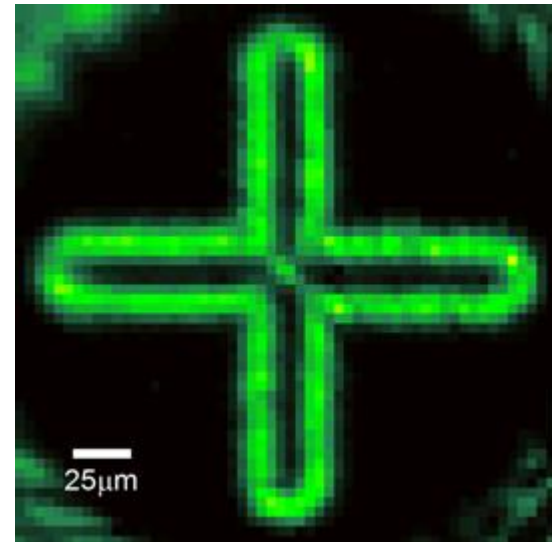
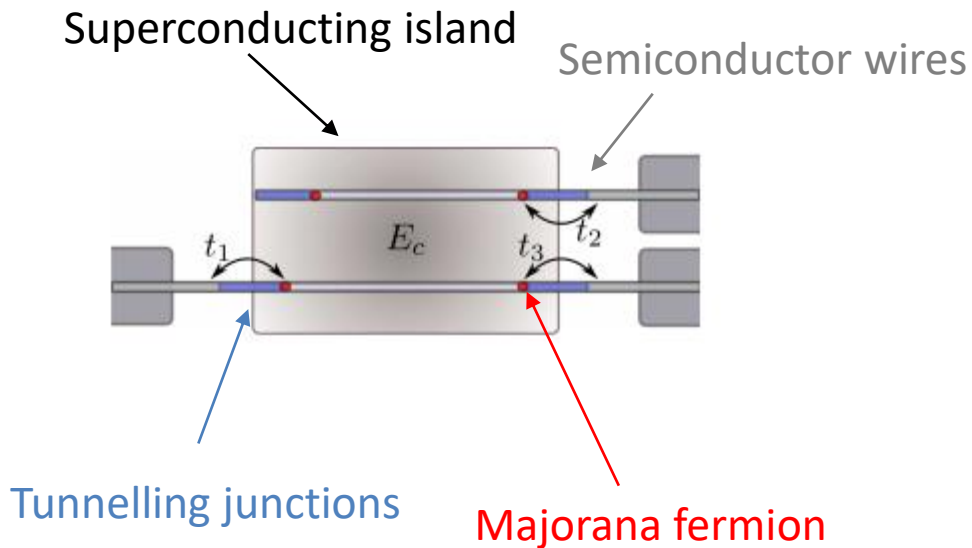
F Buccheri et al,  
*New J. Phys.* **18**, 075012 (2016)

Ring-shaped optical potentials for trapping  
of atoms in a ring-shaped optical potential  
Trapping of atoms in a ring-shaped optical  
potential

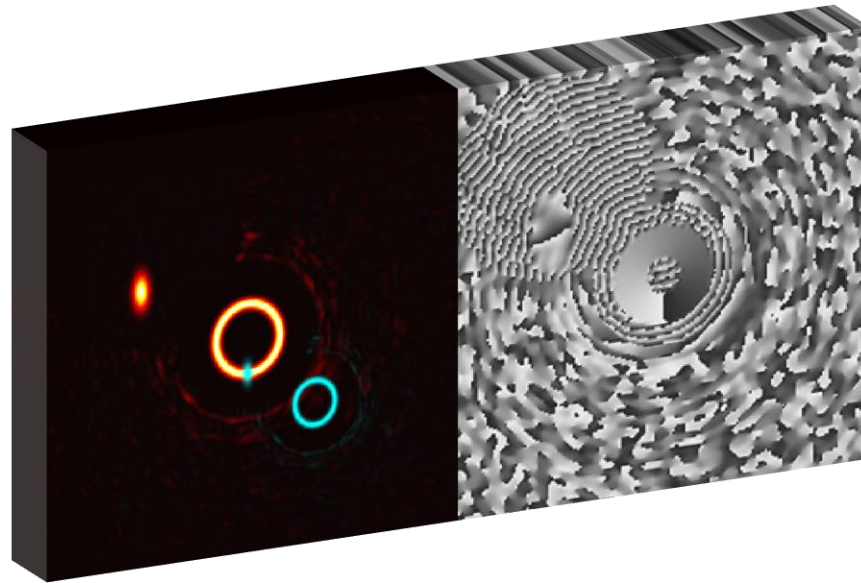
# Topological Kondo Qubits

$$H^\alpha = \int_0^L dx \left[ \frac{\hbar^2}{2m} \partial_x \Psi_\alpha^+(x) \partial_x \Psi_\alpha(x) + \frac{c}{2} \Psi_\alpha^+(x) \Psi_\alpha^+(x) \Psi_\alpha(x) \Psi_\alpha(x) \right],$$

$$H = -i \frac{\hbar v_F}{2\pi} \sum_{\alpha=1}^M \int dx \psi_\alpha^+(x) \partial_x \psi_\alpha(x) - \lambda \sum_{\alpha \neq \beta} \gamma_\alpha \gamma_\beta \psi_\alpha^+(0) \psi_\beta(0)$$



Can you put it all together?



# How can phase exert a force?

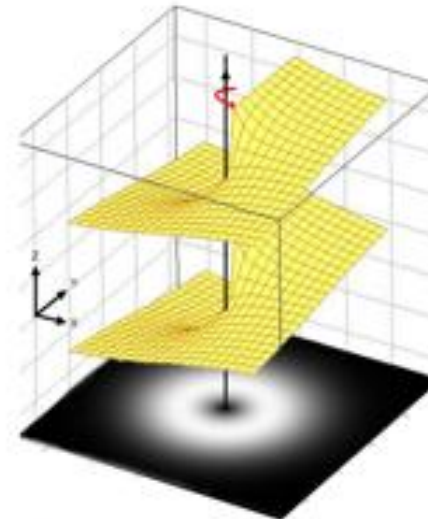
Structured light can possess angular momentum: rotation

$$j = \epsilon_0 \left[ r \times \langle E \times B \rangle \right]$$

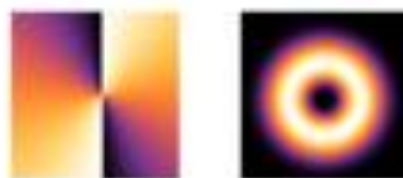
Allen et al Phys Rev A (1992)

**Orbital:** due to inclined wavefronts

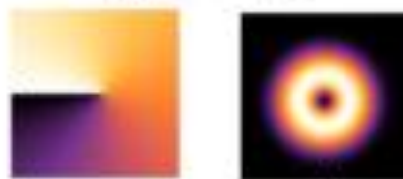
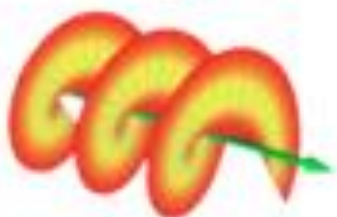
*$l\hbar$  per photon*



Laguerre-  
Gaussian  
(LG) beam



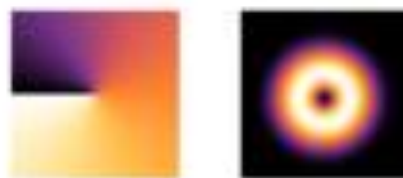
$$m = +2$$



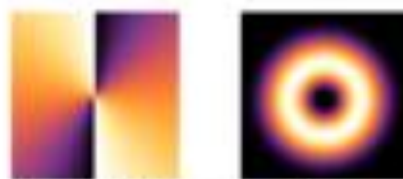
$$m = +1$$



$$m = 0$$

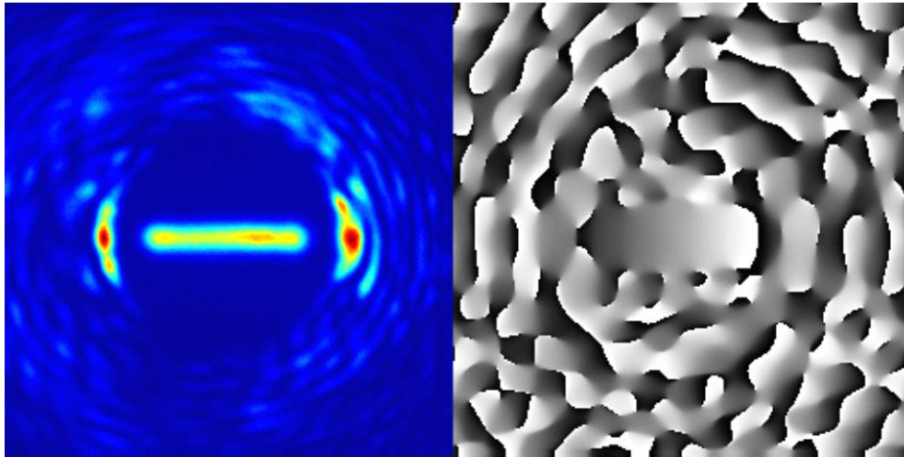


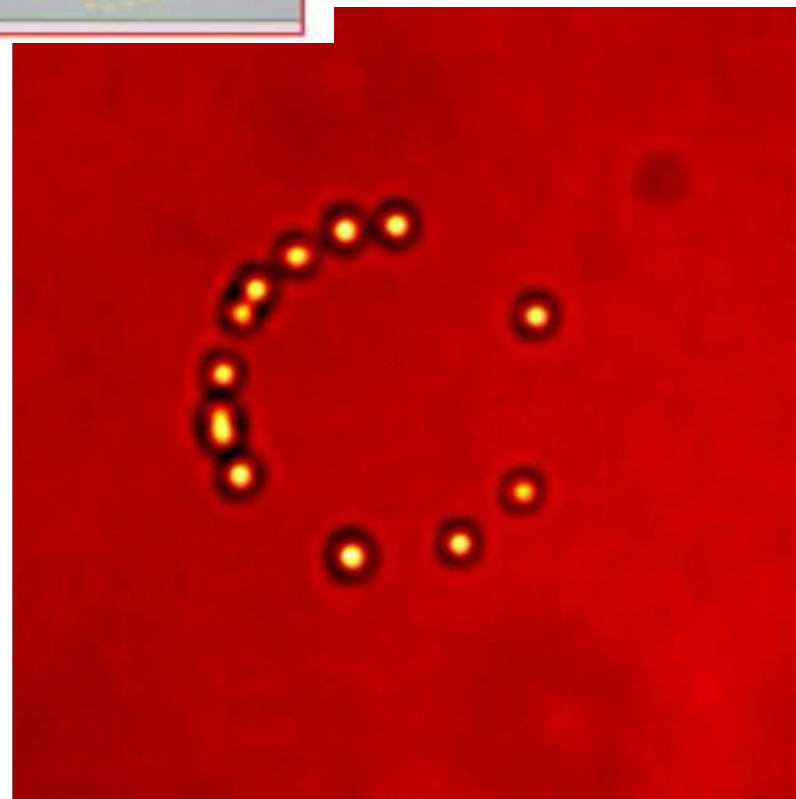
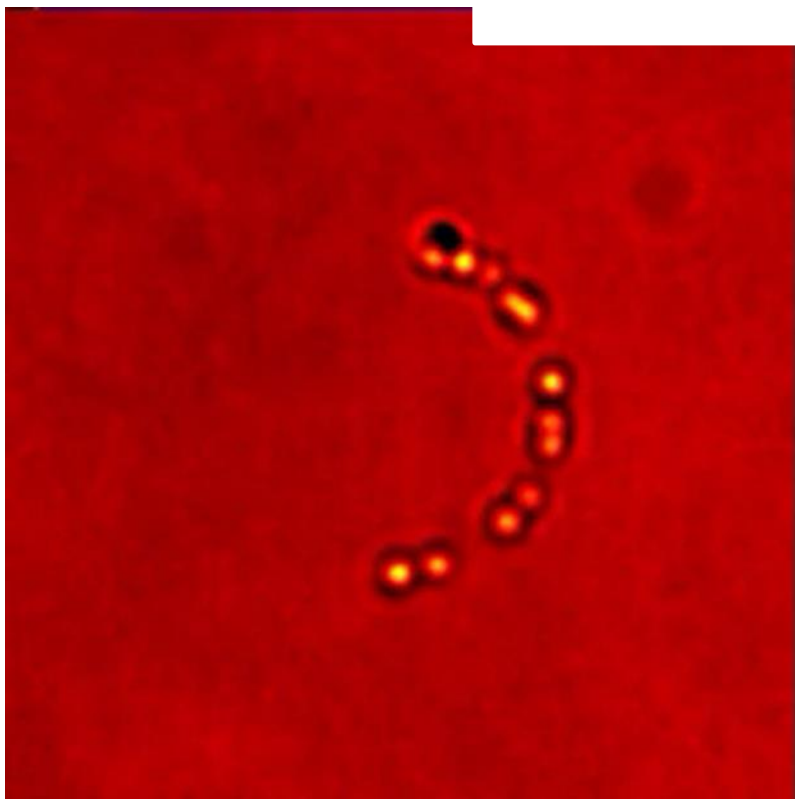
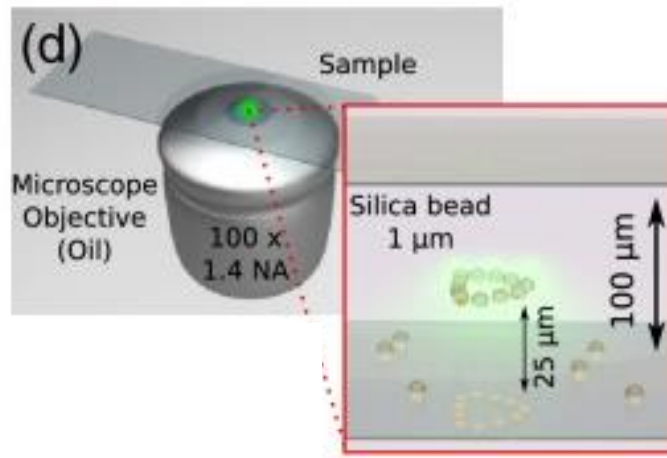
$$m = -1$$

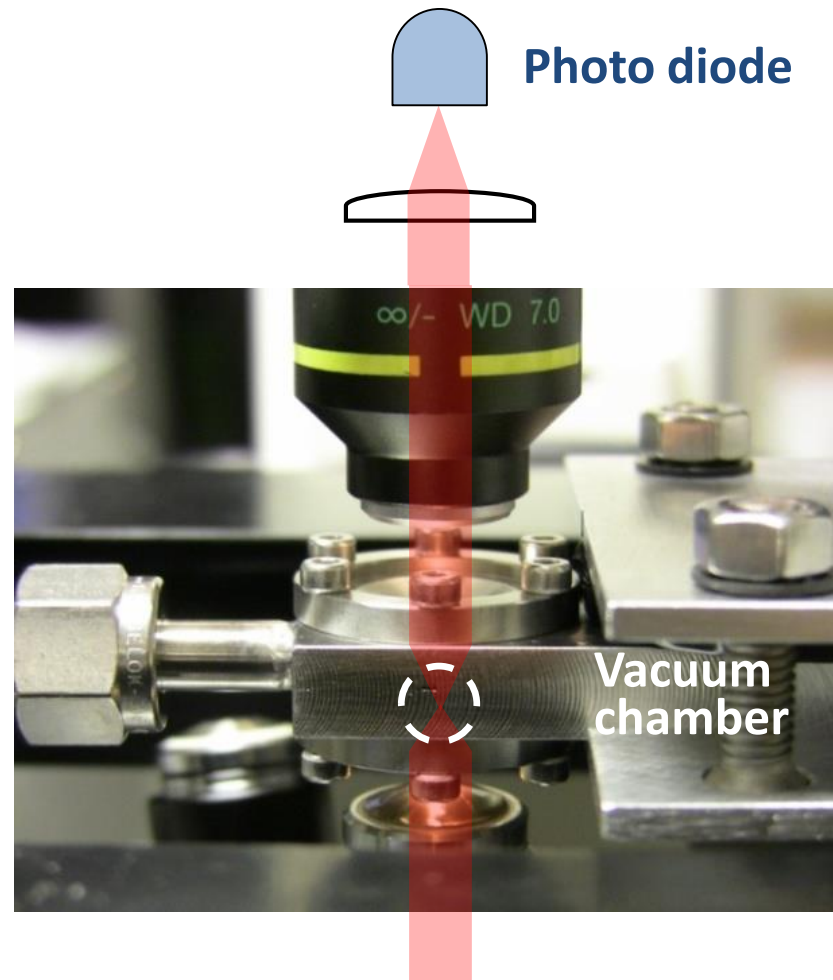


$$m = -2$$

These tilted phase fronts can also exert  
linear momentum

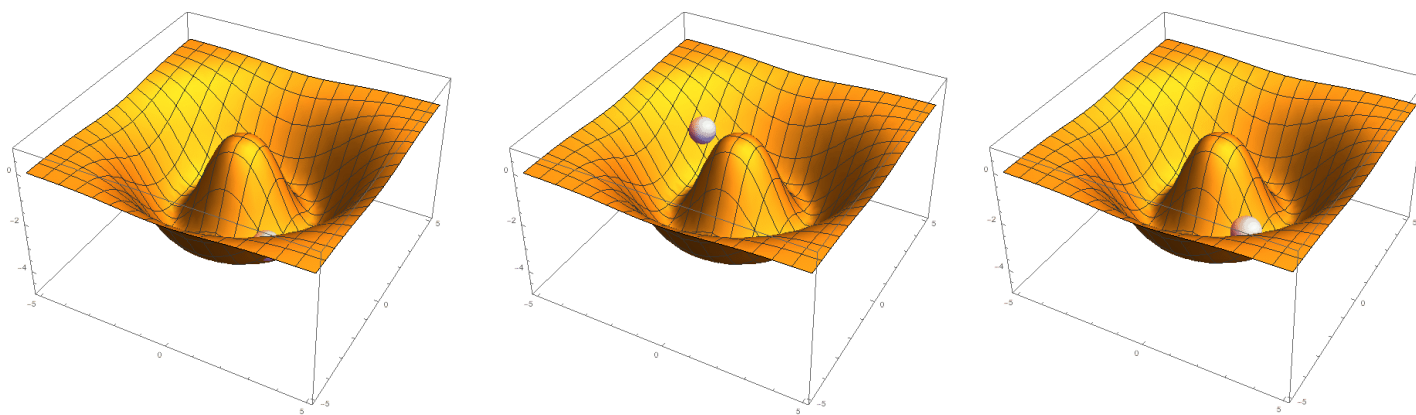






**trapping beam (1070nm)**





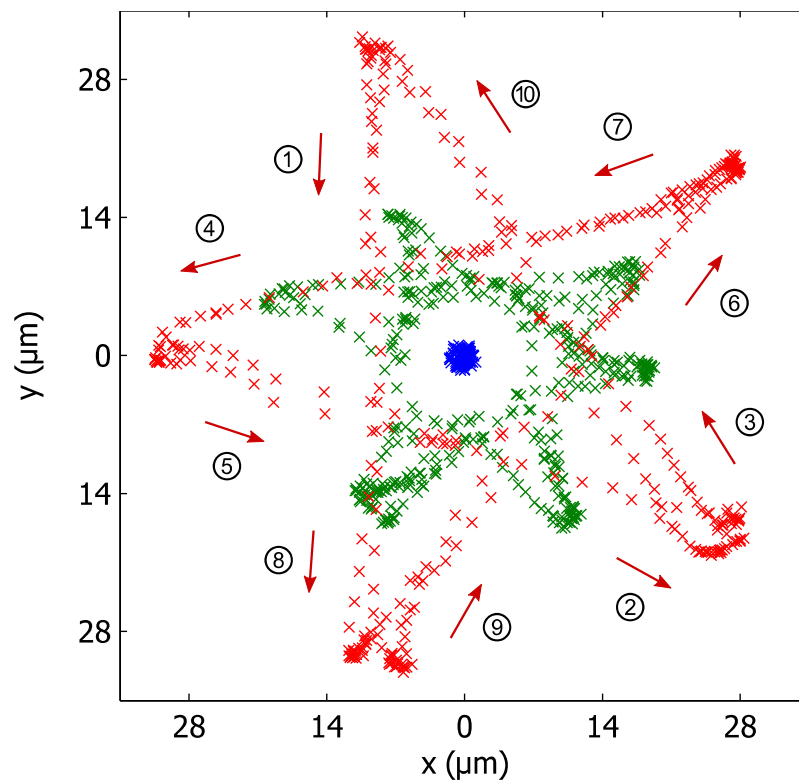
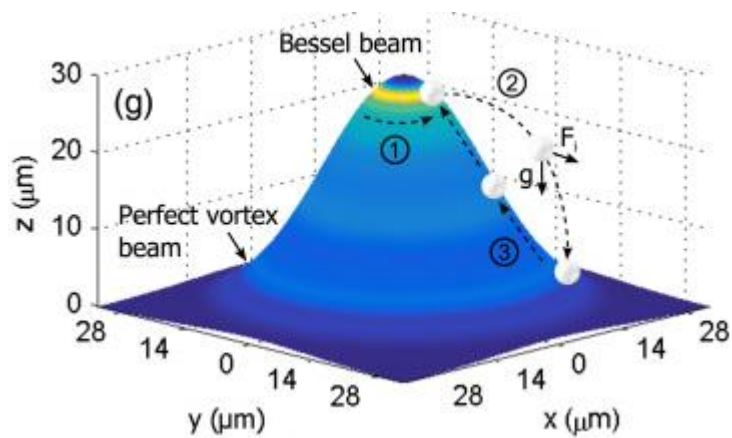
Mazilu et al., Phys Rev A 94, 053821 (2016)

Particle trajectories for

$\ell = 3$  (blue)

$\ell = 10$  (green)

$\ell = 30$  (red)



# Structured light can possess angular momentum: rotation

$$j = \epsilon_o [r \times \langle E \times B \rangle]$$

Allen et al Phys Rev A (1992)

**Spin:** due to polarisation state (rotating E-field)

**Orbital:** due to inclined wavefronts

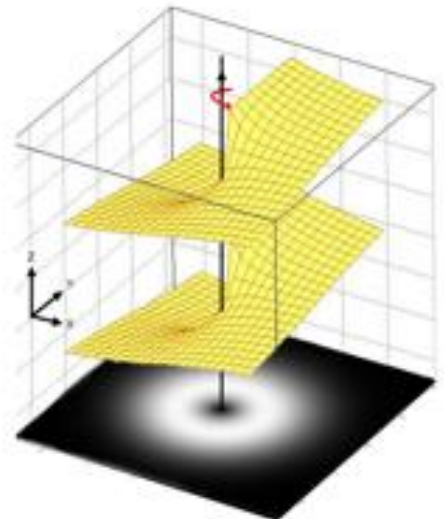
$J_z = +\hbar$

$$|L\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} e^{i(kz - \omega t)}$$

$J_z = -\hbar$

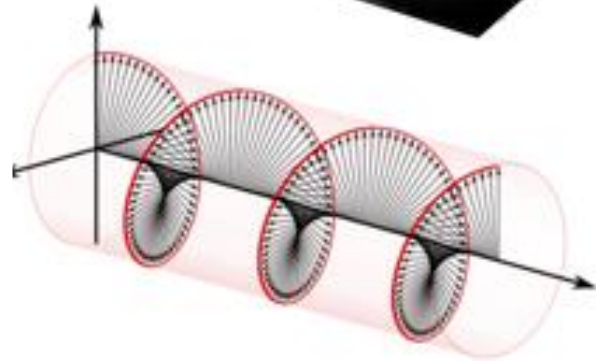
$$|R\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix} e^{i(kz - \omega t)}$$

$\pm \hbar$  per photon

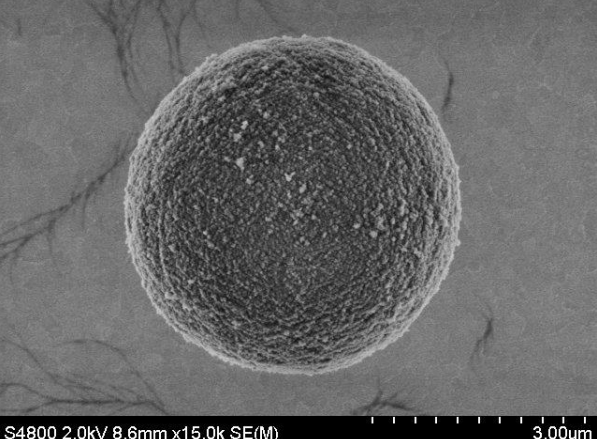


$l\hbar$  per photon

Laguerre-Gaussian (LG) beam



Circularly polarized beam  
Spin angular momentum



BBC NEWS EDINBURGH, FIFE & EAST SCOTLAND

28 August 2013 Last updated at 16:27

**University of St Andrews scientists create 'fastest man-made spinning object'**

Top Stories

- UK storms wash away railway line
- Ten hurt as gas blast fattens homes
- Millions hit by Tube strike chaos
- London Tube strike latest
- UN: Vatican must remove abusers

Features

- Obscure treasures: The dodo skeleton and other strange objects
- Cow v camel: Would you drink coffee made with...

NBC NEWS SCIENCE

TOPICS: Space, Environment, Innovation, Weird science

**QUANTUM PHYSICS**  
Scientists create fastest-spinning man-made object ever

**Scientists create fastest-spinning man-made object ever**

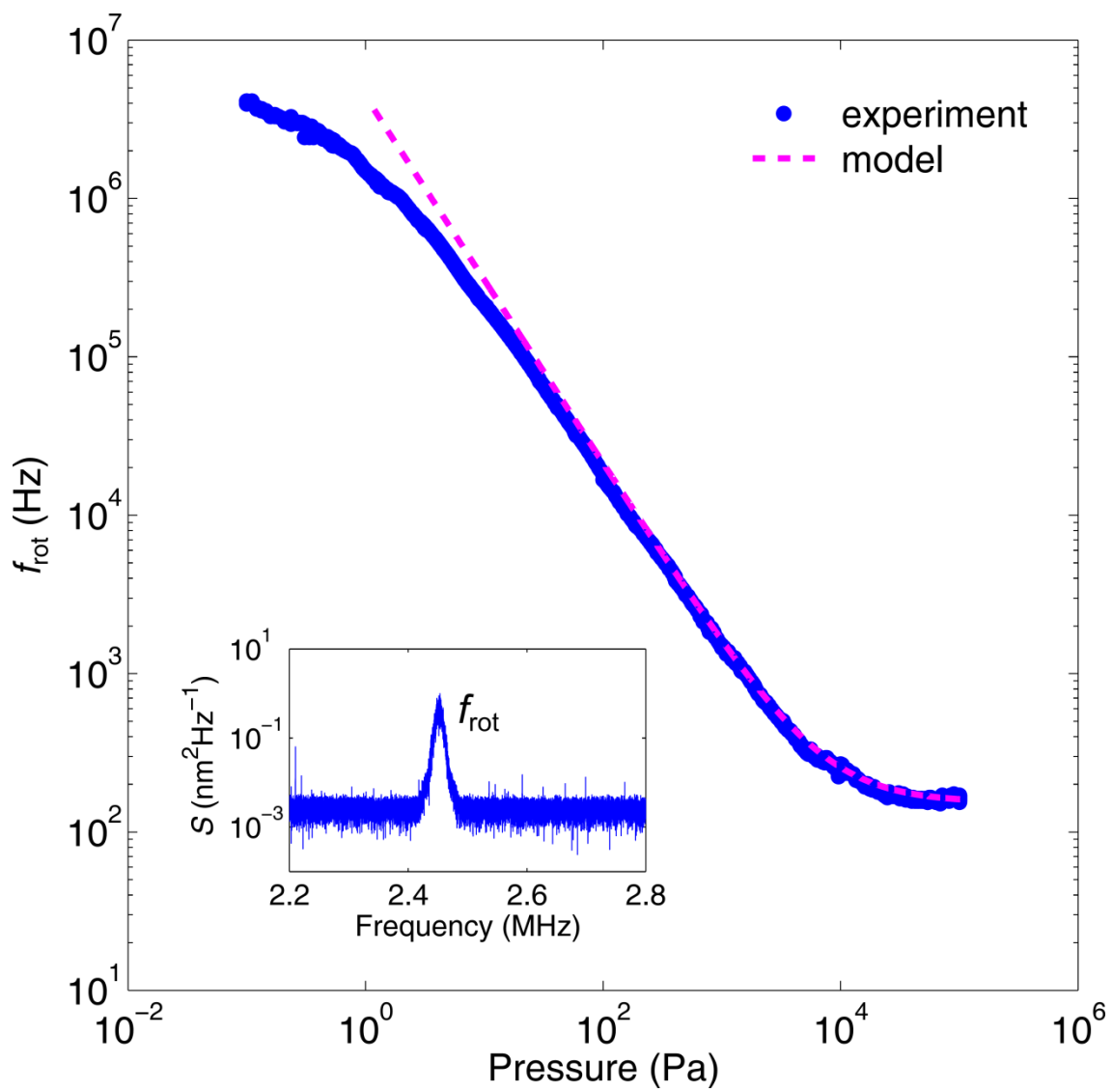
Tia Ghose, LiveScience

Aug. 29, 2013 at 12:39 PM ET

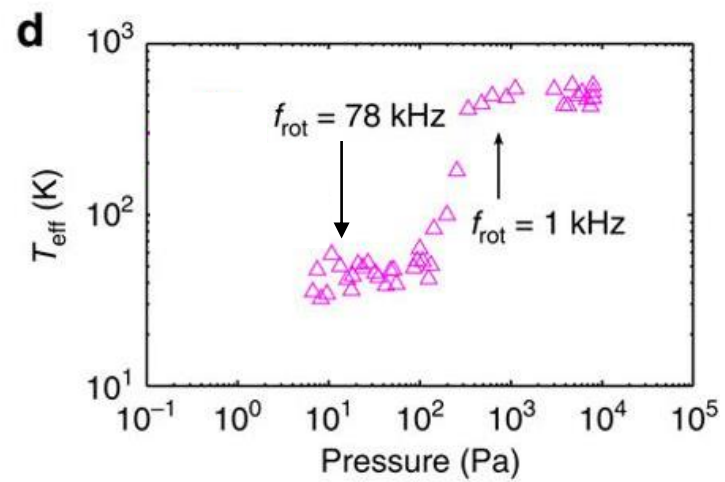
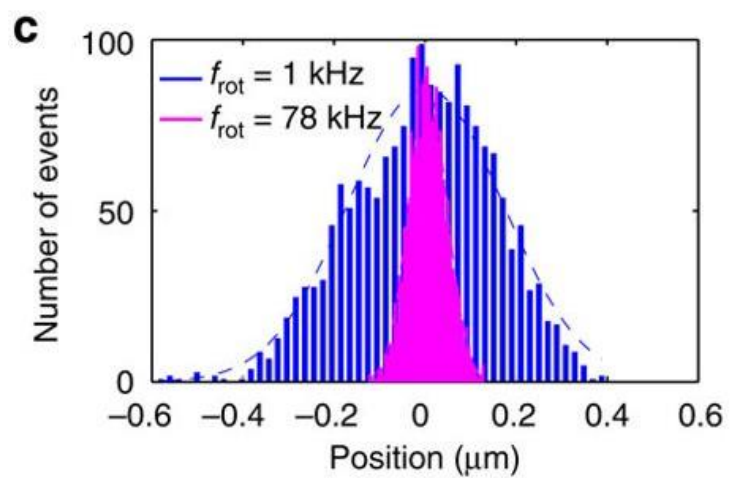
Scientists have created a microscopic sphere and set it whirling at a blistering 600 million rotations per minute.

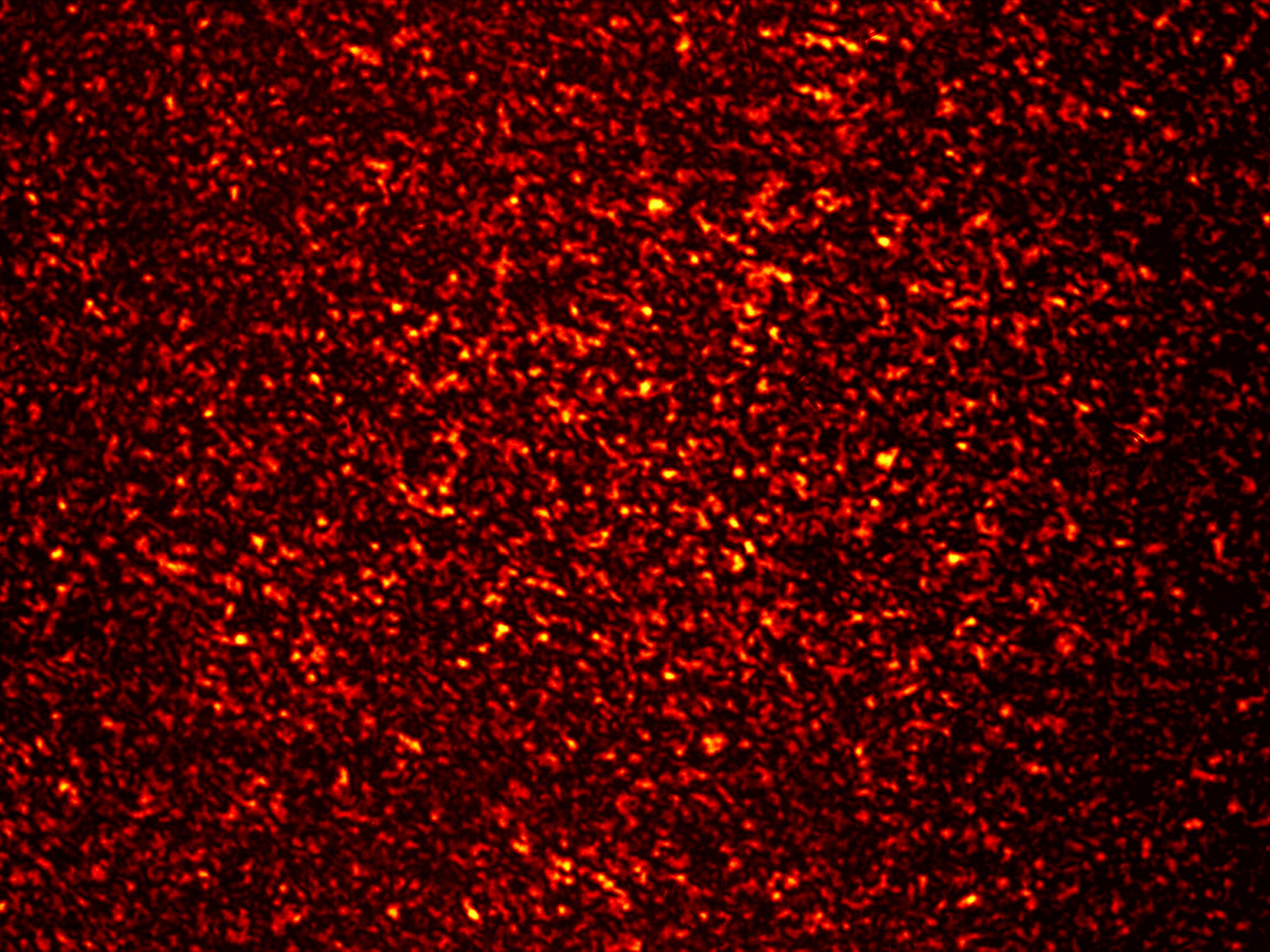
The sphere, which rotates 500,000 times faster than the average washing machine, is the **fastest-spinning object** ever made.

**Arita et al. Nature Comm**  
**4, 2374 (2013)**



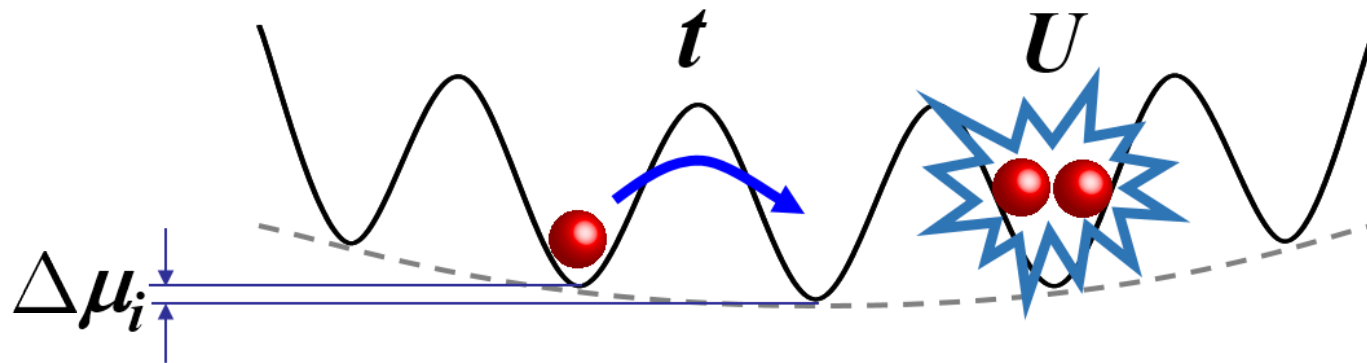
*Guinness book of world records 2015 (fastest man-made rotation)*

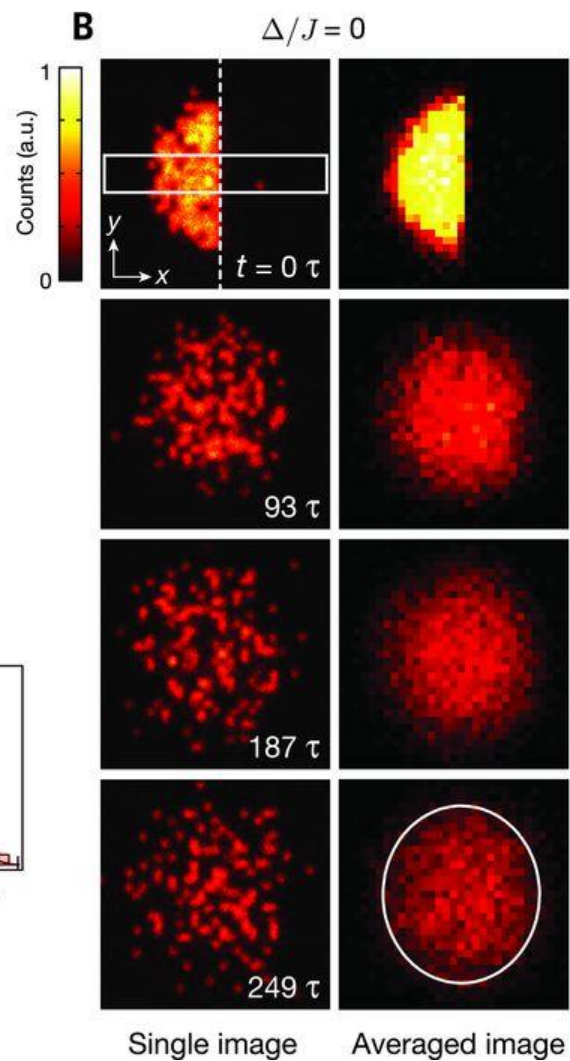




# The Bose-Hubbard Model

$$\mathcal{H}_B = -t \sum_{\langle ij \rangle} (\hat{b}_i^\dagger \hat{b}_j + \hat{b}_j^\dagger \hat{b}_i) - \mu \sum_i \hat{n}_i + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1)$$



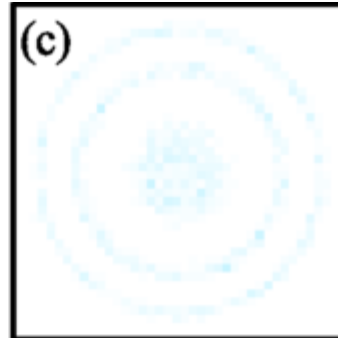
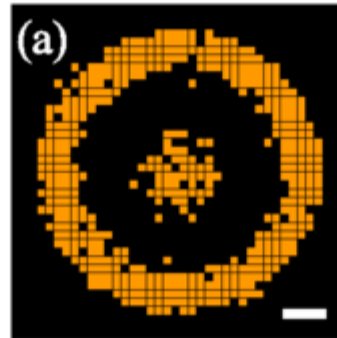
**A****B**

# shameless plug...

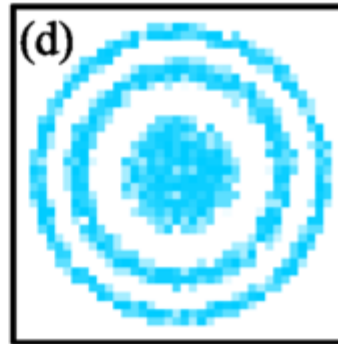
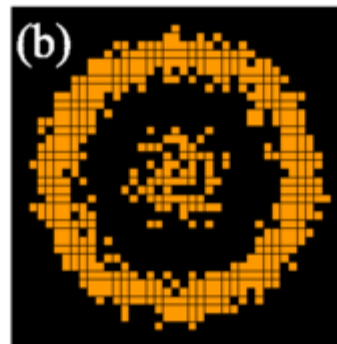
Parity-sensitive  
density

$$Q_{EA} = [\langle n_i^2 \rangle]_{av} - [\langle n_i \rangle]_{av}^2$$

$\delta = 0$   
 $T = 0.07 U/k_B$

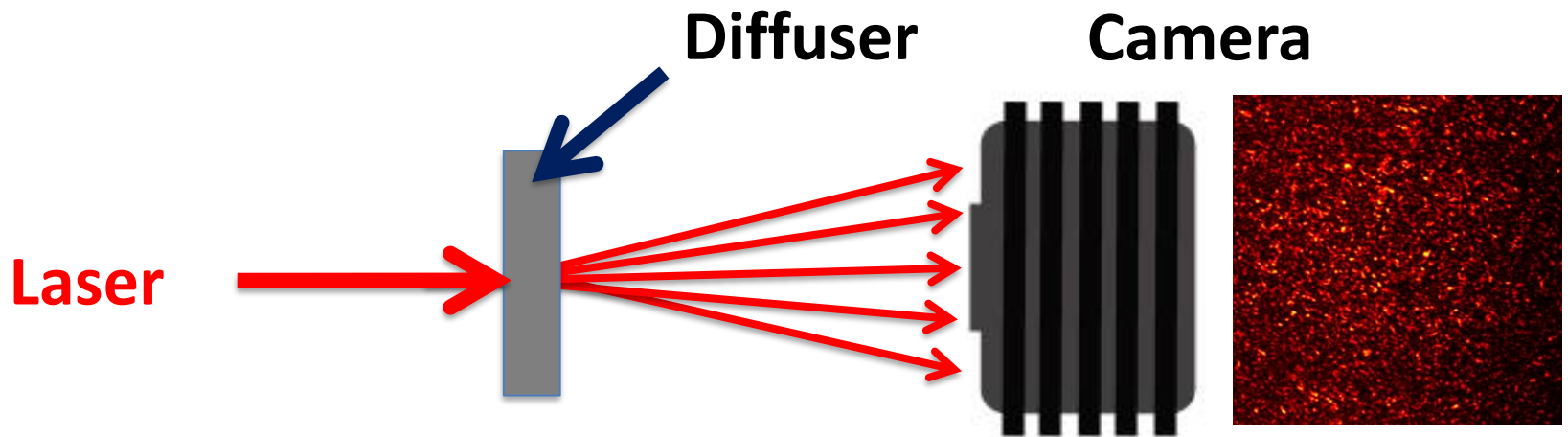


$\delta = 0.3$   
 $T = 0$

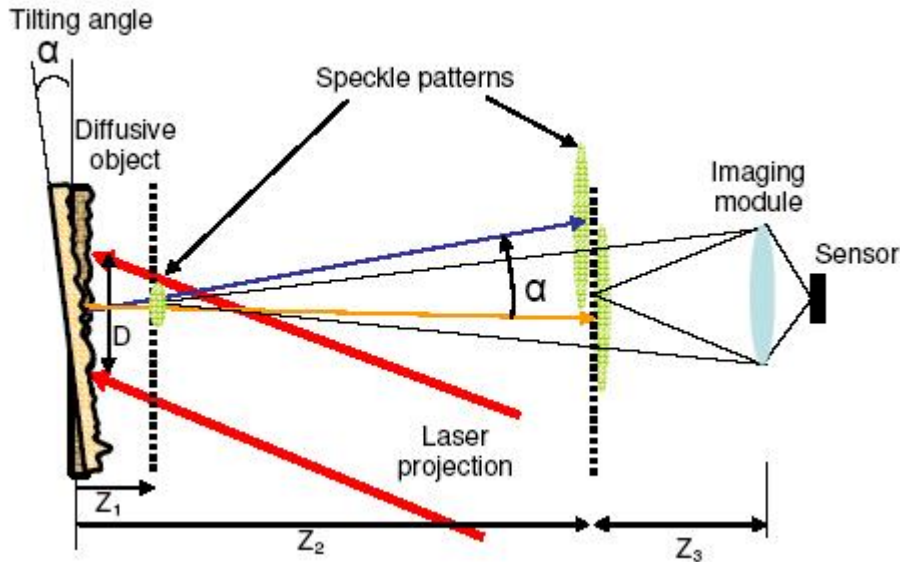




Is speckle useful for anything other than just adding disorder?



# Measuring motion of a subject



Imaged at a distance of 30m

Zalevsky et al, Opt Express **24** 21566 (2009)

Alternatively, apply a vibration to the subject and measure response. Used in detecting:

Faults in airplane wings

Breast tumours

Voids behind walls

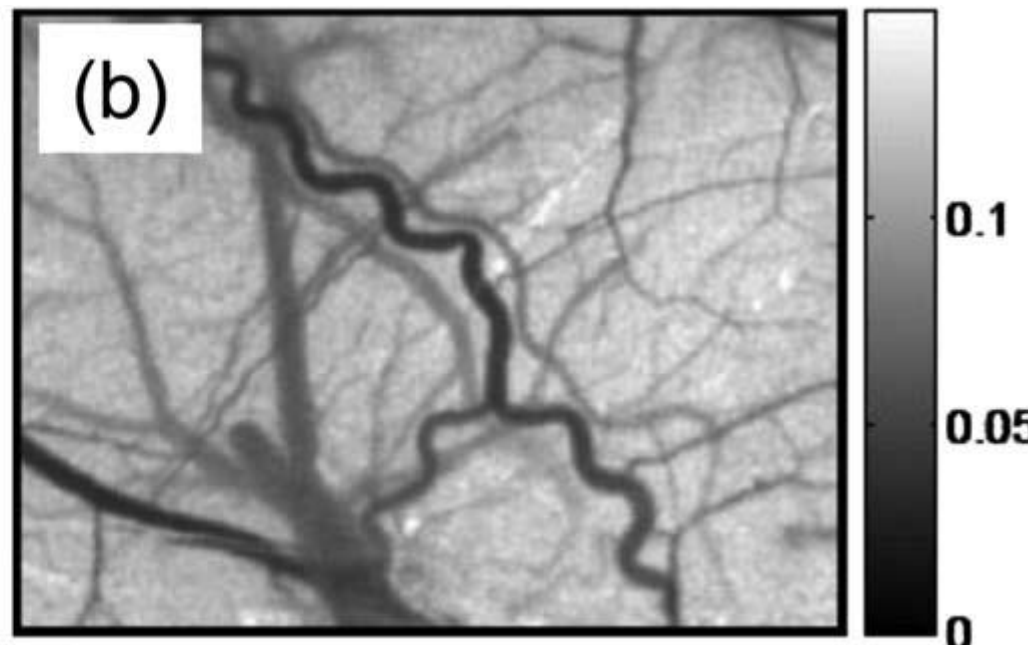
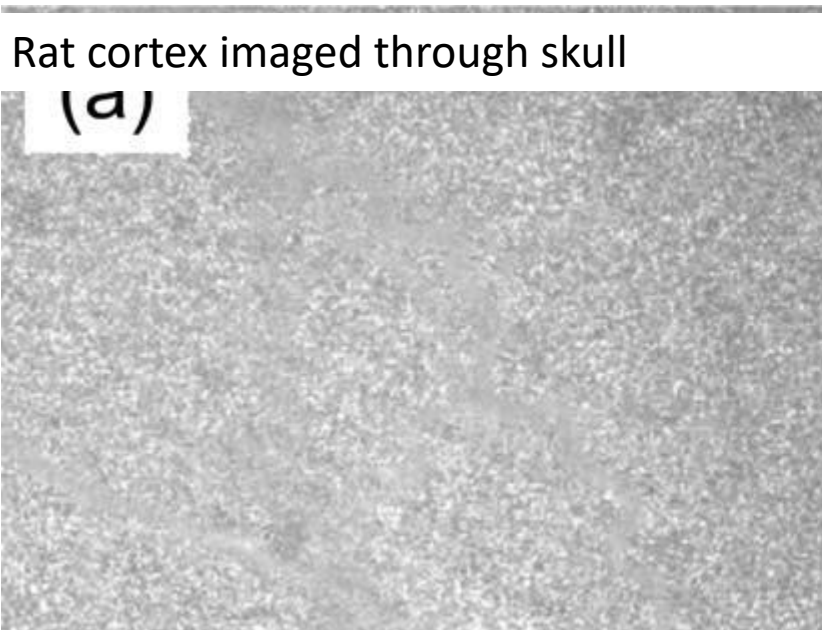
X-Box Kinect

# Measuring motion within the subject

<https://youtu.be/iH90scynV8Q?t=45s>

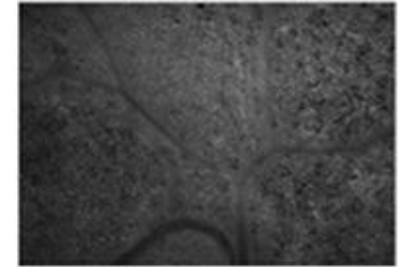
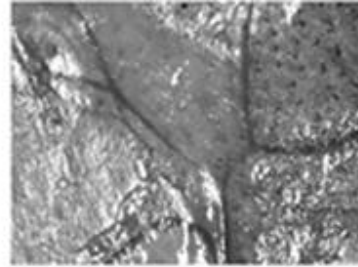
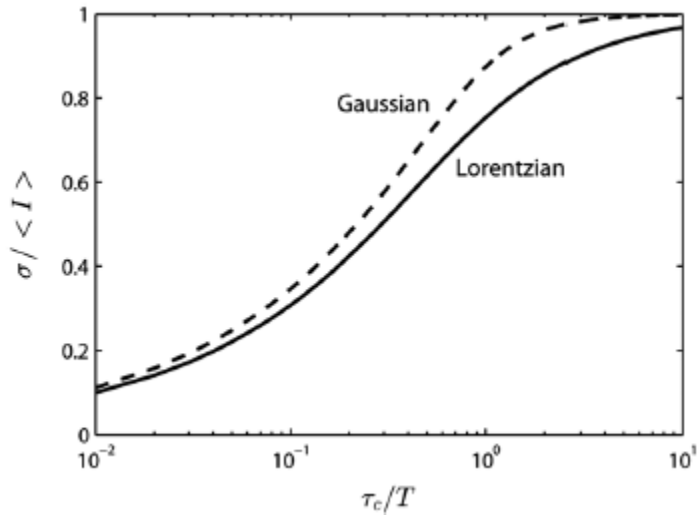
$$K = \frac{\sigma}{\langle I \rangle}$$

Laser Speckle Contrast Imaging / The Biospeckle Laser:  
direct, non-destructive, wide-field image  
-> velocity distribution map



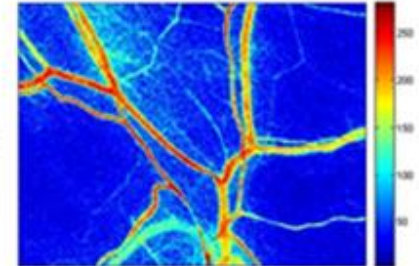
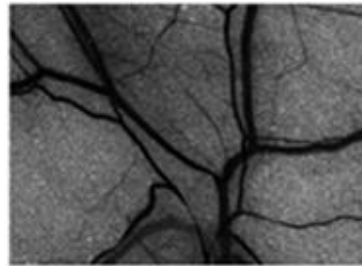
# Measuring motion within the subject

Briers et al, JBO **18** 066018 (2013)



Reflectance Image

Raw Speckle Image



Speckle Contrast Image

Speckle Flow Index Map

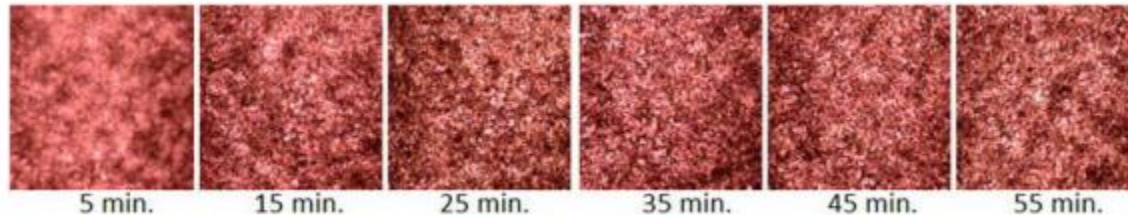
Application areas:

- Cerebral Blood Flow
- Micro-circulation
- Ophthalmology
- Migraine studies

- Dentistry
- Wound and burn assessment
- Monitoring seeds
- Vehicle velocimetry

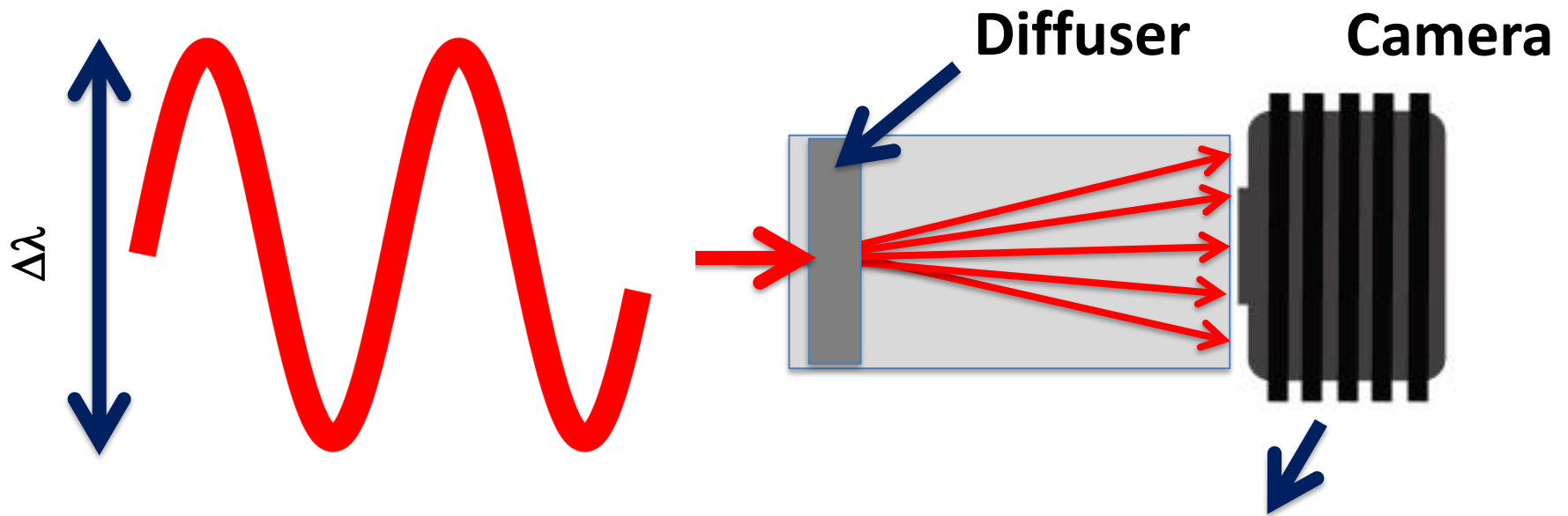
# Measuring motion within the subject

Watching paint dry...



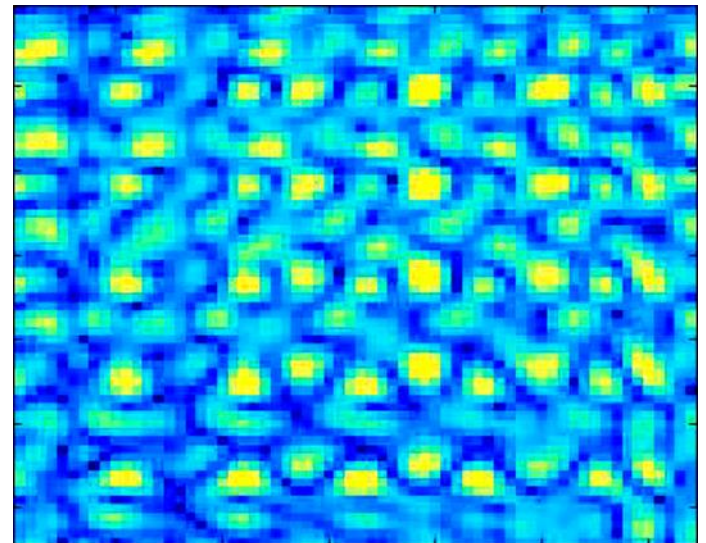


# Concept



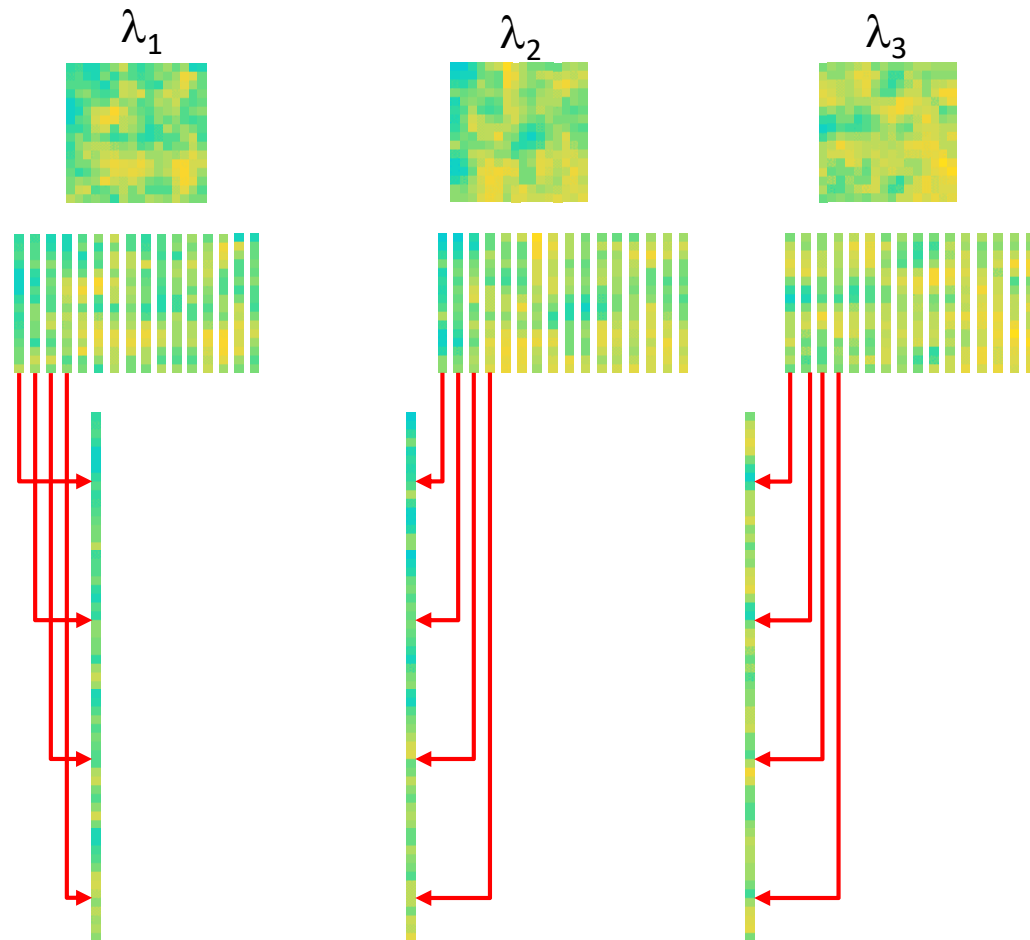
- **Speckle pattern 'randomization'**
  - **Coherence** of field preserved
  - **Interference** marker for wavelength
- **Interference changes with  $\Delta\lambda$**   
yields unique speckle pattern

**But how can we extract  $\lambda$  from the pattern?**



# Wavelength Measurement ABOVE the Correlation Bandwidth

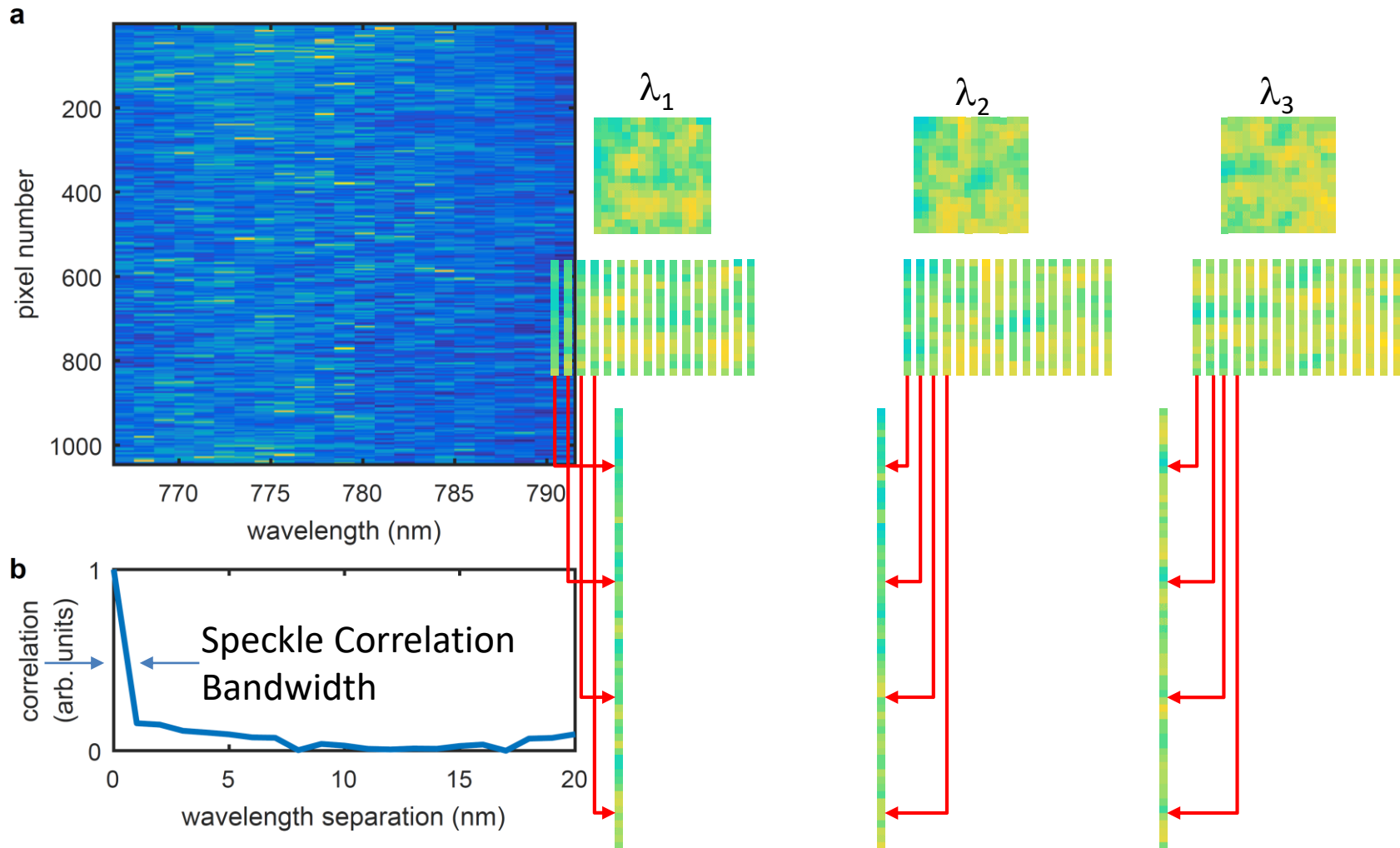
- What is the relationship between the speckle pattern and the wavelength?  
*It depends on the microscopic detail of the scattering medium*
- Take a data-driven approach to the analysis





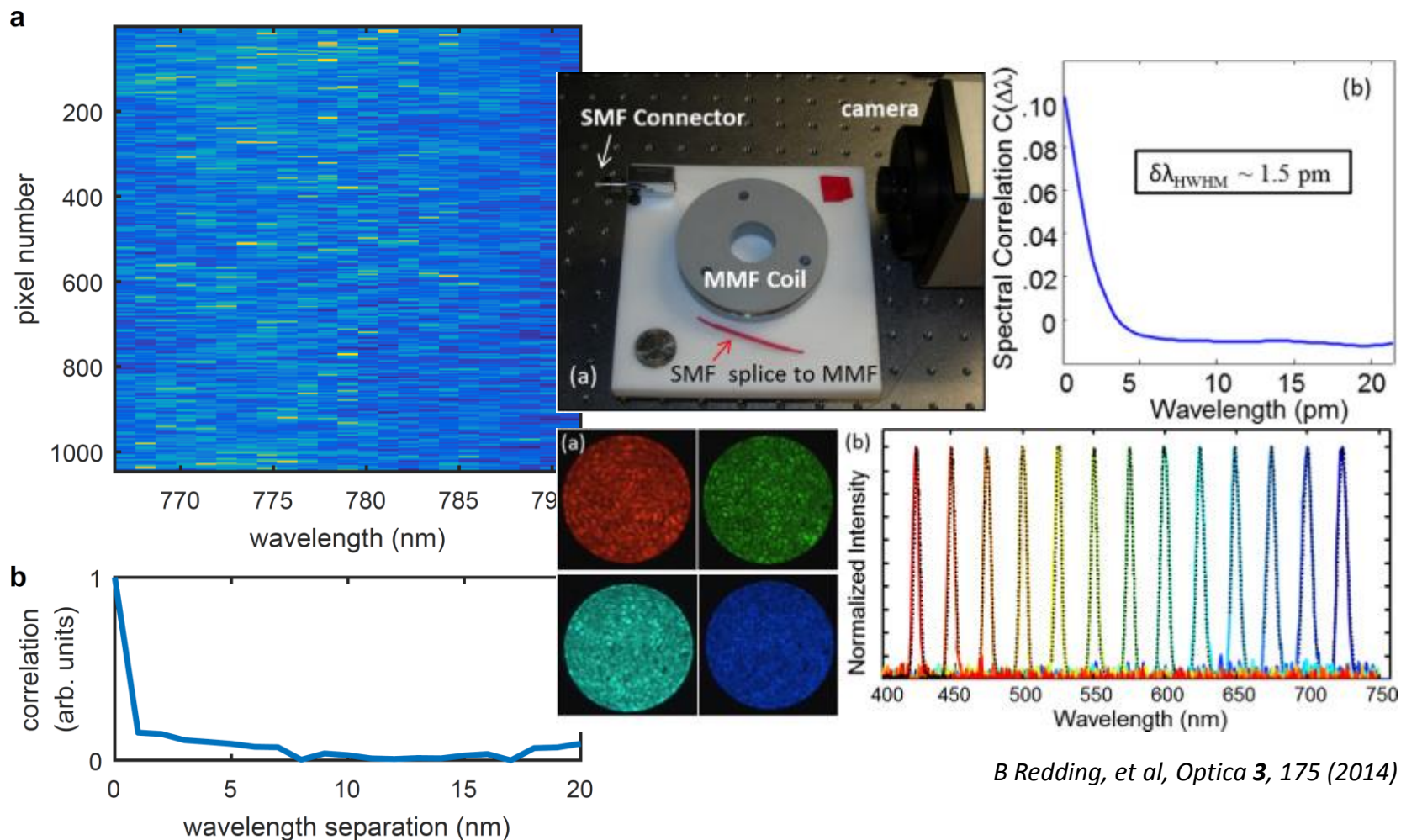
# Wavelength Measurement ABOVE the Correlation Bandwidth

- What is the relationship between the speckle pattern and the wavelength?  
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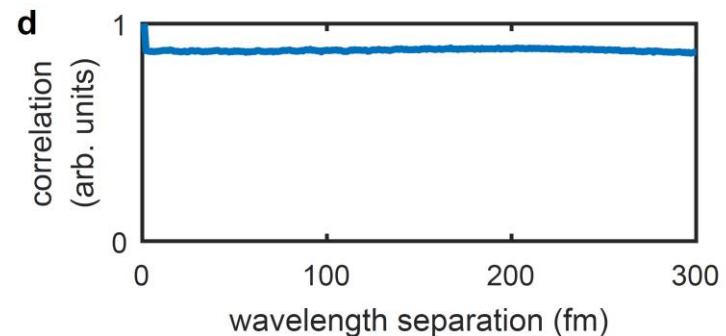
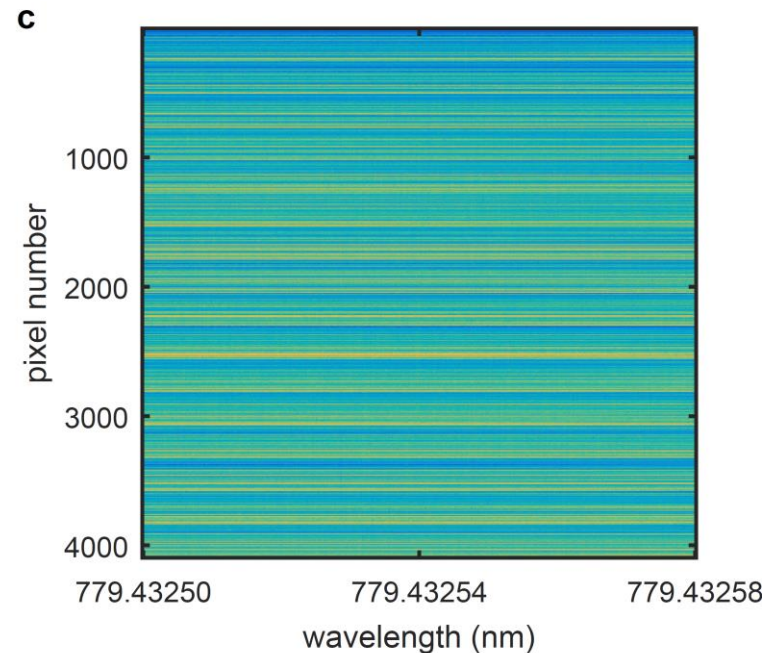
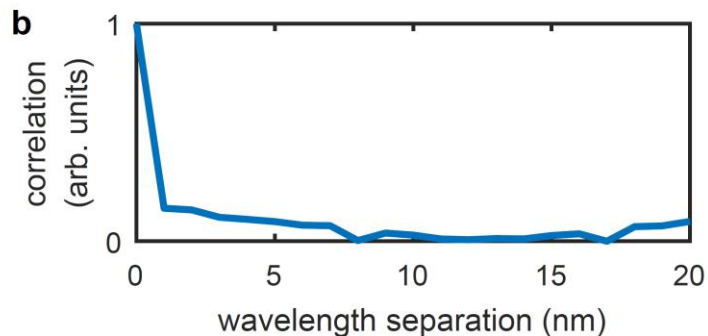
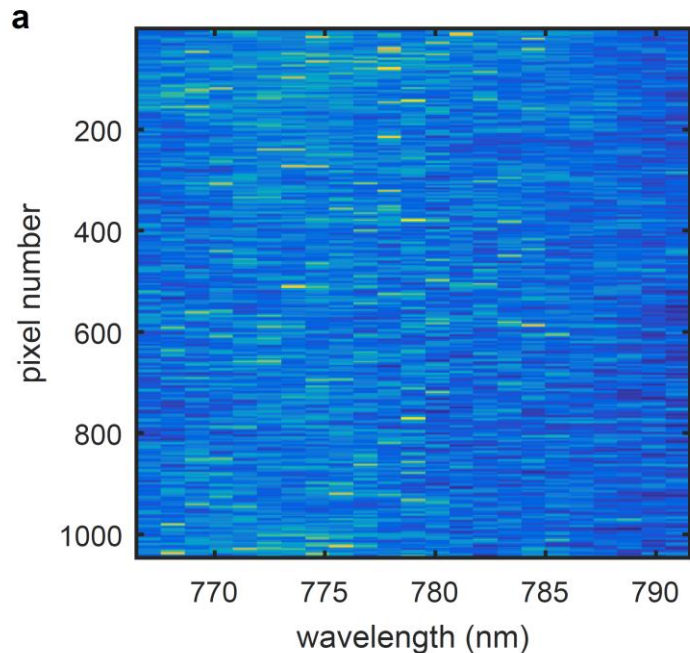
# Wavelength Measurement ABOVE the Correlation Bandwidth

- What is the relationship between the speckle pattern and the wavelength?  
*It depends on the microscopic detail of the scattering medium*
- Take a data-driven approach to the analysis



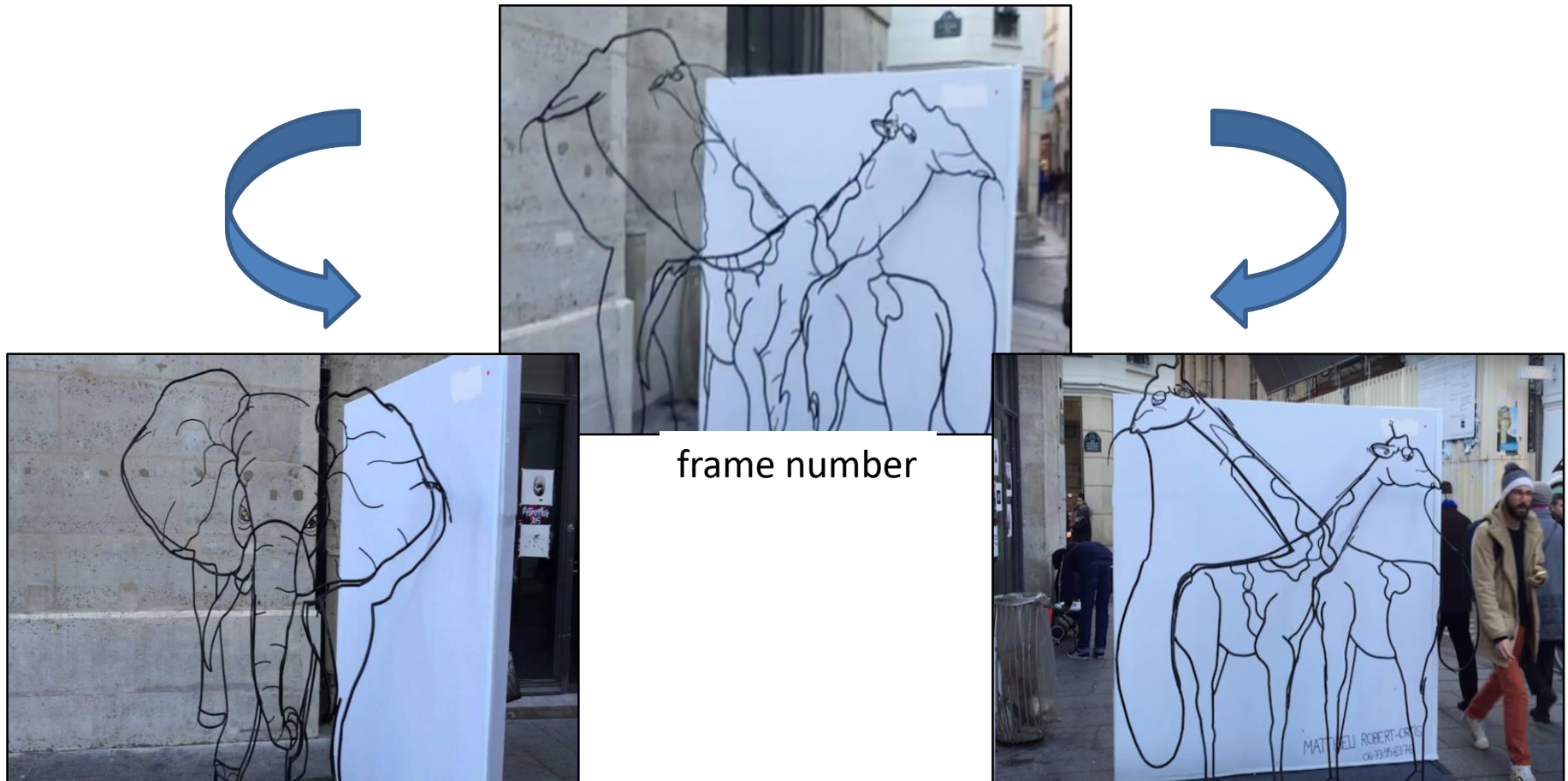
# Limitations of TMM

- What is the relationship between the speckle pattern and the wavelength?  
*It depends on the microscopic detail of the scattering medium*
- Take a data-driven approach to the analysis



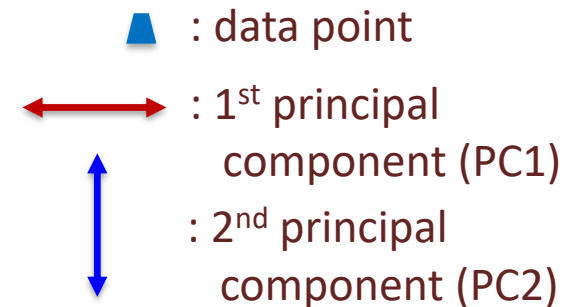
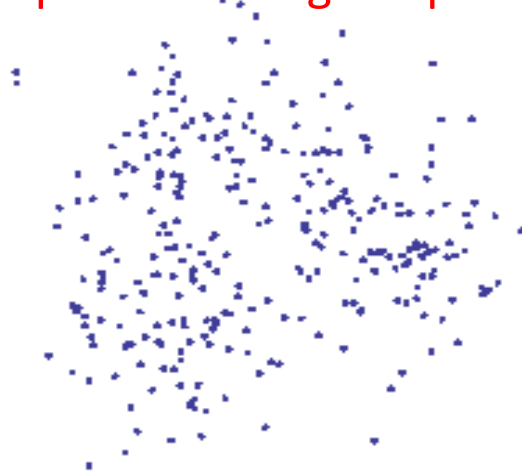
# Can we beat the correlation bandwidth?

- What is the relationship between the speckle pattern and the wavelength?  
*It depends on the microscopic detail of the scattering medium*
- Take a data-driven approach to the analysis  
*Sometimes the answer requires a change of perspective...*



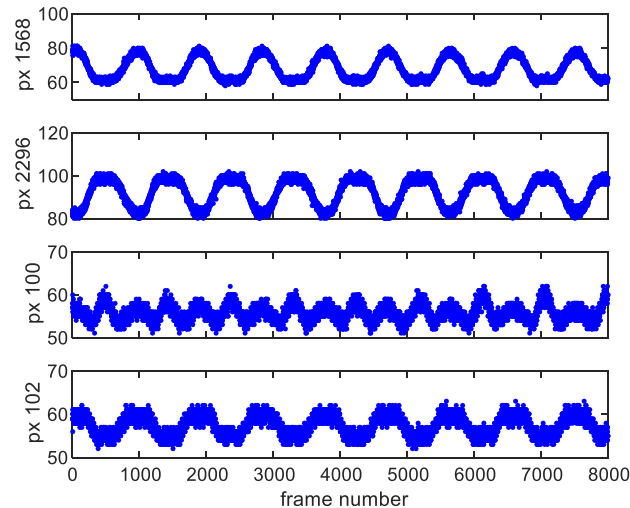
# Principal Component Analysis

- What is the relationship between the speckle pattern and the wavelength?  
*It depends on the microscopic detail of the scattering medium*
- Take a data-driven approach to the analysis  
*Sometimes the answer requires a change of perspective...*



**Principal Component Analysis** is a mathematical process that is defined as a rotation that transforms the data to a new coordinate system, such that the greatest variance by any projection of the data comes to lie on the first coordinate (called 1<sup>st</sup> Principal Component), and so on..

# Principal Component Analysis



- We need to consider the covariance matrix:

$$\sigma_{AB}^2 = \frac{1}{n} AB^T$$

$$\begin{pmatrix} 0.646 & -0.621 & -0.004 & 0.189 \\ -0.621 & 0.630 & 0.010 & -0.189 \\ -0.004 & 0.010 & 0.055 & -0.010 \\ 0.189 & -0.189 & -0.010 & 0.076 \end{pmatrix}$$

- We want to find a new basis, where covariances are zero, and variances are ranked from smallest to largest  
i.e. the eigenbasis!

# Principal Component Analysis

$$\sigma_{AB}^2 = \begin{pmatrix} 0.646 & -0.621 & -0.004 & 0.189 \\ -0.621 & 0.630 & 0.010 & -0.189 \\ -0.004 & 0.010 & 0.055 & -0.010 \\ 0.189 & -0.189 & -0.010 & 0.076 \end{pmatrix}$$



Diagonalize

**Eigenvalues:**

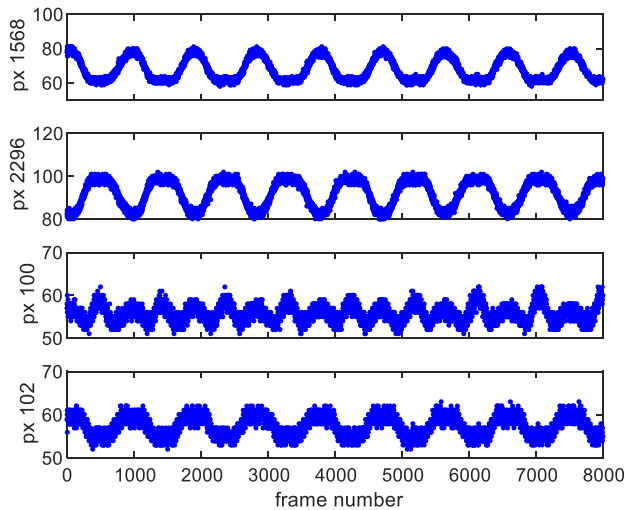
$$\begin{pmatrix} 0.691 & 0 & & \\ 0 & 0.030 & & \\ & & \ddots & \\ & & & 0.009 & 0 \\ & & & 0 & 0.009 \end{pmatrix}$$

**Basis vectors:**

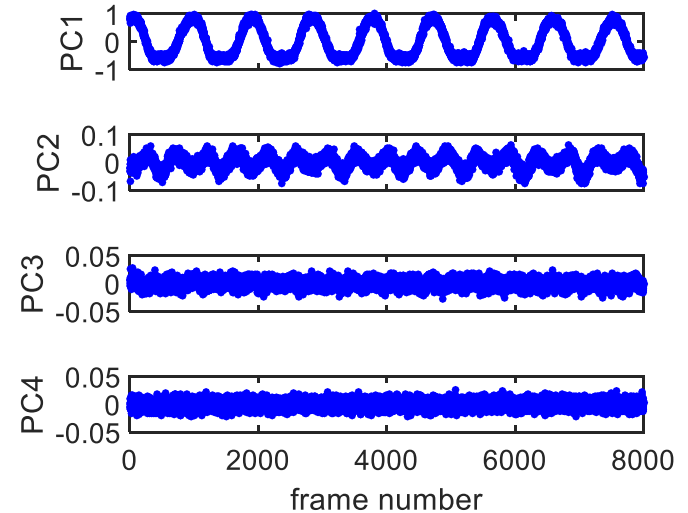
$$\begin{pmatrix} 0.696 \\ -0.687 \\ -0.009 \\ 0.211 \end{pmatrix} \begin{pmatrix} -0.124 \\ -0.053 \\ -0.972 \\ 0.193 \end{pmatrix} \begin{pmatrix} -0.456 \\ -0.209 \\ -0.235 \\ 0.833 \end{pmatrix} \begin{pmatrix} 0.542 \\ 0.694 \\ -0.013 \\ 0.474 \end{pmatrix}$$

Projecting the data onto the new basis gives the **principal components**

# Extracting the Wavelength



Transform



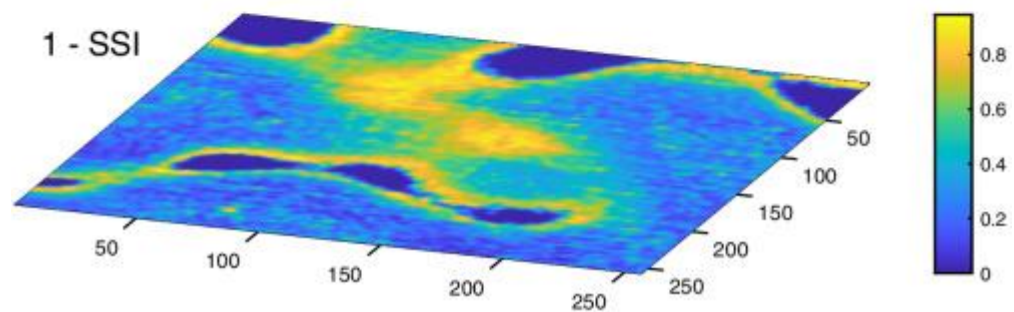
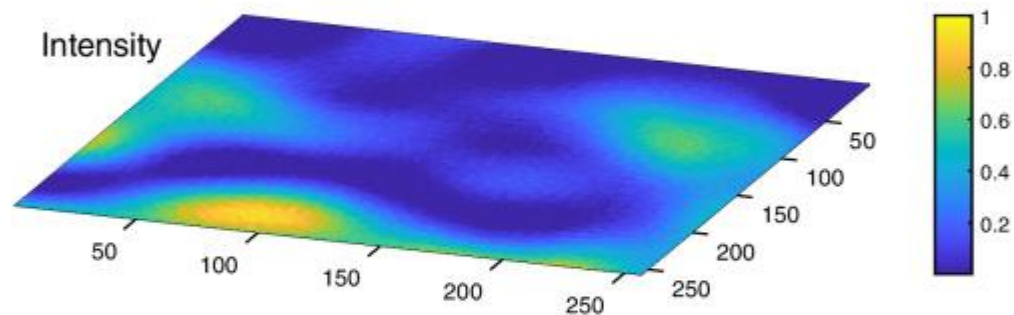
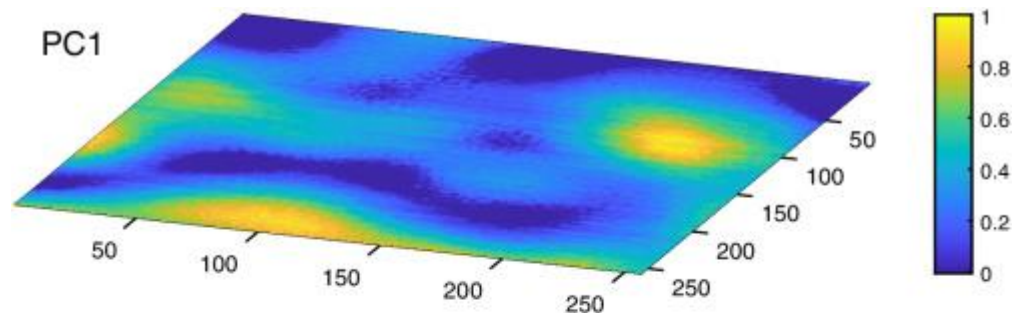
We transform our data to the new basis, comprising its principal components (PCs)

PC1 varies linearly with wavelength  
(Proportionality constant acquired through a calibration measurement)

Importantly, the linear dependence means we can interpolate between calibration points

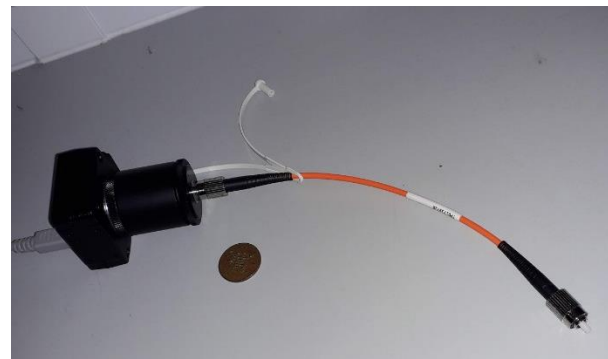
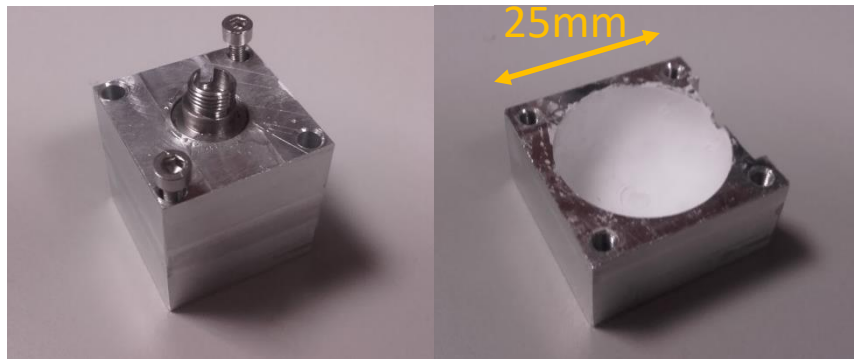
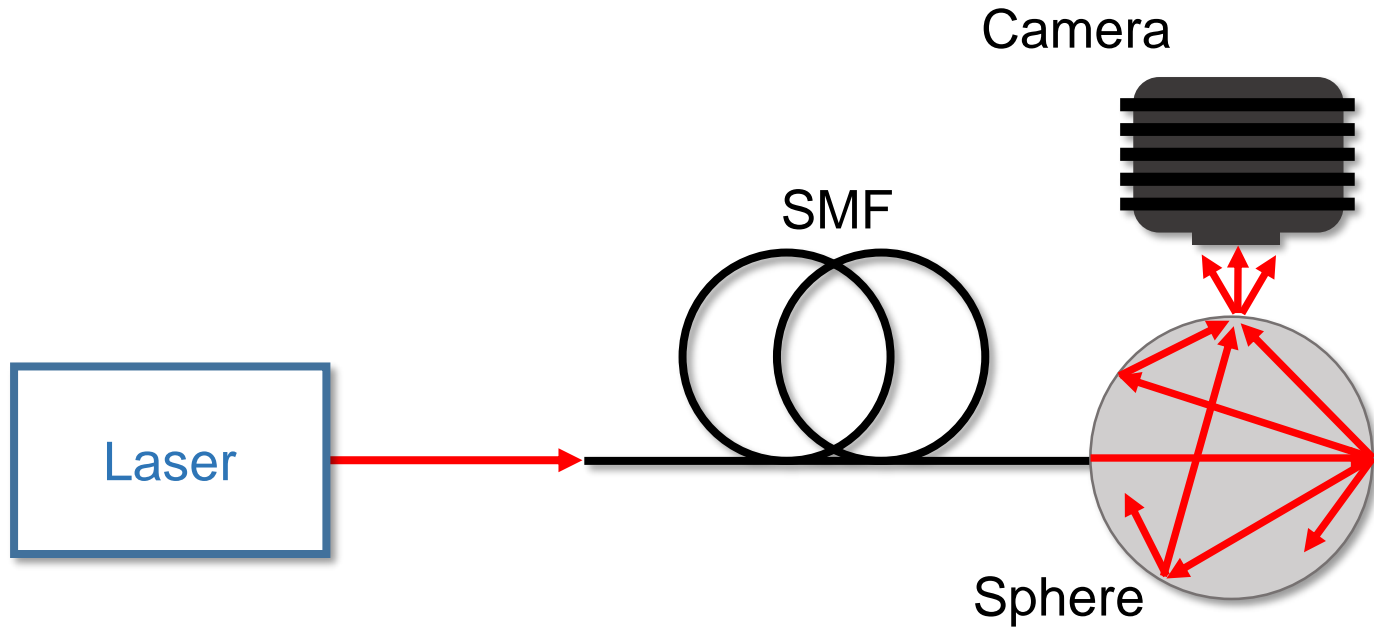


# So what's being measured?



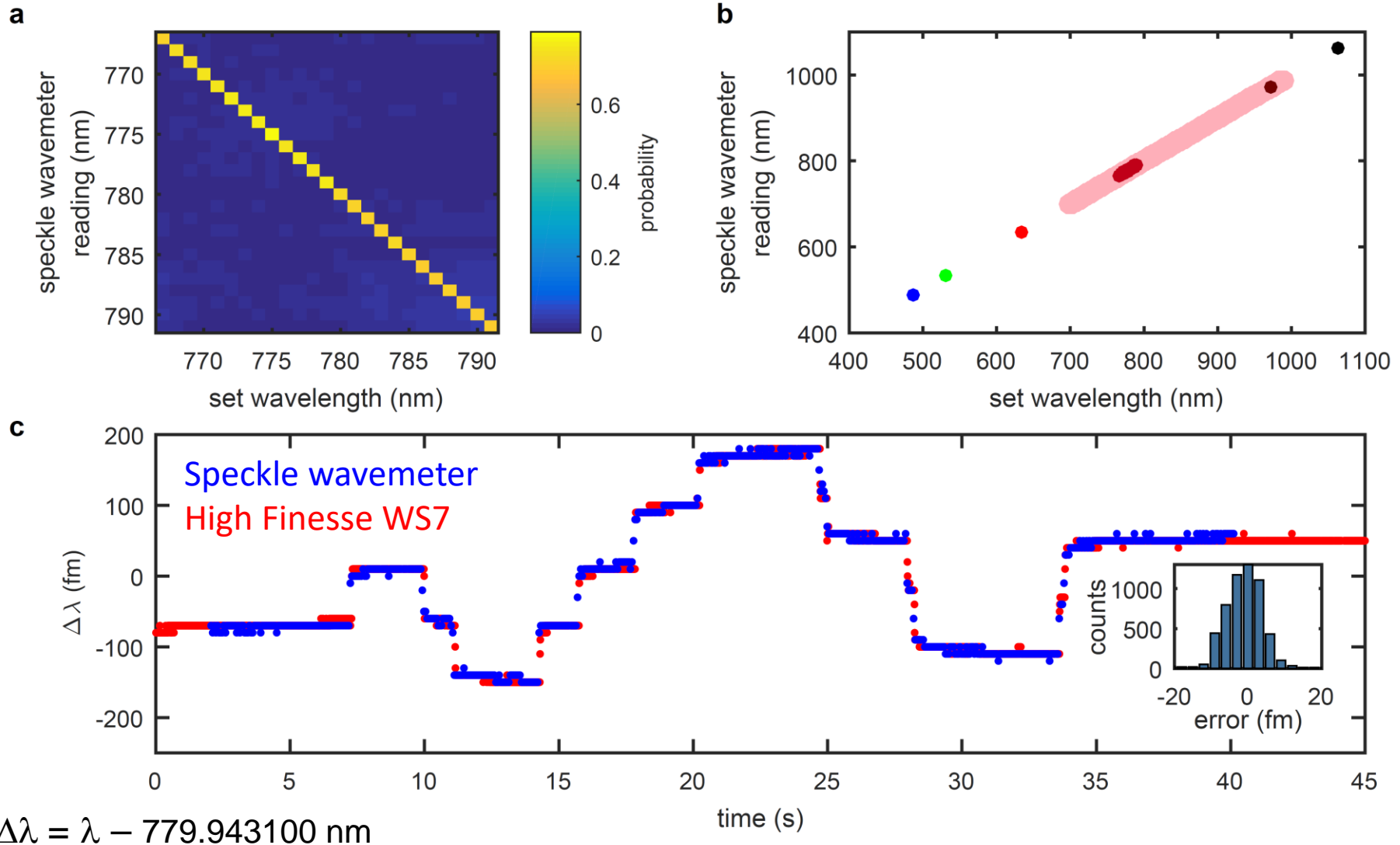
# Speckle Wavemeter

## Setup



# Speckle Wavemeter

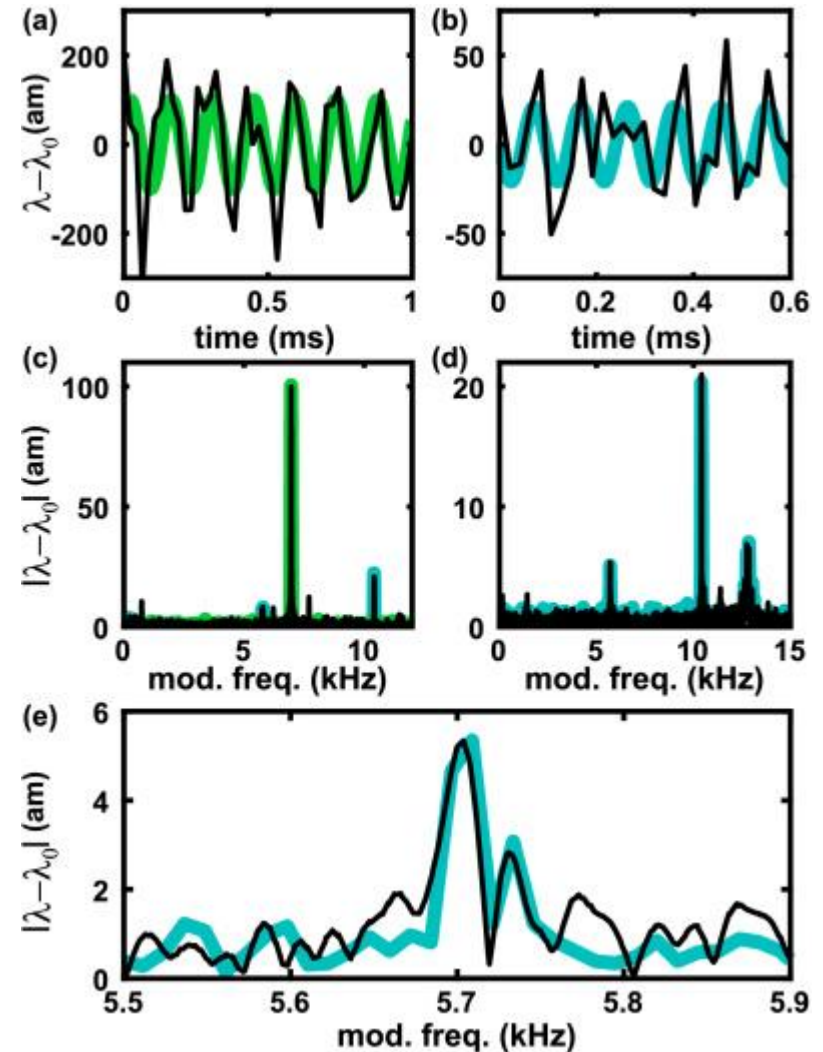
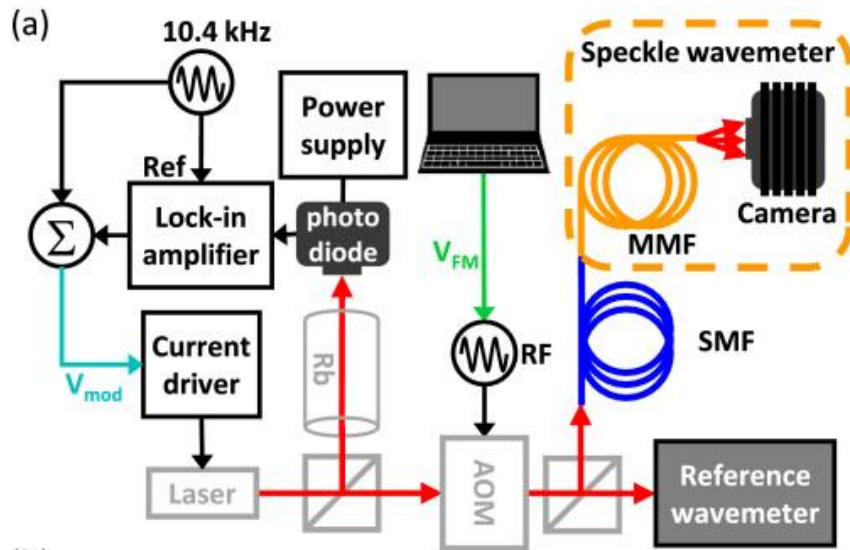
## Broadband performance (using TMM)



# Speckle Wavemeter

## Precision

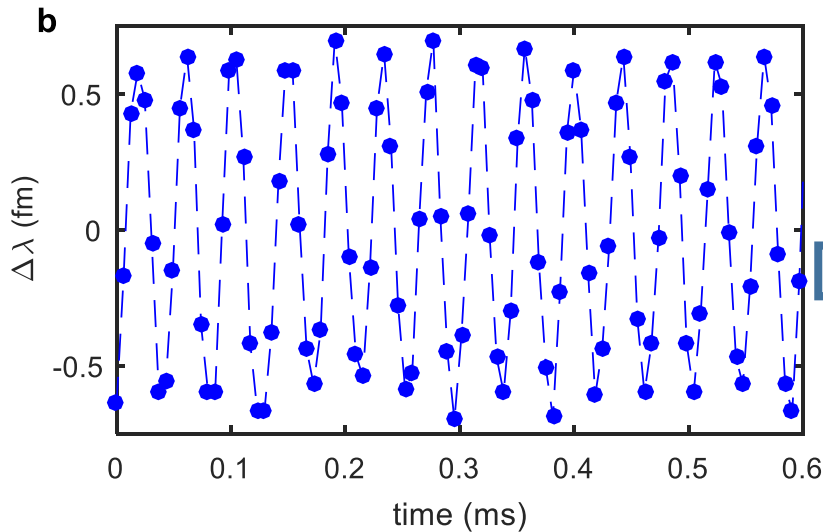
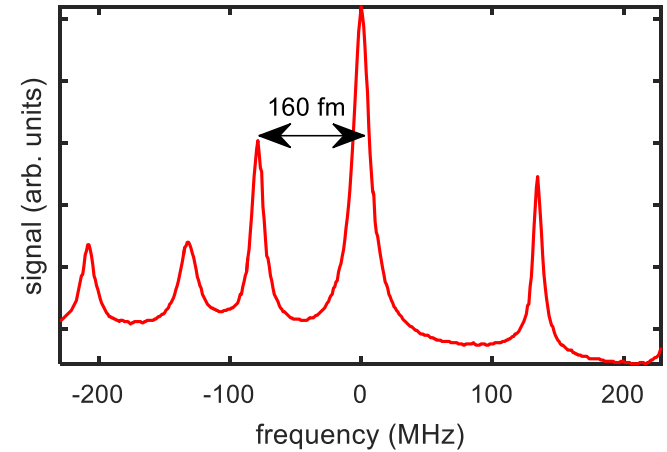
Sinusoidal current modulation of an ECDL



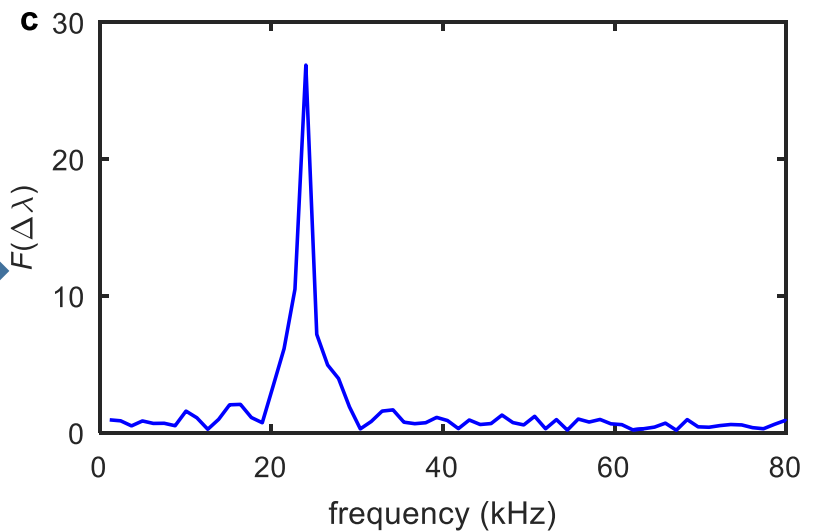
# Integrating Sphere Wavemeter

## Acquisition Rate

Ti:Sa top-of-fringe locked to rubidium spectroscopy with 24 kHz dither to laser current.

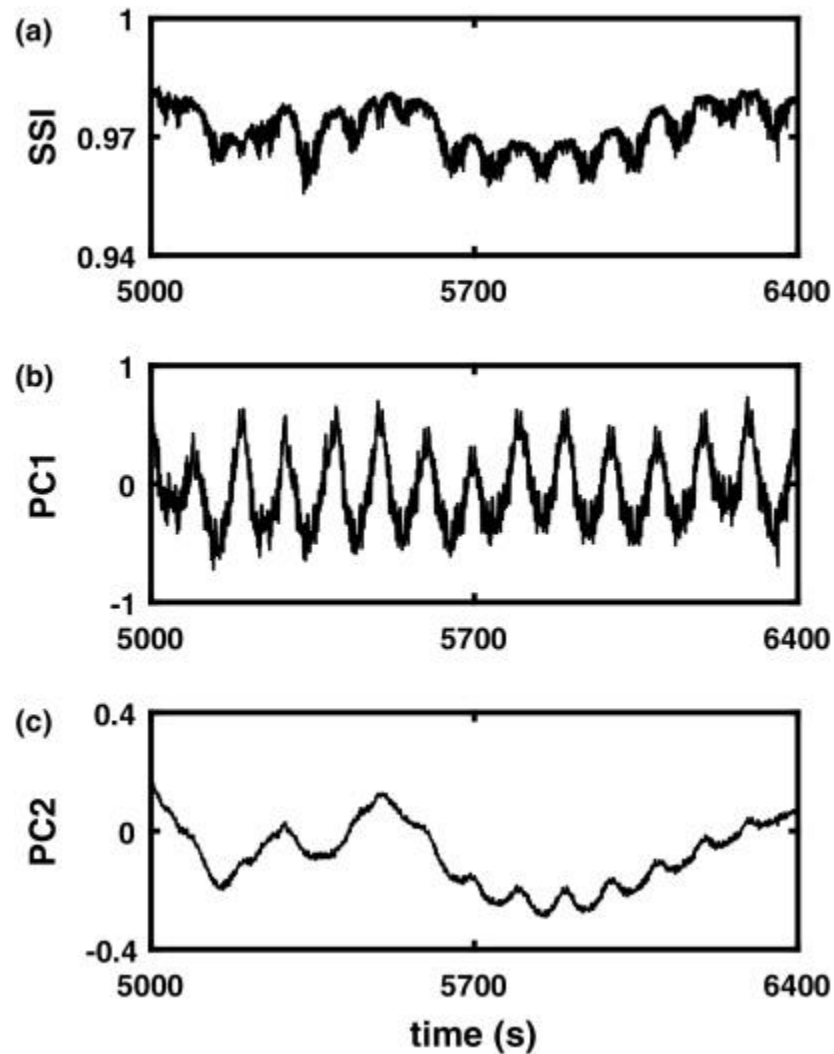


FFT



# Speckle Wavemeter

## Environmental insensitivity



# Integrating Sphere Wavemeter

## Summary

Resolution: 5 am at 780nm

Operating range: vis-nir

(488 - 1064nm demonstrated)

(200 – 2500nm theoretical)

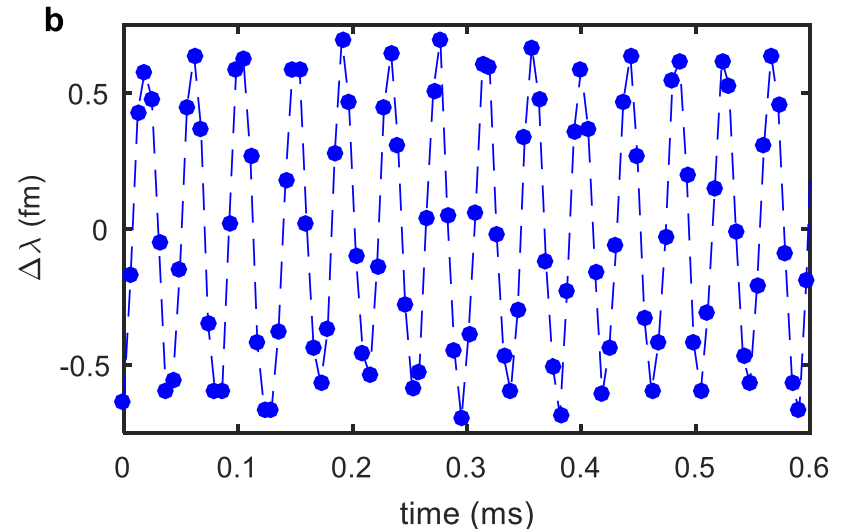
Min input power: < 300  $\mu$ W

Max acquisition rate: > 200 kHz

Calibration stability: 1.5fm/hr

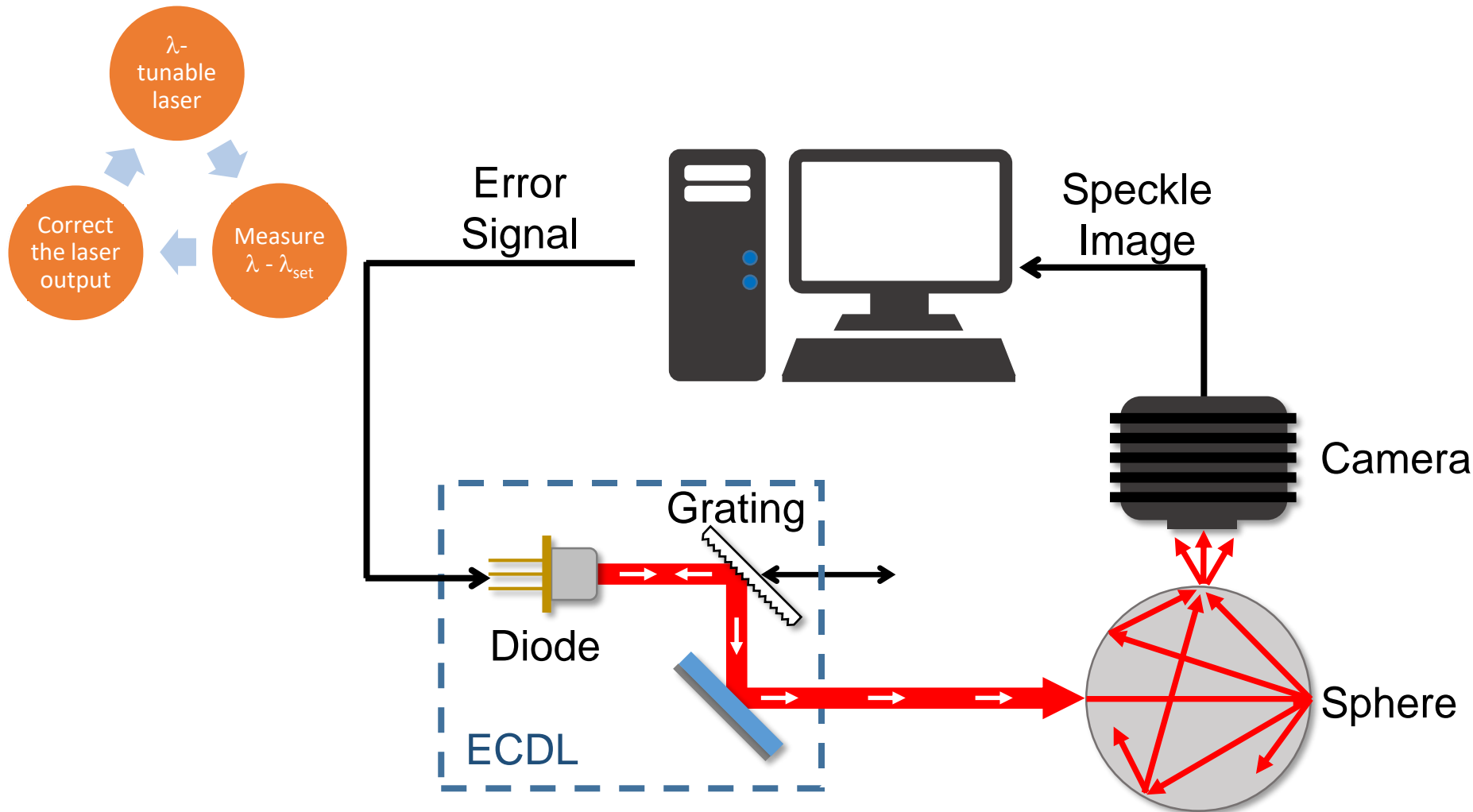
Vibration dependence: not measured

Footprint (typical): 6 x 6 x 13 cm



# Speckle Stabilization

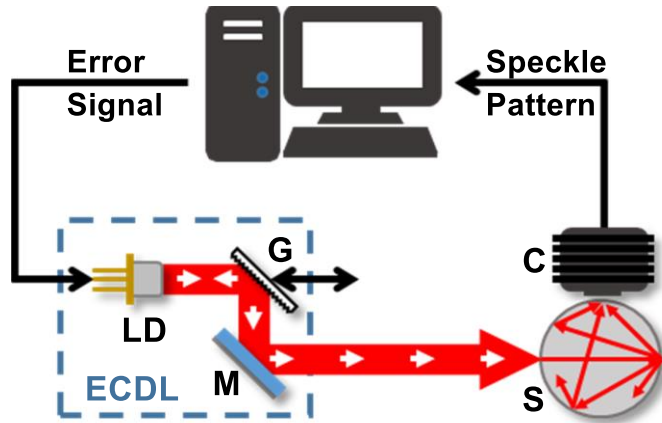
Sometimes, just knowing the wavelength isn't enough, and we really want to control it!



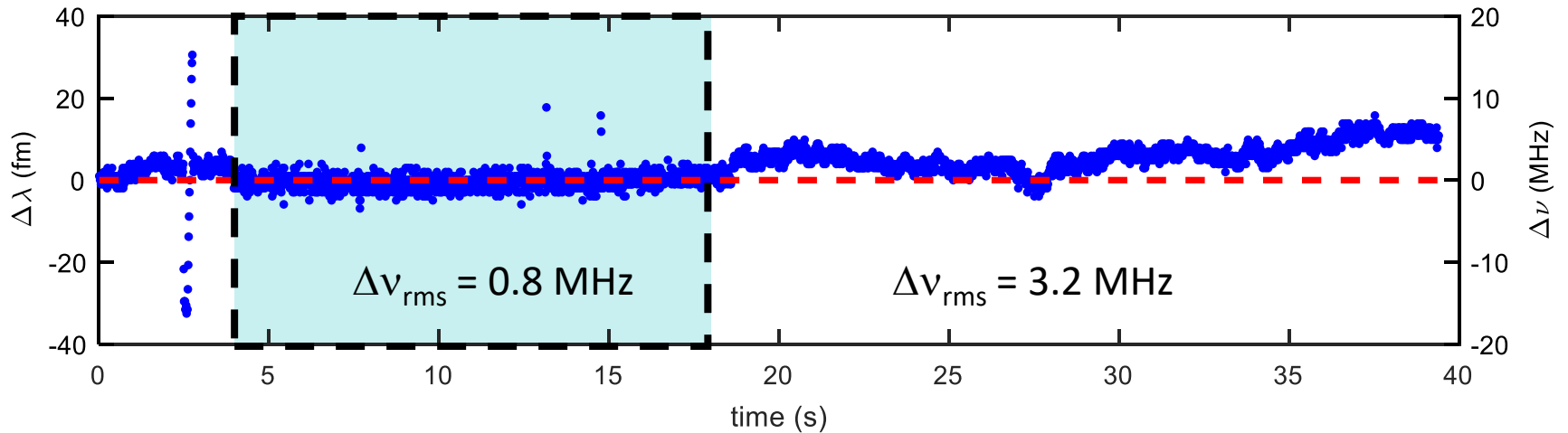


# Speckle Stabilization

a



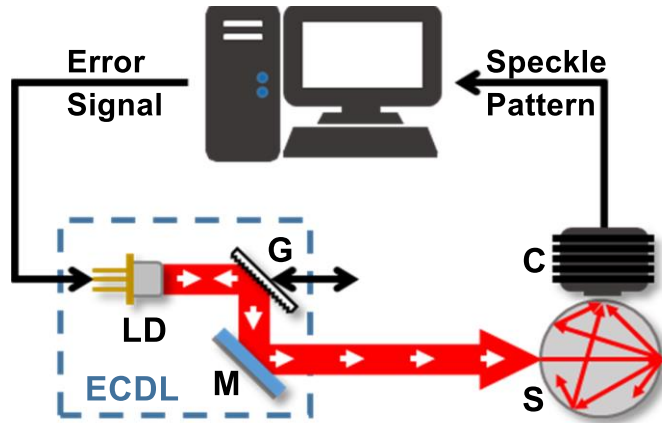
b



$$\Delta\lambda = \lambda - 780.243700 \text{ nm}$$

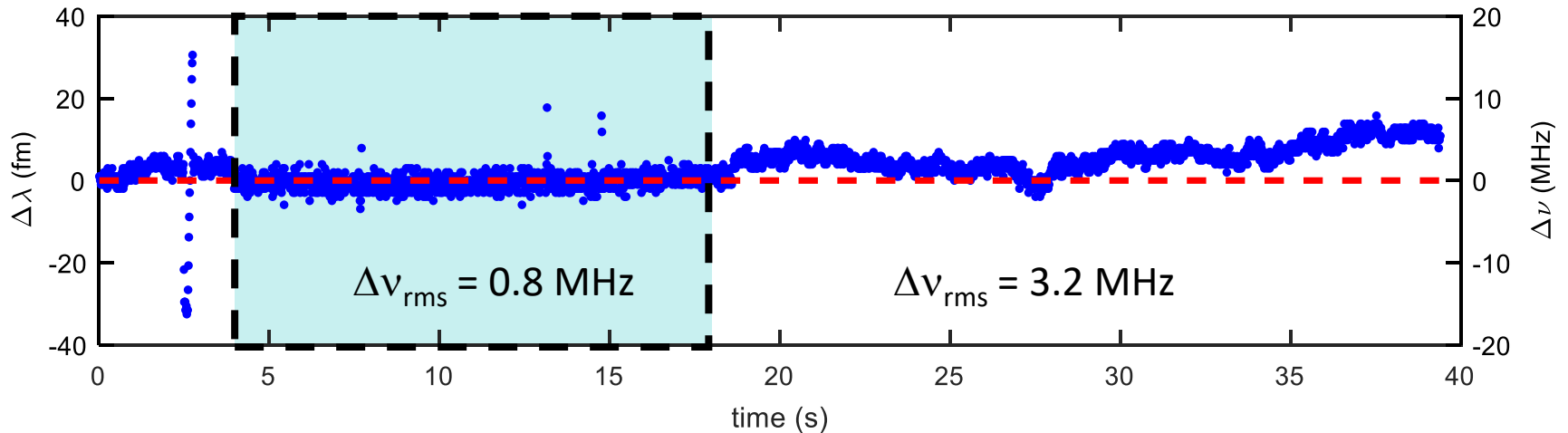
# Speckle Stabilization

a



- Tuning Range: Infinite (arbitrary lock point)
- Capture Range: 30 MHz
- Linewidth: 800 kHz over 10s
- Instability:  $2 \times 10^{-9}$  over 10s, without thermal or vibration management
- Lock Update Rate: 200 Hz

b



$$\Delta \lambda = \lambda - 780.243700 \text{ nm}$$

# Speckle Stabilization:

Riis / Arnold / Griffin group  
Strathclyde

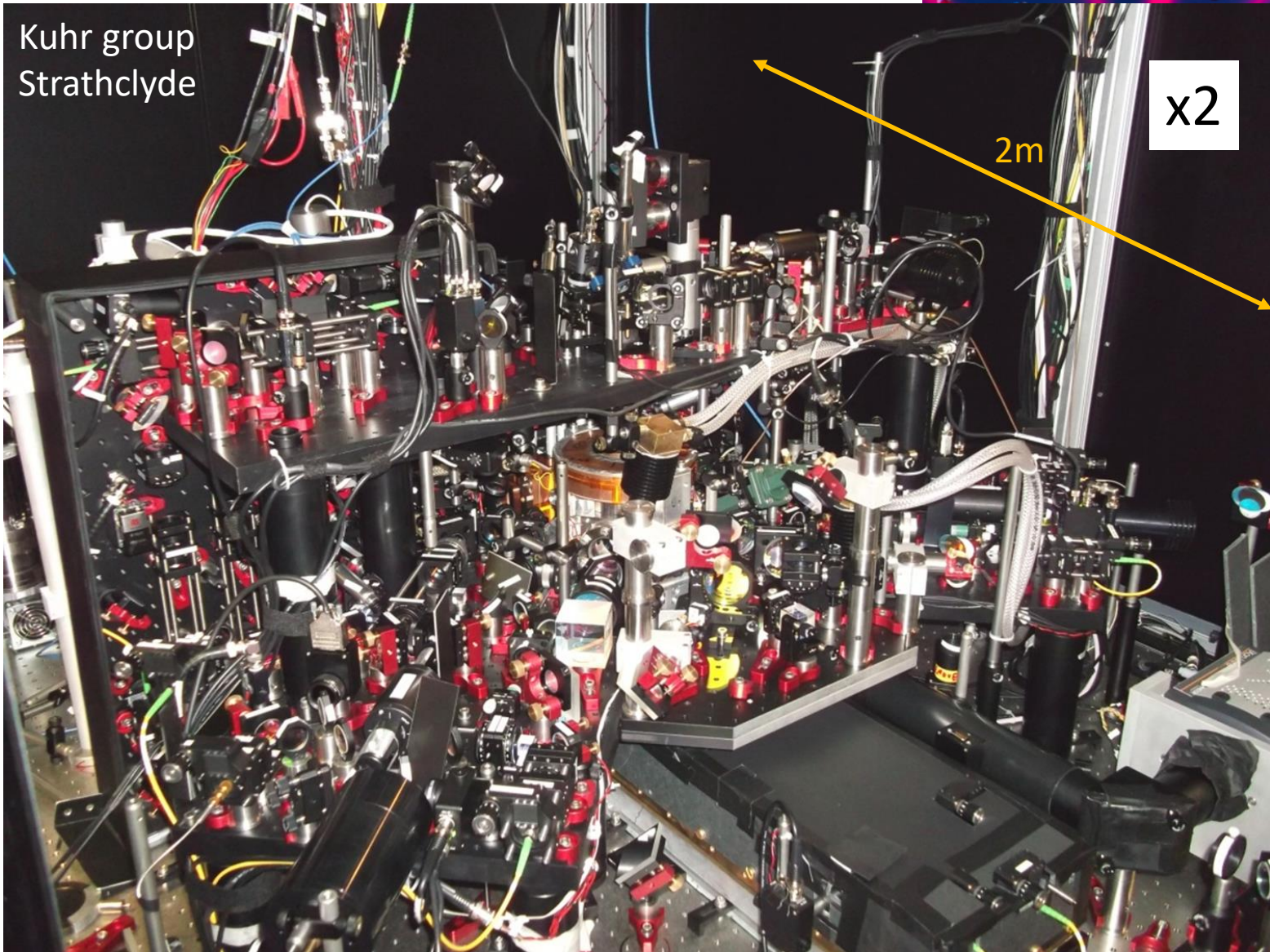
Kuhr group  
Strathclyde

x2

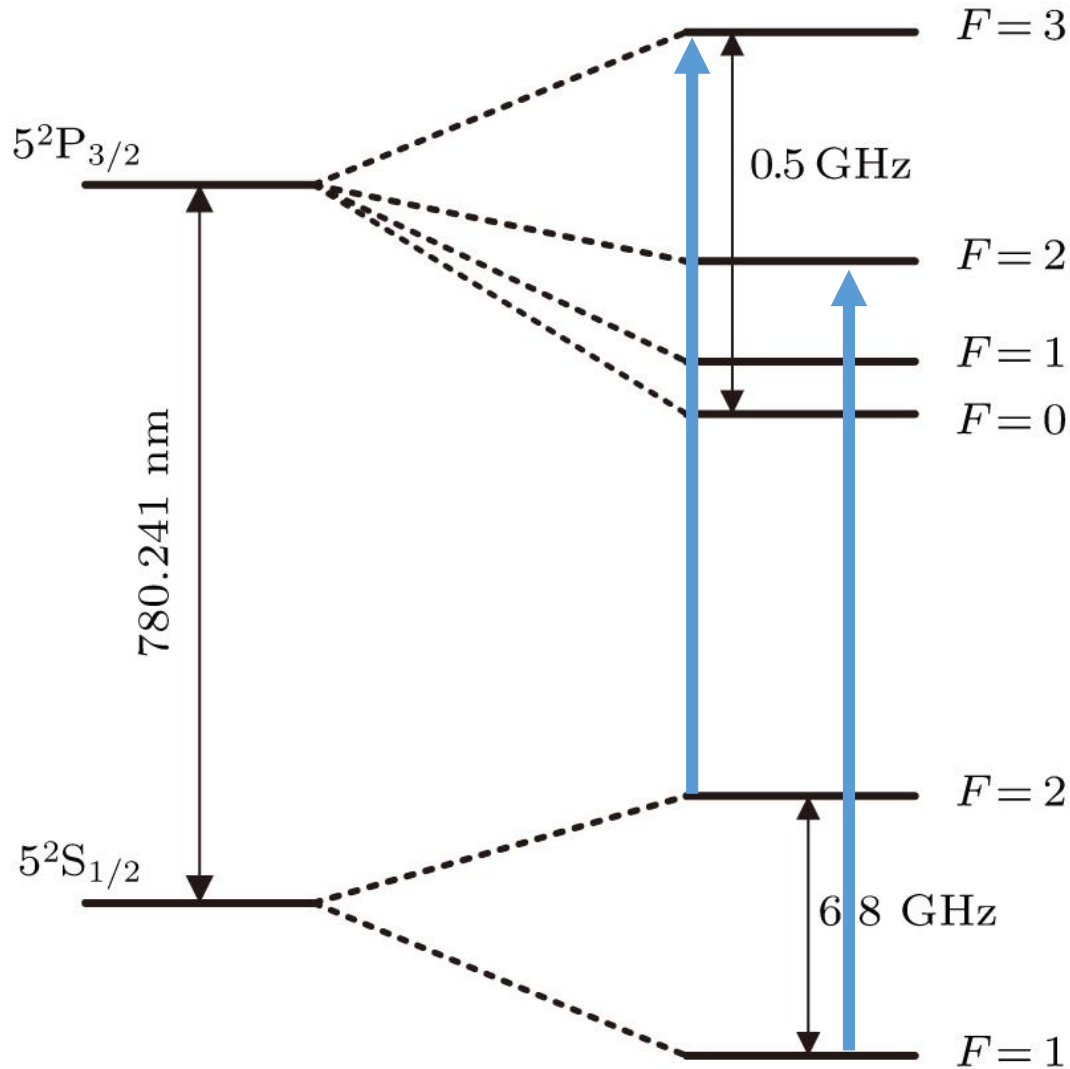
2m

- AS Glass
- Vacuum moat
- NEG coating
- G-MOT reflector
- Si-Glass anodic bond
- Si-Si eutectic bond
- n conductance channel
- getter/LIAD chamber
- chamber

20mm



# Tracking multiple wavelengths



# Find Out More...

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